Dynamic Simulator of Transmission Gear to Damage Gearing Analysis

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Abstract

The dynamic torsional model presented in this paper is driven to the implementation of simulator software of vibration response of gears pairs. The equivalent rigidity of the set is obtained from the dimensional numbers of the gears and is defined during the turn describing the variation during the contact of each pair of teeth, tooth to tooth of the pinion. It makes possible to get a response involving defects in the tooth (crack and shape) and on the assembly (eccentricity and misalignment). The vibration signal is obtained through the time integration of the initial value represented by the model equation of movement, considering the excitations are stationary.

Introduction

One of the emphases of the current research in the field of monitoring gears using vibration analysis is the development of specific techniques for the signal analysis, aiming at the correct diagnosis of the defect or even an evaluation of the quality of the manufacturing and assembly of the gear pair. It is in this context that the present work of simulator development that generates vibratory signals equivalent to the ones measured on the bearings of the shafts of the gear pairs is inserted, allowing to the test of these techniques and its improvement before the proceeding of the experimental testing, which are usually expensive and long.

In the present study, the gear pair is modeled as a system of mass-spring-damping, where the tooth deformation and the twist of the system shaft-gear are considered as elements of storage of potential energy, thus allowing not to consider the flexibility of the bearings, independent of the fact that these are sufficiently rigid structures when compared to the shafts. This choice is particularly possible, because in engaged pairs there is a well definite geometric relation that allows the transformation of small radial or tangential displacements into small equivalent angular variations. Moreover, the highest exciting external force is the torcional moment due to power transmitted by the pair, what reinforces the choice of a torsional model to describe the set.

The model of one degree of freedom thus developed is improved considering that there is a fluctuation in the magnitude of the equivalent rigidity during the gearing process, due dimensional variations provoked by the deformation of teeth in contact, what causes temporary modifications of the involving profile and the consequent oscillation in the radial position of the action line of the engaged pair, which is the point of tangency of the theoretic primitive circles of the gears. To consider this effect, the value of the equivalent rigidity of the system must be calculated step by step throughout a turn of the gear.

Monitoring of speed reducers: advantages and problems

Among all types of machines that have the capacity to transform some type of energy into mechanical energy, some are in a highlighted position, such as electric motors, combustion engines and steam or gas turbines. These engines are usually designed for a nominal velocity operation that varies from a thousand to several thousand rotations per minute. The main reasons for them to operate in high rotation bands are that the high speed produces a notable rate of weight/power and an excellent relation power/ initial cost. Although in many cases, the machines that will be connected to these engines require low speed rotations and high torcional moment. To become compatible to the operational characteristics between the machines and your drive motors, referring to the regimen of rotation and power requirements, it becomes necessary to use some kind of reduction system.

This need provides the wide usage of the gearboxes for running machines in the production line of several industrial segments. Industries of sugar and alcohol, paper and food are very common examples in which the gearboxes have a vital importance in the production.

The maintenance in this type of equipment is considered to be critical because a problem in the transmission of the movement causes an irreversible damage in the production. On the other hand, guided by the necessity of minimizing costs and obtaining the ISO 9000 certification for exportation, these companies have abandoned it's systematic preventive maintenance procedures, where it is executed the disassembly, inspection and repair in an annual basis of all the important equipments, in order to incorporate predictive maintenance techniques, such as techniques based on vibration analysis.

The basic idea of machine monitoring through the analysis of vibrations is that the structure of the machines, when excited by dynamic efforts from its functioning, respond with vibration signals, whose frequency is identical to that one of the provoking efforts and that the vibration signal taken in any point will be the sum of the vibratory answers of the structure in different frequencies of these excitations.

When the machine is in good conditions, these exciting efforts come from the regular movement of its several components. When the equipment gets some kind of wear, it's expected that the levels of all signals tend to increase. When a damages shows up, new components are added to these signals. So, the monitoring procedure initiates with a detailed analysis of the equipment, trying to identify each exciting effort (also called noise source), as well as its importance. Next, the equipment is periodically monitored throughout the application of some technique that allows the identification and the diagnosis of the most important defects.

The classical form of analysis, applicable to the majority of the rotating machines, is done through the spectral analysis, in which it is possible to identify the main sources of noises, as well as, to observe the presence or the appearing of defects, and also the diagnosis of the gravity level of the defect.

Once the presence of the frequencies in the vibration spectrum is identified the diagnosis of defects is done based in the presence of harmonics of this frequency, lateral bands around them, sprouting of other different frequencies and appearing of frequencies that suggest resonance. The diagnosis of the defect level is done based on the level of amplitude of the fundamental noise sources and relative distribution among its amplitudes and its harmonic or side bands.

Specifically in the case of gearboxes, unfortunately, the classical form of analysis based on the spectrum of the vibration signal demands a lot of work and experience to be carried through, especially in the identification of the sprouting of new components and in the detection of amplitude variations. Each one of the mechanical elements of a gearbox such as, gears and bearings, are sources of exciting efforts, that when multiplied by the number of shafts of a gearbox, at the minimum two, produces diverse spectral components. Moreover, once the gearbox is used together with other machines and in environments where there are several external sources of noise there is an increase of the components observed in the spectrum. Another additional factor is the presence of random noise, on high considerably levels, what makes it difficult to identify the components of interest, as well as the perception of the sprouting of new components and variations in the amplitude.

