Analyzing a Simple Prop-Whirl-Flutter Model Using Modern Analysis Tools

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Abstract

The aero-mechanical phenomenon known as prop-whirl flutter has been blamed for a number of aircraft disasters, dating back to the total loss of two Lockheed Electra passenger airliners in 1959-60. Although relatively rare, the phenomenon has been a suspect in smaller plane crashes as recently as 1991 and 2005. In each case, the planes experienced loss of one wing due to fatigue failure of the wing-fuselage interface. Prop-whirl flutter can cause just such a failure due to an undamped coupling of an engine/nacelle/propeller whirl mode with the wing's airfoil flutter mode. Prop-whirl flutter has always been a difficult phenomenon to analyze. It has also been hard for both engineers and engineering students to visualize. Large finite element models have been utilized to represent aircraft wing and nacelle structures, and to analyze for the occurrence of prop-whirl flutter. The introduction of readily available, easy to use, yet highly powerful mathematical analysis tools (such as MATLAB) has changed the analysis process. Relative to the subject at hand, they have made it possible to construct simple matrix equation models representing a wing-nacelle system and to analyze them for potential excitation of the phenomenon.

History of the Prop-Whirl-Flutter Phenomenon

The Lockheed L188 Electra was designed as a commercial transport aircraft with space for up to 100 passengers. As shown in Figure 1, it utilized four wing mounted Allison 501 turbo-prop engines. The first flight for the Electra was in December of 1957. The potential for the aircraft looked good and orders for 144 aircraft had been received by 1959. However, On September 29, 1959 a Braniff Airways Electra disintegrated in-flight near Buffalo, Texas, losing the entire left wing. A nearly identical incident occurred only a few months later, on March 17, 1960. The second crash involved a Northwest Orient Airlines Electra that lost its entire right wing, crashing near Tell City, Indiana. All crew and passengers were lost in both incidents, and the Electra fleet was grounded pending investigations.[1]

The FAA, Lockheed, and Allison Gas Turbine Division of General Motors all participated in the investigation. The official FAA reports now describe the cause of the first accident like this: "Structural failure of the left wing resulting from forces generated by undampened propeller whirl mode."[2] The second crash cause is given as "In-flight separation of the right wing because of flutter induced oscillation of the outboard nacelle."[2] Accident investigators in Texas found evidence that the outboard nacelle on the failed wing had swung as much as 35 degrees out of alignment. Lockheed had designed the nacelle to break away in the event of large unbalances such as a propeller blade loss, and this had not occurred. Rather, the entire wing had separated from the fuselage while the nacelle remained attached to the wing.

Figure 1: The Lockheed L188 Electra Aircraft [1]

Wing overload did not seem a likely cause, because such a structural weakness would have turned up during Lockheed's extensive structural testing, or in the thousands of flight hours that Brannif, Northwest, and Eastern Airlines had seen on the Electras in their fleets. Prior to this time, no one had ever considered that an engine/propeller system whirling mode could couple with a flutter mode of a structural airfoil such as a wing. However, with other explanations seeming even less probable, the investigation turned toward this possibility. Peter Garrison describes the investigation, saying "Lockheed's flutter analysts reprogrammed their computer to include whirl mode, and the mechanism of the accidents began to emerge. By an unlucky coincidence, the whirl mode frequency of the Electra's big four bladed propellers happened to match the flapping frequency of the wing. The propellers, like the child driving a swing ever higher by small movements of her body, had eventually caused the wing to flap so violently that in 30 seconds it broke at the root, without the propeller whirl ever overloading the nacelle structure."[3] This phenomenon became know as prop-whirl-flutter.

It was deduced that damage to one of the engine's four mounts had caused a reduction in the stiffness of the engine/prop system, thus lowering the whirl mode until it was coincident with the wing's flutter mode, precipitating the ever increasing excitation of the wing mode, and ultimately causing the wing failure. Eventually, not only did analysis prove the cause to be prop-whirl-flutter, but full-scale testing of a Lockheed Electra with reduced stiffness engine mounts demonstrated the phenomenon in a wind tunnel. Films taken in 1960 of the Electra wing experiencing prop whirl flutter reside in the archives of the Smithsonian National Air and Space Museum. This video clip can been seen today on the internet at the www.airspacemag.com website. [4] Lockheed eventually redesigned the mount system for the Electra to ensure that reduced stiffness could not cause the whirl mode of the engine/prop to fall into this situation again. However, prop-whirl-flutter remains a concern today. It was implicated in the 1991 crash of a Beechcraft 1900C twin engine turboprop regional airliner.[5] As recently as December, 2005 a Chalk Ocean Airways G-73 Mallard went down near Miami, Florida due to loss of a wing, exhibiting fatigue failure along the wing root. [6] The 58 year old aircraft had been designed in the same era as the Electra.

