

## Update on the Discrete Element Method in Engineering Education

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### **Abstract**

In this paper, the authors discussed some experiences to introduce a Discrete (or Distinct) Element Method (DEM) to engineering graduate students. The authors of this paper were among the researchers who applied DEM in the asphalt mixture study; therefore a course of the application of DEM was designed and scheduled at Texas A&M University – Kingsville (TAMUK) in summers of 2004 and 2005. In addition, the authors applied the DEM in student research projects in TAMUK and Michigan Technological University. In the class education, the objectives were to teach students numerical analysis methods focusing on discrete element method. In order to introduce the advances of the DEM to the graduate curriculum, some materials such as soil, sand, and asphalt mixture materials were introduced to students to better understand the DEM. Students would also be able to use the image processing software (ImagePro Plus) in the advanced material study. This course also involved the virtual experiments concepts using discrete element simulation. The course was organized to include ten major topics such as image processing, discrete element analysis, and finite element analysis, in class computer applications and discussions, homework, research projects, and presentation.

### **Background**

The authors of this paper applied DEM in research [1-8] in the past and therefore the authors planned this course in graduate student education. The DEM was introduced to students as a numerical analysis method. In the course work, students were taught the application of DEM. The Texas A&M University –Kingsville (TAMUK) offered this course to graduate students in summers of 2004 and 2005. Some contents of this paper have been presented in the 18th Engineering Mechanics Conference of the American Society of Civil Engineers in June 2005 [9]. Later, the application of the DEM as a computer aid in an academic geotechnical program was attempted in the Department of Civil and Environmental Engineering at University of Pittsburgh [10]. Some simple examples such as the simulation of a standard laboratory test, different types of

foundations, and different cavities inside a rock mass were introduced to geotechnical students.

### **Objectives of the Course and Research Studies**

The overall course objectives were to teach students numerical analysis methods-focusing on discrete element method (DEM). The student, upon completion of the course, would be able to solve basic engineering numerical analysis and simulation using discrete element numerical method. In order to introduce the advances of the DEM to the graduate curriculum, some materials such as soil, sand, and asphalt mixture materials were introduced to students to better understand the DEM. Students would also be able to use the image processing software in the advanced material study. This course also involved the virtual experiments concepts using the discrete element simulation. In order to help students learn more effectively, a number of research works have been reviewed including the work by Kolb [11], Magin and Reizes [12], Mosterman et al. [13], Penumadu et al.[14], and many others.

The course was organized to include ten major topics such as image processing, discrete element analysis, and finite element analysis, in class computer applications and discussions, homework, research projects and presentations. The class topics include:

Topic 1: introduction

Topic 2: numerical methods in civil/geotechnical engineering

Topic 3: introduction to discrete element methods

Topic 4: geotechnical engineering image processing technique in civil infrastructure (image processing technique for particulate materials)

Topic 5: discrete element analysis/simulation and application, including general formulation of discrete element method, average stress tensor force displacement law, law of motion, boundary and initial conditions, time step, differential density scaling, damping, etc.[15]

Topic 6: contact constitutive models (a stiffness model, a slip model, and a bonding model; and simple viscoelastic model, simple ductile model, displacement-softening model, user-defined contact constitutive models)

Topic 7: advanced implementation issues: cohesive strength adhesive strength, micro-properties

Topic 8: discrete element analysis: biaxial and Brazilian test as well as other simulations

Topic 9: particulate materials modeling application

Topic 10: asphalt mixture examples

Topic 11: geotechnical engineering application

Topic 12: students' presentation of numerical methods in civil/geotechnical engineering application (research projects) and final exam

## Identification of Concepts in Course Study

### *Numerical Methods*

In this class, the numerical methods such as calculus and fundamentals, solution of nonlinear equations, interpolation and polynomial approximation, curve fitting, numerical differentiation, numerical integration, solution of differential equations, solution of partial differential equations, eigenvalues and eigenvectors, and numerical optimization were not introduced to students since most of graduate students already have the background. In order to provide background information of different numerical methods, finite difference methods (FDM), finite element methods (FEM), boundary element methods (BEM), and DEM were briefly discussed. Then the DEM was studied in the remainder of the semester.