For this reason, to detect and the follow up of the evolution of fails in gear systems, most of the times, it's required the application of more dedicated techniques of analysis. Among them, it is widely used the time synchronous average**,** demodulation**,** statistic analysis, techniques based on the distribution time / frequency and time / scale, where the quality of the gearing is analyzed, step by step, throughout a turn of the gear, based on some indicator derived from the statistic treatment of the signal. All of these techniques are specific, and focus towards a certain defect or type of defect and mainly demand a specific procedure for the acquisition of the vibration signal and its pre processing**.**

Considering that during the development of these techniques it is very important the possibility of having ideal signals, that represent an specific defect, whose intensity and importance of the defect might be controlled since its more incipient level until a high severity condition that would lead to the damage of the equipment. It justifies the development of simulators of vibration signal of the gear pairs, which even not being exactly perfect, allow the controlled generation of signals referring to ideal pairs and with progressive severity defect.

Vibration in gearboxes: sources of noise

Considering a gear pair without defects, the several sources of noise derived from its regular functioning, which can be identified through an analysis of the spectrum of signal frequency:

Turn of the Shafts: refers to a signal always observed that provokes, for each shaft, a continuous periodic response with frequency identical to its rotation in cycles for second. This signal is caused by the residual assembly deviations and small flexion of the shafts due to the radial loading.

Gear mesh frequency: it generates for each engaged pair a periodic response, whose frequency is equals to the product of the turning frequency of the shaft in which the gear is fixed by the number of its teeth. This signal comes from the deviations relative to the perfect profile of teeth, caused by both the deformation of the tooth under loading and by an unequal wear.

Harmonics of gear mesh frequency: components always observed whose frequency is equal to two, three or four times the gearing frequency, with low amplitude. These harmonics come from residual angular misalignment during assembly.

According to Penter & Lewis [1] the fails on mechanical reducers occur mainly in the gears with 60% of the incidences, followed by the fails in the bearings with 19%, in shafts with 10%, in boxes with 7%, in fixings with 3% and in retainers with 1%.

The classical symptoms associated to the most common defects, in other words, defects located in the gears, shafts and bearings that totalize a sum of 89% of the incidences of fails on a mechanical reducer are angular misalignment, eccentricity and damages in the gear teethes [2].

Shaft with angular misalignment: significant presence of harmonics of frequency of misaligned shaft turn, in special of the frequency of gearing of the fixed gears. The second or third harmonics appear with the amplitude equal or higher than the basic one, in these terms, when one of them exceeds 150% of the value of the basic amplitude the misalignment cannot be tolerated.

Eccentricity: if there will be any gear out of center due to an non adequate assembly, machining mistakes or shaft deformation, it is expected that its gearing frequency shows up with high amplitude and in particular with "lateral bands" from the modulation process. These lateral bands are equally spaced frequencies to the right and to the left of the gearing frequency and its harmonics whose difference is the frequency of shaft turn in which the gear this fixed. If the frequency amplitude or some lateral band exceeds 70% of the value of the basic one, the eccentricity should not be tolerated.

Damages in the teeth: the punctual damages that may be in the teeth are: pitting, crack and teeth brake. All these are characterized by a located loss of rigidity of the defective tooth, that it in such a way reflects a change in the amplitude, as well as in the phase of the vibration signal, during the period of gearing of the problematic tooth. These changes occur due to a located modulation that reflects in an increase of the lateral bands present in the spectrum. When some of these defects are extensive, it occurs a fast and abrupt change in the force applied to the tooth. This fact may excite resonant frequencies of the system shaft-bearing [[3, 4]]. These resonances are due design matters, located between the frequency the shaft turn and of gearing and come followed by lateral bands spaced of the frequency of the shaft turn in which the problematic gear is fixed.

Torcional model of a gear pair

An engaged pair, represented in the figure (1), is completely defined by knowing the numbers of teeth of the motion gear Z1 and moved Z2 besides its module m. The primitive diameters D1 and D2 are calculated by multiplying the module by number of teeth. The dynamic model of one freedom degree adopted for its modeling presented in figure (2) is formed by two discs connected by the gearing rigidity, gearing damping and excitement due to the gearing errors.

According to this torsional model there is no lateral motion in the centers of the two gears. It can be considered that the rigidity of the gearing depends on the number and position teeth position, which are in contact, in a definite moment, being, therefore a periodic function of relative angular position of the gears. In this model the error of transmission is still considered, which represents geometric errors in the teeth profiles and errors of assembly of the engaged pair. The transmission error is defined as being the difference between the current and ideal positions of the conduced gear, and will be shown as a linear displacement throughout the action line.

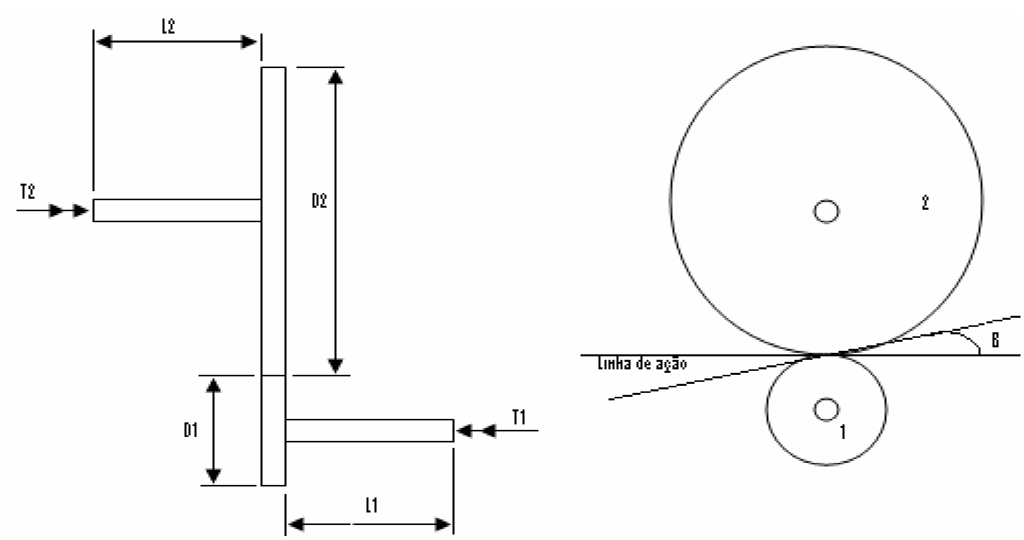

Figure 1. Representation of an engaged pair in its action line to a pressure angle β.