The aerodynamic effect of propeller yaw angles has been examined as far back as Freeman's report from Langley Field's aeronautical laboratory in 1931.[7] Houbolt and Reed addressed the subject of prop-whirl-flutter directly, in the Journal of Aerospace Sciences, following the Electra crashes, in 1962.[8] Reed updated the study in 1967 [9] while Buschow and Kane [10] and Mayoral [11] have addressed the issue more recently, in 1995-96. But perhaps the most thorough discussion of the prop-whirl-flutter phenomenon was provided by Kunz in 2002. [12] The introduction of readily available, easy to use, yet highly powerful mathematical analysis tools (such as MATLAB) has changed the analysis process. Relative to the subject at hand, they have made it possible to construct simple matrix equation models representing a wing-nacelle system and to analyze them for potential excitation of the phenomenon.

A Simple Model of Prop-Whirl-Flutter

Let us try to construct a simple model of an airplane wing so that we can examine the phenomenon known as prop-whirl-flutter. Assuming the wing to be grounded at the fuselage, the wing can be modeled as three lumped masses and inertias along the wing, with each mass having the coordinates shown in Figure 2. This is an extreme simplification of the system and modern tools like MATLAB are capable of handling much more involved models. However, for purposes of this paper the three mode wing model will be adequate to demonstrate the approach and will lead to matrix equations fully capable of filling the page. It must be recognized that the approach can easily be extended to an increased number of degrees of freedom.

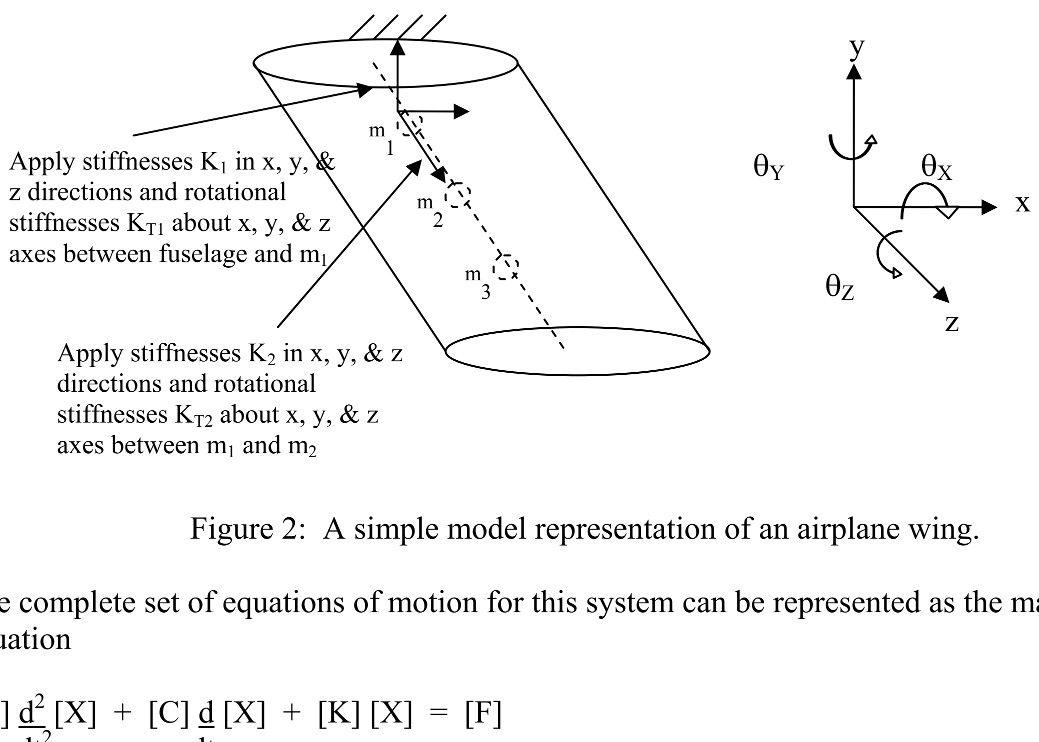

Figure 2: A simple model representation of an airplane wing.