The DEM is a quite different approach compared to other numerical methods. There are a number of applications of particle physics involving large discontinuous deformations of the particulate media. Some of the examples may include, asphalt concrete and Portland cement concrete manufacturing, aggregate producing, grain transportation and many others. **Figure 1** shows the two typical particulate materials in civil engineering – aggregate (manufactured and natural) and asphalt concrete. In the figure, each aggregate is a piece of particulate material and the particulate medium may deform as a solid, flow as a fluid, or behave as individual particles [15]. All of these “phases” may play important roles in the analysis, yet at present there is no model available to account for these different characteristics of the particulate material behavior. In order to describe the particulate mechanics problems, a model which simulates the material as a collection of individual particles that interact only at inter-particle contact points are referred as discrete element method (DEM).



a. Manufactured and natural aggregates prepared in lab



b. Asphalt mixture cores prepared in Michigan Tech



c. Aggregates in an asphalt plant in Michigan



d. Asphalt concrete construction in Michigan

Figure 1. Particulate materials and construction of asphalt mixture

### *Discrete Element Method*

Cundall [16] introduced discrete element methods (DEM) for the analysis of rock-mechanics problems and since then it has been developed in granular material analysis [17-32]. The original application of this method was to investigate the behavior of granular materials, where a specimen containing several hundred particles was tested numerically. Discrete element methods have received considerable attention in the past 20 years. Many others studied granular materials or particulate materials after discrete element methods were introduced [15, 33-42]. The application has been introduced in different fields especially the asphalt mixture field [43-48].

The discrete element algorithm is a numerical technique, which solves engineering problems that are modeled as a system of distinct, interacting, and general-shaped (deformable or rigid) bodies or particles, subject to motion and deformation. The calculation cycle in PFC 2D is a time-stepping algorithm that requires the repeated application of the law of motion to each particle, a force-displacement law to each contact, and a constant updating of wall locations. Contacts, which may exist between two balls (i.e., discrete elements in 2D) or between a ball and a wall, are formed and broken automatically during the simulation. The calculation cycle is illustrated in [Figure 2](#).

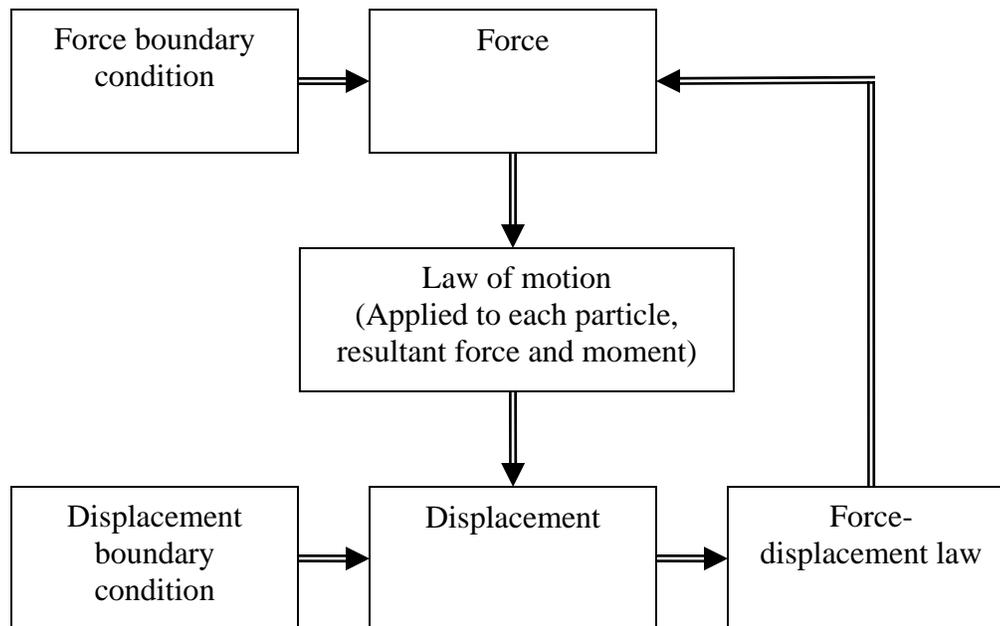


Figure 2: Discrete Element Calculation Cycle

### Application of DEM in Course Work

Class demonstrations included different packing methods for granular materials and discrete element coding as well as the simulations. Students liked to see different packing techniques by programming. The Brazilian test, similar as an indirect tensile test (IDT) in hot mix asphalt was introduced to students to evaluate the compressive and tensile stresses in a specimen as well as the tensile strength of the specimen. Some other discrete element simulations such as tip-loaded cantilever beam, collisions with a particle assembly, biaxial test, core flow versus mass flow hopper, dynamics of a beam-column structure, granular flow from a hopper, mine block-caving process, and rockfall [15] were introduced to students using the advanced multi-media facility in the classroom. Students were able to conduct the same simulation in class. Therefore, they were able to repeat the modeling and simulation procedures. Students were also asked to do similar simulations by changing some boundary conditions or other parameters in the codes.