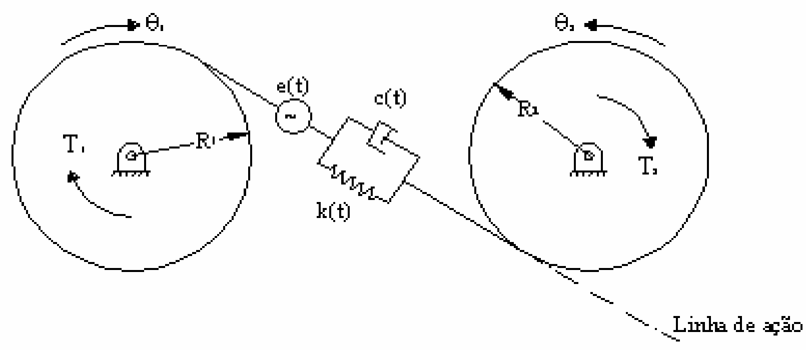

Figure 2. Dynamic Model of a gear pair.

This model will have as entries the geometric and functional definition of the engaged pair, its characteristics of functioning and the presence or absence of any defect in the gears or assembly. The solution of the motion equation will be the vibratory response of the set and it represents vibration signal as if it would be obtained in a measurement with accelerometer mounted over the bering of its shaft. The excitement is established as an overlapping of the excitement due to the normal functioning of the engaged pair that will be continuous force acting in the direction of the action line of the gearing, added to the excitations due to the noise sources inherent to the eventual defects among the ones that were already listed.

In order to derive the expressions of the mass, damping and equivalent rigidity to the considered dynamic model, it is considered the balance of forces in the contact plan between the teeth, considering that the transmission of the force is carried according to the direction defined by the action line of the coupling, according to the representation on figure (2).

The equivalent rigidity of the system, k(t), is resultant of the addition of effects due to the shaft twist effects, teeth flexion and deformation in the contact plan of teeth. Among these three contributions, the displacement due to twist of the shafts is most significant. The deformation in the contact plan induces the variation in the position of the action line, and the displacement due to teeth flexion is small. In this paper on will focus only the effects due to the twist of the shafts and the variation of the action line position, once they're dominant in the transmission.

Considering the set already stabilized, turning and transmitting a continuous torcional moment, it is possible to derive the equation of the movement for vibration analysis, assuming that the action of a force F(t) applied in the contact point of the teeth, provokes a condition of displacement $(\delta, \dot{\delta}, \ddot{\delta})$ in the direction of the action line and due to the torsion of the shafts, an angular displacements in the gears: $(\theta_1, \dot{\theta}_1, \ddot{\theta}_1)$ and $(\theta_2, \dot{\theta}_2, \ddot{\theta}_2)$, since $r_1\theta_1 + r_2\theta_2 = \delta$, where *ri* is the radius of the primitive circumference of gear i. As it is in balance:

$$F = \frac{T_1}{r_1} + \frac{T_2}{r_2} \tag{2}$$

where the torque $T_i = k_i\theta_i + I_i\ddot{\theta}_i = \frac{k_i}{r_i}\delta + \frac{I_i}{r_i}\ddot{\delta}$.

Substituting this result in the equation of balance:

$$F(t) = (\frac{k_1}{r_1^2} + \frac{k_2}{r_2^2})\delta + (\frac{I_1}{r_1^2} + \frac{I_2}{r_2^2})\ddot{\delta} \tag{3}$$

Comparing the terms with the equation of movement, the rigidity and the equivalent mass can be identified:

$$k = (\frac{k_1}{r_1^2} + \frac{k_2}{r_2^2}) \text{ e } m = (\frac{I_1}{r_1^2} + \frac{I_2}{r_2^2}) \tag{4}$$

where: $k_1 = \frac{\pi G d_1^4}{32 L_1}$, $k_2 = \frac{\pi G d_2^4}{32 L_2}$, $I_1 = \frac{m_1 r_1^2}{2}$ e $I_2 = \frac{m_2 r_2^2}{2}$.

Rigidity of the Gearing

A brief observation on the equation (4), allows the conclusion that the rigidity equivalent of the system is strongly dependent on the radius of the primitive circle of the gears of the engaged pair. Thus, the contact and flexion deformations of the teeth will interfere on the actual value of the rigidity, in such a way that will provoke a fluctuation of the actual value of the primitive radius during the gearing action, which would be in the intersection of the center line and the tangent between the primitive circumferences of the gears, when provoking deviations between the actual and theoretical teeth position. In addition, it must be considered that the rigidity of the gearing depends on the number of teeth pairs that are in contact in a certain instant (covering factor), which is the function of the teeth number of each gear and pair module.