The complete set of equations of motion for this system can be represented as the matrix equation

$$[M] \frac{d^2}{dt^2} [X] + [C] \frac{d}{dt} [X] + [K] [X] = [F]$$

where [X] is an eighteen element column vector containing, from top to bottom, the translational coordinates at element 1 (x_1, y_1, and z_1) then the rotational coordinates of element 1 (θ_{x1}, θ_{y1}, and θ_{z1}) followed by the similar coordinates at element 2 and element 3. The mass and inertia matrix, [M] would be an 18 by 18 diagonal vector where the first six terms, corresponding to the first element would be m_1, m_1, m_1, J_{x1}, J_{y1}, and J_{z1}, where m_1 is the mass of lumped element number 1, and the J terms are the rotational mass inertias about the x, y, and z axes at element 1. Terms for the subsequent elements would be of the same form. The [C] and [K] matrices would be damping and stiffness matrices, respectively. These would be banded diagonal matrices, while [F] would contain the forcing functions and moments. This matrix could easily be enlarged by adding additional elements to the model of Figure 2. However, each new element will drive the size of the matrix up by six terms, such that a four element system would require a 24 by 24 matrix, etc. This rapidly becomes unwieldy and in reality, if we want a "simple" model to work with, we need to reduce the degrees of freedom and the size of the matrices if possible.

Thus, let us return to the system for the model shown in Figure 2, and consider the wing stiffness. In the axial (z) direction, the wing will be very stiff. Also, the stiffness in the fore-and-aft direction (x) will be stiff relative to the stiffness in the vertical direction (y) due to the large area moment of inertia about the wing's Y axis relative to the wing's X axis. Since prop-whirl-flutter is a reduced stiffness phenomenon, the direction of greatest interest is the one where the stiffness is lowest already. So to simplify the problem to a manageable level, let us reduce it to only the y direction translational degrees of freedom. Likewise, twisting stiffness about the x and y axes would be high relative to the twisting stiffness about the z axis. So by the same logic, let us reduce the rotational degrees of freedom to rotation about the z axis, which will correspond to the wing's torsional deflection, or twisting. Now the equations of motion reduce to the following:

$$\begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & J_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & J_3 \end{bmatrix} \begin{bmatrix} y_1'' \\ \theta_1'' \\ y_2'' \\ \theta_2'' \\ y_3'' \\ \theta_3'' \end{bmatrix} + \begin{bmatrix} C_1+C_2 & 0 & -C_2 & 0 & 0 & 0 \\ 0 & C_{T1}+C_{T2} & 0 & -C_{T2} & 0 & 0 \\ -C_2 & 0 & C_2+C_3 & 0 & -C_3 & 0 \\ 0 & -C_{T2} & 0 & C_{T2}+C_{T3} & 0 & -C_{T3} \\ 0 & 0 & -C_3 & 0 & C_3 & 0 \\ 0 & 0 & 0 & -C_{T3} & 0 & C_{T3} \end{bmatrix} \begin{bmatrix} y_1' \\ \theta_1' \\ y_2' \\ \theta_2' \\ y_3' \\ \theta_3' \end{bmatrix}$$

$$+ \begin{bmatrix} K_1+K_2 & 0 & -K_2 & 0 & 0 & 0 \\ 0 & K_{T1}+K_{T2} & 0 & -K_{T2} & 0 & 0 \\ -K_2 & 0 & K_2+K_3 & 0 & -K_3 & 0 \\ 0 & -K_{T2} & 0 & K_{T2}+K_{T3} & 0 & -K_{T3} \\ 0 & 0 & -K_3 & 0 & K_3 & 0 \\ 0 & 0 & 0 & -K_{T3} & 0 & K_{T3} \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \\ y_2 \\ \theta_2 \\ y_3 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \\ F_3 \\ M_3 \end{bmatrix}$$

Where the m terms are the masses at the selected element locations, and the J terms are the rotational inertias about the z axis at each element location. The C and K terms are the damping and stiffness in the y direction at each element and the C_T and K_T terms are the rotational damping and stiffness about the z axis at each element.