### Students Homework and Research Projects

Students were asked to learn by practice. The assignments included different basic modeling concepts. For example, in the discrete element modeling study, students were asked to conduct a very simple simulation of a two-particle system model (i.e., two discrete element in series), in which they used a stiffness model, a slip model, and a bonding model, respectively. Simple verification problems were assigned to students to learn the programming technique including particle and geometry generation, contact

laws, and displacement and contact force monitoring. Discrete element modeling may have different shapes of elements such as circular, disk, oval, spherical, or even irregular elements. In this class, only circular elements were used.

Figure 3 shows a compaction simulation, which was one of the simple assignments. Students needed to generate the boundary geometries (walls) and specific gradation of aggregate particles (a typical coarse aggregate gradation of a type of asphalt mixture). Then a compaction procedure will be applied. Students monitored the contact force generated in the particle-particle contacts. The dark lines indicate the compressive contact force, where the thickness of the line indicates the magnitude of the contact force.

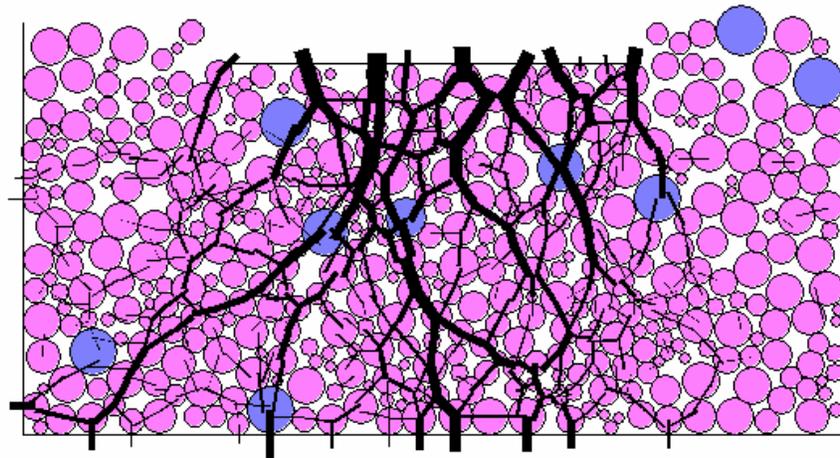


Figure 3: A compaction of granular material with a gradation of grains (the dark lines indicate the compressive contact force magnitude and direction)

Mini-research projects were assigned to students. The assignments required students use discrete element method to conduct the following tasks: compaction simulation of sand particles, coarse aggregate sieving simulation, a mix of different soil compaction and some other complicated engineering problems.

### **Advanced Topics**

Advanced topics introduced to students included the most recent modeling technique using discrete element models [5, 15, 49, 50]. For example, cohesive strength and adhesive strength in cohesive material such as asphalt mixture were explained in detail so that students had a better chance to understand the micro-properties of the material. The micro-properties and macro-properties of the material were illustrated to students by using the example from recent research work [49]. A clustered DEM approach, or so-called microfabric discrete element modeling (MDEM), was introduced to students to analyze asphalt mixture microstructure. Figure 4 shows the discrete elements of a piece of asphalt mixture, where various material phases (e.g., aggregates, mastic) are modeled with bonded clusters of discrete elements. Particularly, discrete element modeling was first applied in the asphalt mixture microstructure using a number of discrete elements to

represent the aggregate and mastic, where the mastic was assumed to be a combination of asphalt and aggregate finer than 2.36 mm. Figure 4 was produced by simulating the complex aggregate shapes with modeling inclusions, such as aggregates, with a “mesh” of small discrete elements.