Özgüven [5], Amabili and Rivola [6] and Kuang and Lin [7], presented studies indicating that due to gear design standardization, such fluctuation of the equivalent rigidity may be approximated by an overlapping between the theoretical rigidity *km*, calculated for the values of theoretical radius of the primitive circumferences of the gears, with a variable parcel *ka* that is between 5% and 15% of *km*. In addition, it can be assumed that, during a gearing period, *Tg*, defined as the passed time between the initial contact of a certain pinion tooth with a gear tooth until its way out of the gearing, the equivalent rigidity of the model is:

$$k(t) = k_m + k_a(t) \tag{5}$$

where: $$k_a(t) = \delta(t) k_a; \quad \delta(t) = \begin{cases} 1, & se \quad 0 \le t \le \dfrac{T_e}{2} \\ -1, & se \quad \dfrac{T_e}{2} \le t \le T_e \end{cases}$$

As result of this model, the equivalent rigidity of the engaged pair *k(t)* will be a function defined as square shaped wave, with a frequency equal to the gearing frequency of the pair. Its amplitude of variation reflects the index of the engaged pair cover, it means that for the gearing of a single tooth at a time it is used *ka = 0,15km*. For more adequate coverings, it is used *ka = 0,10km.* For helicoidal gears, *ka = 0,05km* is used.

The modeling of the variation of the rigidity on the gearing cycle of each tooth through a square shaped wave assumes an abrupt variation of it, what does not really corresponds to the reality. It's going to be demonstrated in this paper that such modeling introduces an effect just like a series of repeated impacts on the dynamic system. This effect can be explained when its verified that step function is discontinuous, in other words, that its transition value is instantaneous. The assumption that the rigidity of the gearing varies in a less abrupt way is closer to the reality, which means that its transition value is not instantaneous.

An alternative to avoid the discontinuity effect is to represent the variation of the gearing rigidity using a co-sine function. In this case, the variable parcel of the rigidity is described as: $k_a(t) = \cos\left(2\pi f_G.t + \dfrac{\pi}{2}\right)k_a$ where f_G is the gearing frequency of the gear pair.

The critic on the usage of the equation based on co-sine to calculate the rigidity variation of the gearing, it's that fact that its is lighter than the actual rigidity variation observed through the usage of modeling by finite elements Brauer, Jesper [[8]]. In an intermediate case, the usage of cycloids curves is possible for adjusting the higher and lower levels of the square shaped wave.

In figure 3 the usage of cycloids curves is presented where the period related to one cycle of rigidity variation will be subdivided into six intervals. The first one relates to the interval of time in which the rigidity goes from its medium value to the maximum. In the second interval the rigidity remains its maximum value constant, in the third one it returns to its medium value, in the fourth it goes from the medium level to the maximum, in the fifth it remains constant on minimum value, and in the sixth it returns, from the minimum value to the medium one.

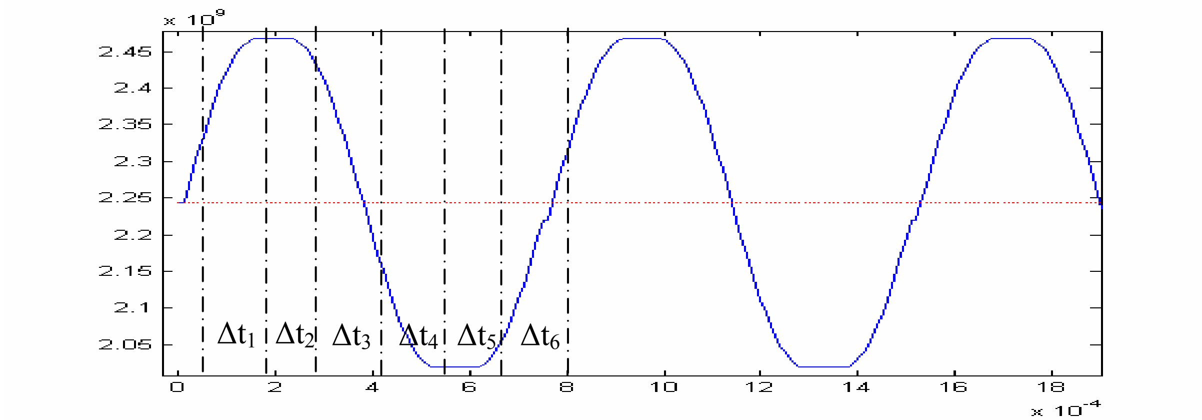

Figure 3. Adjustment of rigidity variation of the gearing using cycloids.

The intended advantage of using cycloids curves to describe the variation of the rigidity between the minimum and maximum values is the possibility of controlling the speed of this transition. The situation of intervals of time transition tending to zero, it approaches to the case of the square shaped wave, and it reflects gearings with more wear where impacts occur. For transitions times tending to ¼ of the action period of the tooth, it approaches to the case of the co-sine, what would be the lighter gearing possible to obtain.

In any formulation, the presence of a crack in a determined tooth, or even the break of a tooth, may be simulated by applying a percentage reduction of the value of *km* during the period of gearing of the defected tooth. A light crack is simulated by applying a 5% to 10% reduction in the value of *km*. A severe crack may be simulated by a 20% to 30% reduction. The loss of a tooth in an engaged pair with covering factor of 2.0 will be simulated by a 50% to 40% reduction of *km* value.

Excitement Definition *F(t)*

The excitement force *F(t)*, will be the sum of the constant force due to the power transmission with the ones due to the transmission errors $e_i(t)$, referring to the composition of the geometric errors on the teeth profiles and the assembly errors f the engaged pair. In this case its general formula is:

$$F(t) = \frac{T_1}{r_1} + \sum k(t) e_i(t) \tag{6}$$

where: $T_1 = \frac{30W}{\pi N_1}$, *N* is the rotation of the motion shaft in cycles per minute and W is the power transmitted in Watts.

The assembly out of the center of one or both gears results in a tangential excitement, towards the direction of the action line of the gearing, which can me modeled by:

$$e_j(t) = \xi_j sen(\omega_j t) \tag{7}$$

where: $\xi_j = \sqrt{\frac{r_g{}^2 \cdot e}{r_g + e} - \left(\frac{r_g \cdot e}{r_g + e}\right)^2} \cdot \frac{e}{r_g}$, where r_g is the primitive radius of the gear and e the eccentricity.