Now let us add a new reference frame associated with the engine mass, m_4, which we will locate coincident with m_2. We will also add the propeller, m_5, as shown in Figure 3. We will set up coordinates at masses 4 and 5 which are consistent with the coordinates at masses 1, 2, and 3. For simplicity, let us treat the engine cg (mass m_4) as being coincident with the wing mass m_2. However, we will need to allow flexibility between the engine and the wing to allow us to simulate the change in flexibility due to engine mount effects.

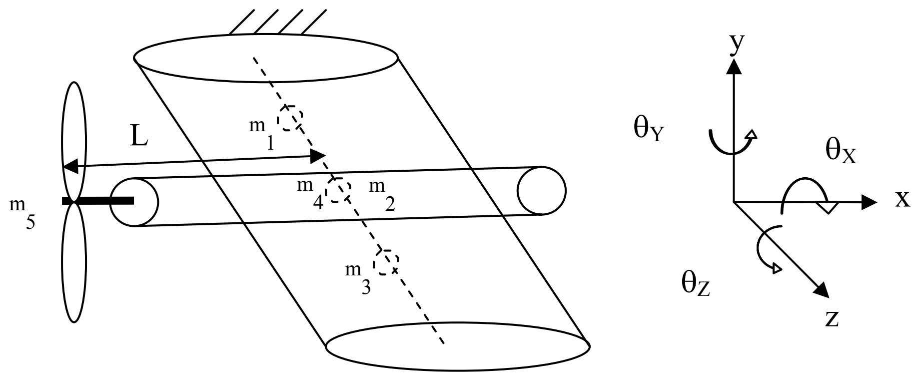

Figure 3: The simplified wing model with engine/nacelle and prop added.

If we let m_4 move rotationally relative to m_2, then it will allow us to simulate the change in flexibility due to engine mount stiffness. Mounts would be stiff in rolling about the x axis, but would permit rotation about the y and z axes, which would represent engine nacelle whirl. Next, let us recognize that the degrees of freedom at the prop (m_5) are dependent variables which can be expressed in terms of the degrees of freedom at m_4. This means that we do not need a separate set of coordinates for the prop. The terms θ_{4y} and θ_{4z} would represent the rotations at node 4 about the y and z axes, respectively.

Donald Kunz of old Dominion University has performed a very thorough development of the whirling of an engine and propeller.[12] Here his propeller equations are converted into our coordinate system.

$$[\, I_{yy} + \tfrac{1}{2}(3-\cos 2\beta) + 4S_b(L-d)\sin\beta + m_5(L-d)^2 \,]\, \theta_y''$$

$$- I_{yz}\, \theta_z'' + (2\Omega\cos^2\beta)\theta_z' + (\omega_y^2 I_{yy})\theta_y = M_{\theta y}$$

and

$$[\, I_{zz} + \tfrac{1}{2}(3-\cos 2\beta) + 4S_b(L-d)\sin\beta + m_5(L-d)^2 \,]\, \theta_z''$$

$$- I_{yz}\, \theta_y'' - (2\Omega\cos^2\beta)\theta_y' + (\omega_z^2 I_{zz})\theta_z = M_{\theta z}$$

where β is the blade pre-cone angle, Ω is the prop rpm, d is the rotor undersling, S_b is the blade inertia, and the ω terms are the natural frequencies of the nacelle. Next, we need to couple these engine/prop equations of motion with the wing equations of motion already developed. Terms θ_y and θ_z from the equations above would use node 4 as their origin and would relate to θ_{4y} and θ_{4z} from the model.

However, these equations are rather complicated, involving blade precone angle, and rotor undersling, to create a truly general expression for the engine and prop description. For our purposes, let us assume that the precone angle and undersling are both zero as they would have been for the Electra. This simplifies these equations to the following:

$$[\, I_{yy} + ½(3\text{-}1) + 4S_b(L)(0) + m_5L^2]\ \theta_y''$$

$$- I_{yz}\ \theta_z'' + 2\Omega\theta_z' + \omega_y^2 I_{yy}\theta_y \;=\; M_{\theta y}$$

and

$$[\, I_{zz} + ½(3\text{-}1) + 4S_bL(0) + m_5L^2]\ \theta_z''$$

$$-I_{yz}\ \theta_y'' - 2\Omega\theta_y' + \omega_z^2 I_{zz}\theta_z \;=\; M_{\theta z}$$