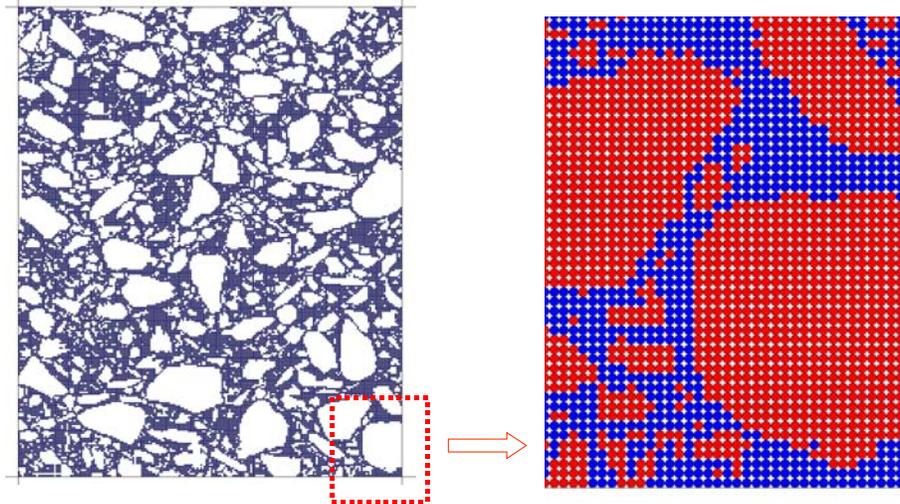


Figure 4: Micromechanical model of asphalt mixture using discrete element method

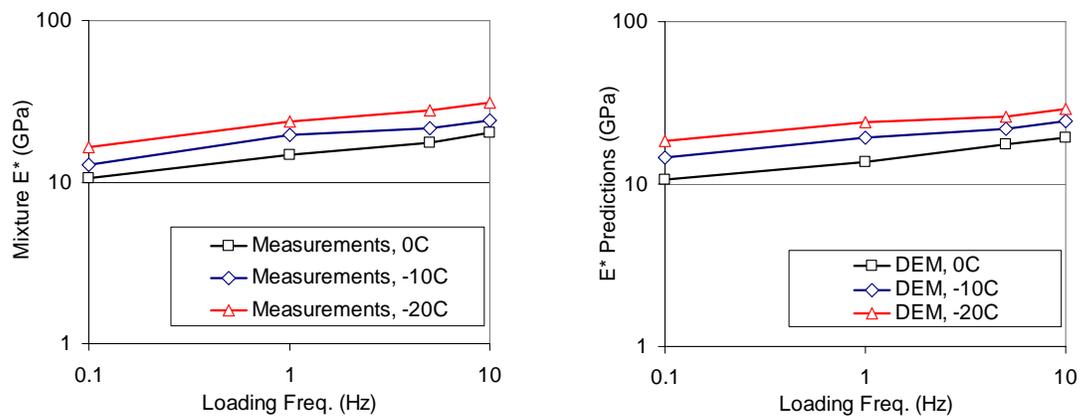
### **Application in Graduate Student Projects**

The authors have introduced the application of the DEM in graduate student projects at Texas A&M University –Kingsville and Michigan Technological University [51]. In Texas, five graduate students completed their master degree research projects using the discrete and finite element modeling simulation and online tools:

- Microstructure modeling of asphalt mixture using finite elements
- Asphalt mastic modulus prediction using discrete and finite element modeling technique
- A simple asphalt pavement overlay model using discrete and finite element modeling technique
- Identification of particle location for a discrete element model (EE department).

In addition, several student papers have been prepared for publication. A Ph.D. student and two postdoctoral researchers at Michigan Tech is currently conducting further research to develop the DEM in asphalt mixtures so that a friendly user interface and functional predictive tool can be possibly available.

As part of the learning outcome, some students were able to modify the models to conduct an asphalt mixture beam loading study by trimming the specimen to a beam shape and an asphalt pavement permanent deformation study by applying a viscoelastic contact model. Some students were able to use the discrete element modeling technique to conduct water contaminant study and sediment study at the TAMUK campus. Using the models in Figure 5, students were able to utilize the virtual laboratory simulation -the discrete element simulation, by preparing the input parameters measured from the laboratory [5, 7, 49]. Figure 5 shows a typical composition between the lab measurements and the discrete element prediction of the mixture complex modulus across a range of test temperatures and loading frequencies. It is found that the students trained in this class were able to understand the basic engineering experiments.



a. Measurements of the mixture complex modulus

b. Discrete element prediction of the mixture complex modulus

Figure 5. The measurements and the discrete element prediction of the mixture complex modulus across a range of test temperatures and loading frequencies

## Summary

In this paper, the authors presented a Distinct or Discrete Element Method (DEM) course offered at Texas A&M University –Kingsville (TAMUK). In the course work, students were taught the theory and application of DEM. The course served a variety of functions in the curriculum. First, students learned an advanced numerical analysis technique in this course, which covered basic knowledge in discrete element methods for engineering purpose. The second function was to introduce the students to a wide range of issues common to all disciplines of engineering, including basic engineering problem solving methods, computer operations, computer programming, presentation, and writing reports. The third function of this course was to introduce research concepts such as the micromechanical modeling approach in discrete element methods to students at a high academic level.

The course provided students' working knowledge and practical skills in discrete element analysis theory and application, with a specific focus on aggregate-asphalt mixture simulation and analysis. Students learned to use discrete element (and finite element) software in engineering analysis and simulation. This course offered students a very solid background in basic concepts, engineering problem solving methods, and interesting research topics.

### **Future Work**

In the future, the authors plan to arrange this course in fall or spring semesters so that students have more time to be familiar to computer programming practice using the discrete element method. In addition, it is always a good idea to introduce this course to all the engineering Ph.D. students so that they can apply the knowledge in their research topics. The authors are planning to integrate DEM study in Ph.D. student research projects at Michigan Tech. Currently, a Ph.D. student and several researchers are conducting research with this method in the pavement material areas.

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