$\omega_j = \frac{2\pi N_j}{60}$, where N_j is the rotation of the shaft in cycles per minute.

The tooth shape deviation, which were considered in this paper are the total deviations according to the standard DIN 3969 - Concepts and parameters of definition in cylindrical tooth wheels with involving type gearing. It refers to a composition between the total deviations of profile and the division deviations and of the teeth thickness. This deviation is usually proportional to the angle of the gear turn, but they don't necessarily follow an entire relation. Thus they may have the same frequency of an assembly out of center (shaft turn), or some value less than two times the turn frequency. These deviations are the cause the "ghost frequencies" frequently observed when it is analyzed the vibration signals of gear pairs. This paper considers that the total shape deviation will excite the frequency of the assembly phase, which is a "ghost frequency" that, according to López, Orts and Lozano [[9]], may be calculated by the following equation:

$$\omega_{fe} = \frac{Z_1(2\pi N_1)}{60 N_e} \tag{8}$$

In this equation, N_e is the number of assembly phases defined as the number of the pinion turns to repeat the engagement of the tooth 1 of the pinion with the tooth 1 of the crown. The excitement amplitude due to the assembly error will be:

$$e_e(t) = \gamma_e sen(\omega_{fe} t) \tag{9}$$

where the value of γ_e depends on the gearing precision. Usually $4\mu m \leq \gamma_e \leq 7\mu m$.

If we substitute in equation (6) the obtained results for the excitements referring to the assembly out of center and shape deviation, the excitement force to be considered in the equation of movement, according to the developed model will be:

$$F(t) = \frac{30W}{r_1 \pi N_1} + k(t)e_1(t) + k(t)e_2(t) + k(t)e_e(t) \tag{10}$$

Signal Acquisition

The vibratory response of the engaged pair may be obtained by integration through the time interval between $t_0 = 0$ and $t_f = T_R$, the problem of the initial value:

$$m\ddot{x} + c\dot{x} + k(t)x = F(t) \quad (11)$$

under conditions: $x(0) = x_0$ e $\dot{x}(0) = v_0$

The problem may be solved by applying Runge-Kutta's technique of integration over an equivalent system of ordinary differential equations of order, obtained by the application of Euler's variable transforming. Defining two other new variables $x_1 = x(t)$ and $x_2 = \dot{x}(t)$, substituting them in the equation of movement and isolating to $\dot{x}_2(t)$, we will have a system first order differential equations:

$$\begin{aligned}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= -\frac{c}{m}x_2(t) - \frac{k(t)}{m}x_1(t) - \frac{F(t)}{m}
\end{aligned} \quad (12)$$

Based in this formulation, it was implemented a MatLab® routine for simulation that makes the integration using the sub-routine "ode45" of the MatLab® Library. Since the objective is to simulate the signal just like it would be obtained by measuring over the bering, the integration step *Δt* is pre-defined equals to the inverse of the sampling frequency *fs* that would be regulated in the device to acquire the signal. The total time of integration *TR, time record*, is defined by multiplying the number of points necessary to describe the signal simulated by the integration step.

Influence of the rigidity calculating model on the simulated signal

The developed simulator was applied to generate the vibration signal of an engaged pair whose gears are manufactured in carbon steel, with module *m* = 1. The pinion has 44 teeth, and the gear 95. The width of the teeth is 30mm. The distance from the pinion to the connection point is *L1* = 200mm, and from gear to the connection outside point, *L2* = 135mm. The set transmits 3800W with input shaft rotation of *N1* = 1800 rpm. For these data, the frequency of turn of the input shaft is 30Hz, the output shaft 13,89Hz, the gearing 1320 Hz, the number of assembly phases is *Ne* = 9 and the excitement force of the system is constant, *F(t)* = 1649,42N.

In order to allow a qualitative comparison of the results, signals had been simulated, considering perfect gears, without defect, where the amplitude of gearing rigidity variation *ka* is 10% of the average theoretical rigidity *km*. The rigidity of the gearing was calculated using each one of the three presented proposals: square shaped wave, co-sine and cycloids. In the square shaped wave case two approaches had been used: pure square shaped wave, using the definition offered by the equation 5, and Fourier's series approximation, which is a very common form used for such problems. All the cases had been simulated with an integration interval $\Delta t = 10^{-4}$s, corresponding to a sampling rate of 10 kHz of the acquisition system. The number of integration steps adopted was of 10240 points.

Rigidity of the gearing as square shaped wave

In representation 4 it is presented the behavior of the rigidity variation as a square shaped wave. Figure 5 presents vibratory signal corresponding to a turn of the gear, obtained with by using the simulator, whose corresponding spectrum of frequencies is shown in representation 6. Observing the spectrum of frequencies, we can verify that, besides the frequency of gearing 1320 Hz, the presence even with small amplitude values, of its first and second harmonics, as well as structures around of these frequencies, mainly in the gearing frequency area, that are characteristic of the existence of strong noises. It's important to highlight that the gearing frequency appeared as a result of the fluctuation of the gearing rigidity implemented in the model, since it is not considered the existence of any type of exciting external force with frequency equal the gearing or any other.

In the vibratory signal abrupt variations of amplitude are clearly noticed, as if transient efforts had occurred. This is caused by the abrupt variation between the maximum and minimum rigidity and vice versa. The conclusion is that the approach of the variation of rigidity in a square shaped wave form is not satisfactory to describe the behavior of the system, because it introduces characteristics as if transient forces existed acting on the set.