Which reduces to

$$[\, I_{yy} + 1 + m_5L^2]\ \theta_y'' - I_{yz}\ \theta_z'' + 2\Omega\theta_z' + \omega_y^2 I_{yy}\theta_y \;=\; M_{\theta y}$$

and

$$[\, I_{zz} + 1 + m_5L^2]\ \theta_z'' - I_{yz}\ \theta_y'' - 2\Omega\theta_y' + \omega_z^2 I_{zz}\theta_z \;=\; M_{\theta z}$$

Redefining terms, I_{cross} for I_{yz} and I_{prop} for $(I_{yy} +1 +m_5L^2)$ and recognizing that for a symmetric prop I_{yy} and I_{zz} are the same, the equations become

$$I_{PROP}\theta_y'' - I_{CROSS}\theta_z'' + 2\Omega\theta_z' + \omega_y^2 I_{PROP}\theta_y \;=\; M_{\theta y}$$

and

$$-I_{CROSS}\theta_y'' + I_{PROP}\theta_z'' - 2\Omega\theta_y' + \omega_z^2 I_{PROP}\theta_z \;=\; M_{\theta z}$$

Now we will incorporate this simplified relationship into the previously developed equations of motion, and simplify the forcing functions of the system to the unbalance force in the prop, F_u. The moments $M_{\theta y}$ and $M_{\theta z}$ above can be rewritten as a combination of forces about the y and z axes as $F_u d\cos\theta_x$ and $F_u d\sin\theta_x$ where θ_x is a reference angle relative to the x axis. We get the equations of motion given below. At this point we must also add the torsional stiffness term representing the rotational stiffness of the engine about an x axis through the cg (located at node 2). If we assume symmetric stiffnesses, then the K_t at node 4 about the z axis is the same as the K_t at node 4 about the y axis and we can refer to both as simply K_{t4}. This is then applied to the appropriate degrees of freedom to get the following set of equations of motion:

$$\begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & J_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & J_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I_{PROP} & -I_{CROSS} \\ 0 & 0 & 0 & 0 & 0 & 0 & -I_{CROSS} & I_{PROP} \end{bmatrix} \begin{bmatrix} y_1'' \\ \theta_1'' \\ y_2'' \\ \theta_2'' \\ y_3'' \\ \theta_3'' \\ \theta_{4y}'' \\ \theta_{4z}'' \end{bmatrix} +$$

$$\begin{bmatrix} C_1+C_2 & 0 & -C_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & C_{T1}+C_{T2} & 0 & -C_{T2} & 0 & 0 & 0 & 0 \\ -C_2 & 0 & C_2+C_3 & 0 & -C_3 & 0 & 0 & 0 \\ 0 & -C_{T2} & 0 & C_{T2}+C_{T3} & 0 & -C_{T3} & 0 & 0 \\ 0 & 0 & -C_3 & 0 & C_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & -C_{T3} & 0 & C_{T3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\Omega \\ 0 & 0 & 0 & 0 & 0 & 0 & -2\Omega & 0 \end{bmatrix} \begin{bmatrix} y_1' \\ \theta_1' \\ y_2' \\ \theta_2' \\ y_3' \\ \theta_3' \\ \theta_{4y}' \\ \theta_{4z}' \end{bmatrix} +$$

$$\begin{bmatrix} K_1+K_2 & 0 & -K_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_{T1}+K_{T2} & 0 & -K_{T2} & 0 & 0 & 0 & 0 \\ -K_2 & 0 & K_2+K_3 & 0 & -K_3 & 0 & 0 & 0 \\ 0 & -K_{T2} & 0 & K_{T5} & 0 & -K_{T3} & 0 & -K_{T4} \\ 0 & 0 & -K_3 & 0 & K_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & -K_{T3} & 0 & K_{T3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & K_Y & 0 \\ 0 & 0 & 0 & -K_{T4} & 0 & 0 & 0 & K_Z \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \\ y_2 \\ \theta_2 \\ y_3 \\ \theta_3 \\ \theta_{4y} \\ \theta_{4z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ F_u d\cos\theta_x \\ F_u d\sin\theta_x \end{bmatrix}$$

Where to simplify the matrix, we have assigned the following relationships

$$K_{T5} = K_{T2} + K_{T3} + K_{T4}$$

$$K_y = \omega_y^2\, I_{PROP} + K_{T4} \qquad K_z = \omega_z^2\, I_{PROP} + K_{T4}$$

Results

The equations just developed were examined using MATLAB, which is one of several new analysis tools capable of performing such calculations quickly and efficiently. The vertical response of the engine cg can be plotted as a function of time. Figure 4 shows a typical response occurring when the propeller and engine are responding to a forcing function that is not coincident with the nacelle whirl and wing flutter modes. By judicious selection of the system parameters, it is possible to bring the wing and propeller modes and forcing function close enough together that the response begins to grow in an unbounded manner. An example of this is shown in Figure 5. This is the sort of response that would have occurred in the Electra crashes when the engine mount failure caused the two modes to occur in conjunction. One can easily see how the responses could rapidly grow large enough to cause wing failure.