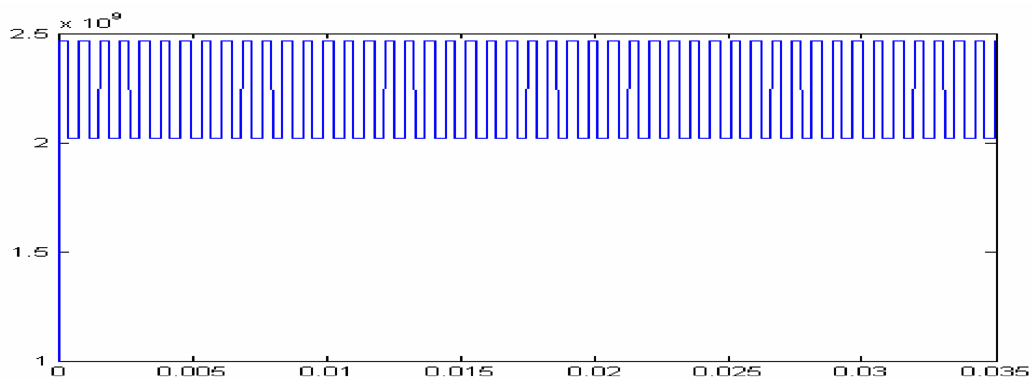

Figure 4. Variation of k(t)as a squared shape wave.

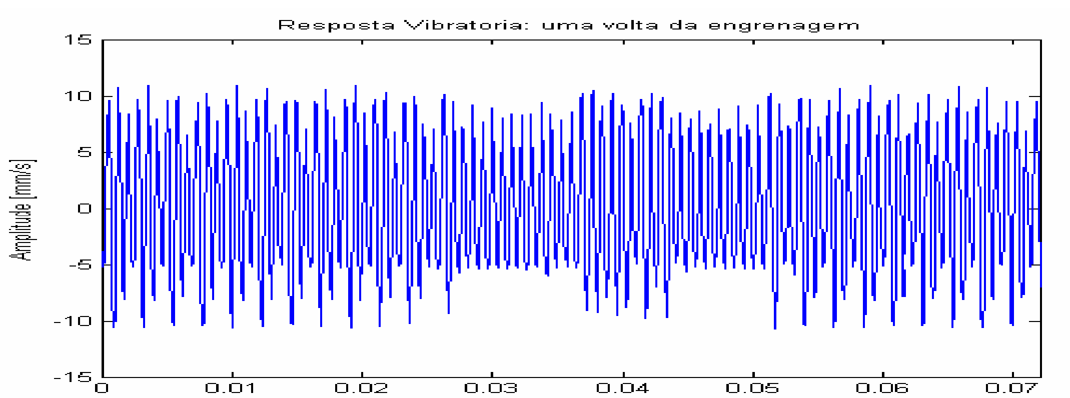

Figure 5. Simulated vibratory signal.

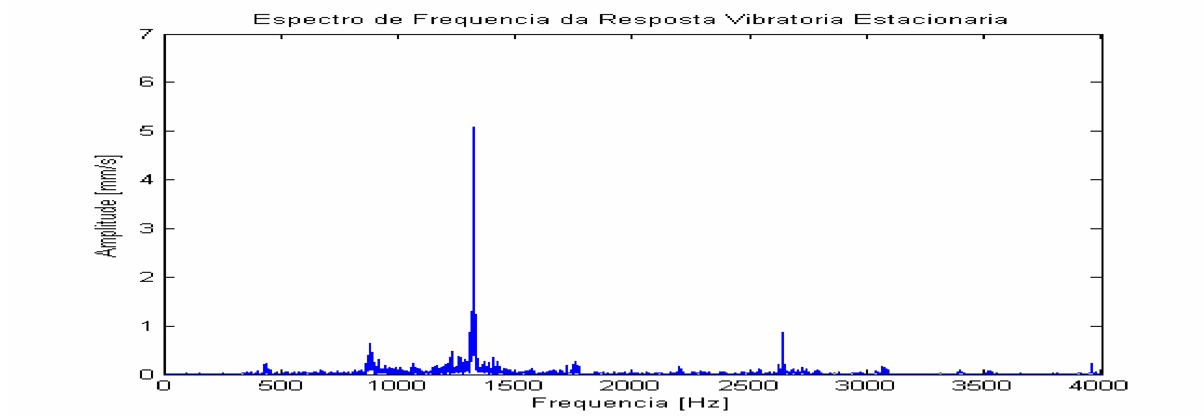

Figure 6. Spectrum of frequencies corresponding to the signal of figure 4.

Gearing rigidity approximated by Fourier series

In figure 7 below, it is presented the variation of the rigidity as a square shaped wave is presented by first 17 terms of Fourier's series. In representation 8 the vibratory signal obtained is presented and in representation 9 the corresponding spectrum of frequencies. In representation 7, we can notice that the approximated square shaped wave presents some "ripples" even though the approximation is good.

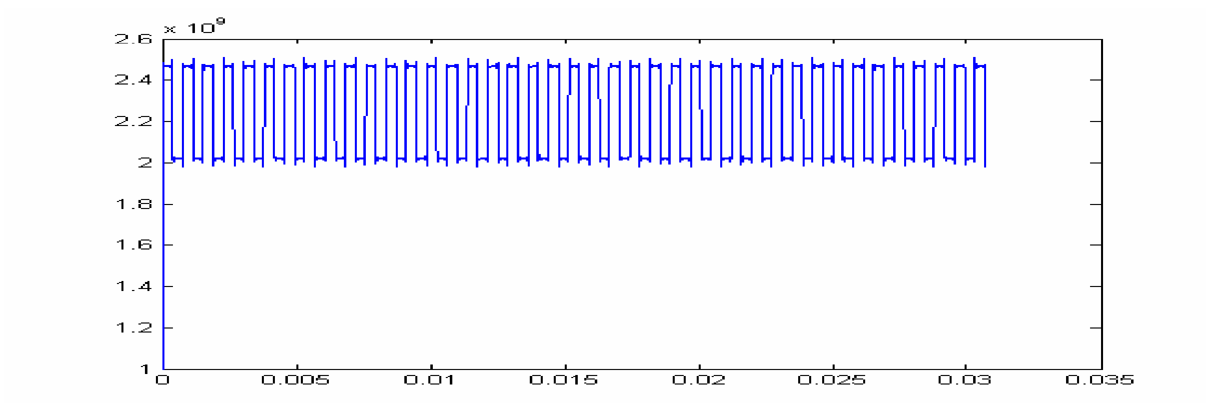

Figure 7. Variation of k(t) as a squared shaped wave approximated by Fourier's series.

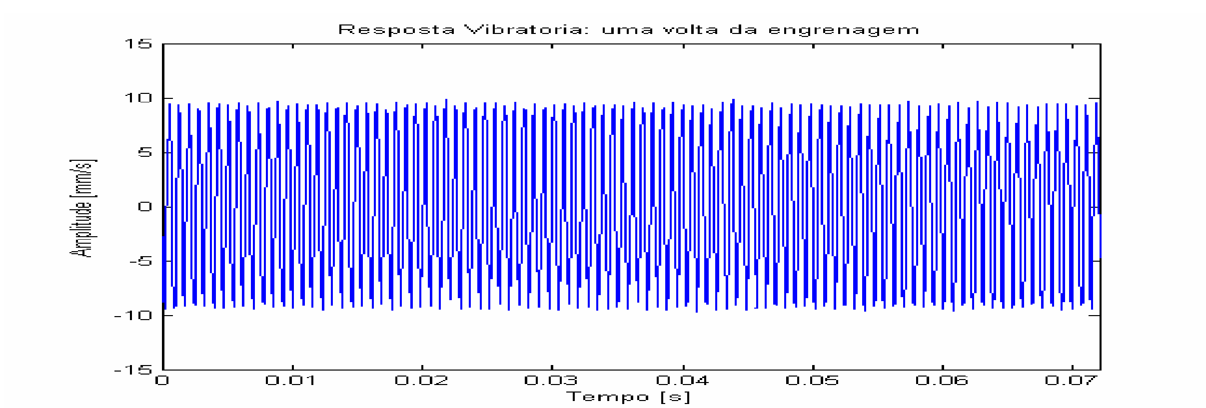

Figure 8. Simulated vibratory signal.

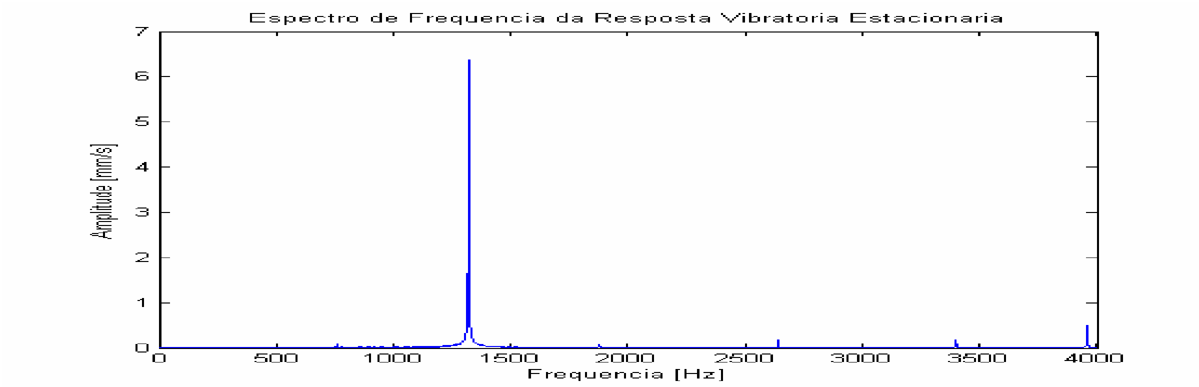

Figure 9. Spectrum of the frequencies of the simulated signal.

Analyzing the spectrum of frequencies in the representation 9, it can be verified that it is sufficiently clean, specially related to the gearing frequency. However, we can observe small amplitudes marking its first and second harmonics among other frequencies. These frequencies are, probably, originated by the "ripples" observed in the modeling of the rigidity variation of the gearing. However, when it gets closer to the squared shaped wave, non continuous, the result turned better due to an addition of continuous functions.

Rigidity of the gearing as co-sine function

Another way to calculate the variation of the rigidity through the time, would be by approximating it as a co-sine, making the appropriate corrections at the beginning, in such a way that *ka(t0)=0*. Figure 10 represents the variation of the value of *k(t)*, considering the rigidity calculus from a co-sine form wave. In this formulation, besides the gears the gearing will be perfect without shocks. The simulated signal and its spectrum of frequencies are presented in figures 11 and 12 respectively.