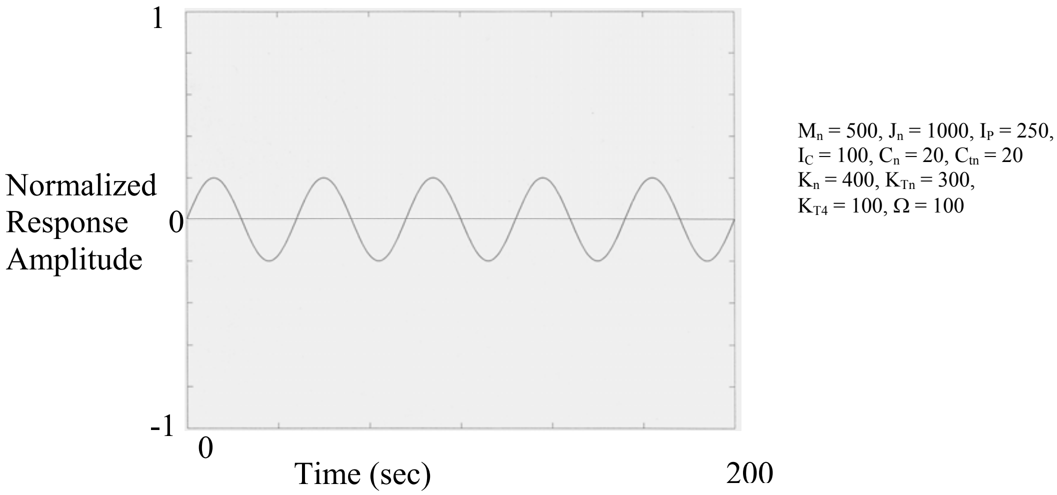

Figure 4: Steady state response of engine under normal conditions

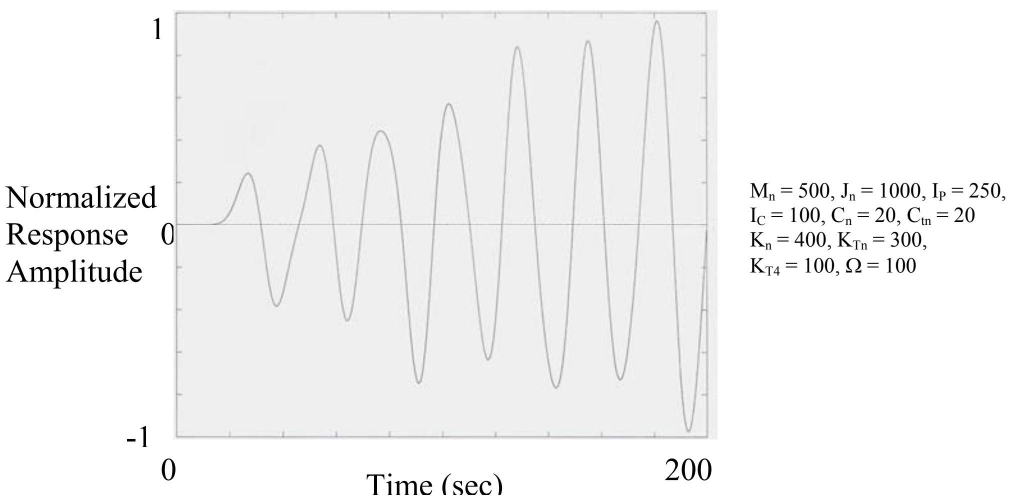

Figure 5: Response growing unbounded as prop-whirl-flutter modes come into conjunction

Conclusions

It is possible to build a simple representation of the equations of motion for a wing/propeller system which can be used to demonstrate the disastrous potential effects of the prop-whirl-flutter phenomenon. Today's new analysis routines make it easy to look at the response of these model equations using various input parameters. Combining these makes it easier than ever before to demonstrate the effects of the prop-whirl-flutter phenomenon.

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