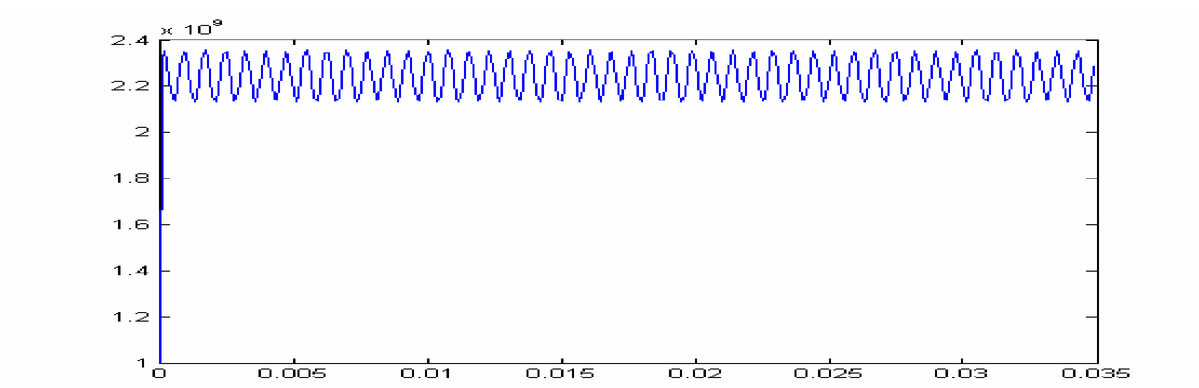

Figure 10. Variation of k(t) as a co-sine function.

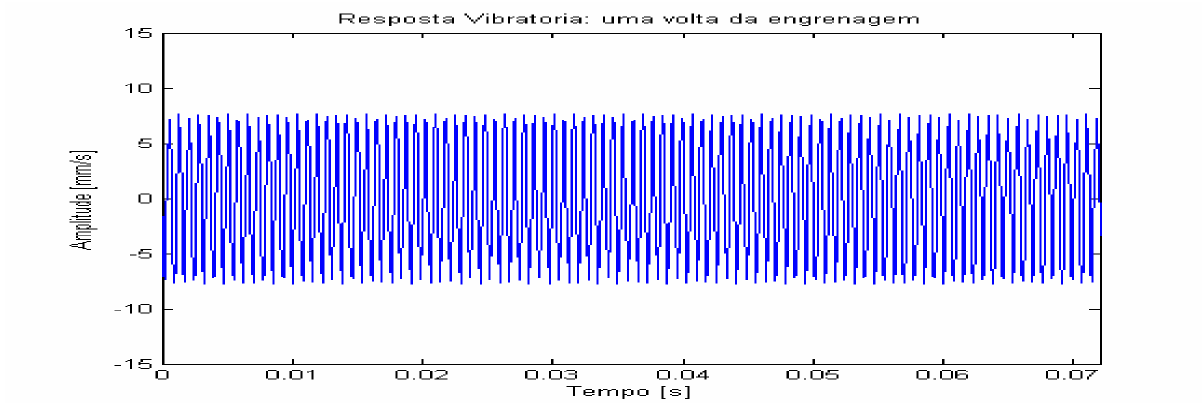

Figure 11. Simulated vibratory signal.

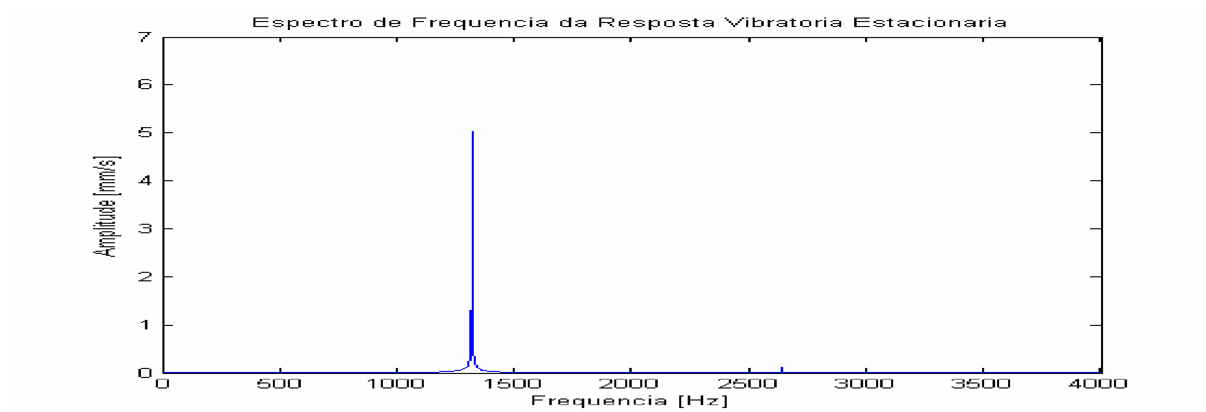

Figure 12. Frequency Spectrum of the signal presented in figure10.

The spectrum of frequencies is perfectly clean, and presents only the frequency of the gearing in 1320Hz. It refers to an ideal condition, where any alteration in the spectrum will only occur if any excitement force, corresponding to the presence of a certain defect which may want to be introduced. In spite of the idealization, it is a good way to approximate the rigidity, if the objective is to study the sensibility or the comparison of different techniques for the diagnosis of a considered defect.

Rigidity of the gearing modeled with cycloid curves

When the fluctuation of the gearing rigidity was approximated by a square shaped wave, the result turned to be inadequate because this modeling introduces the effect similar to the action of repeated impacts on the system, what is due to the discontinuity of this function. The use of the approach by Fourier series approximates the square shaped wave thru an addition of continuous functions, eliminating almost completely the effects of the abrupt variation between the maximum and minimum values of the rigidity. The opposite case is to consider the gearing rigidity as a co-sine function. In this situation the function is light and continuous, and it offers a very idealized result.

The proposal of using a combination of cycloids curves to describe the variation between the extreme values of the gearing rigidity, is the possibility of choosing the time interval that will be used to make the transition from the minimum to the maximum value of the rigidity and vice versa. This way, it is possible to define the severity in which the rigidity varies and to establish how similar to the square shaped wave, more severe, or to the function co-sine, lighter, the system is. It allows the simulation of one more residual defect, which is the clearance.

For this study case, the level of severity of the shock was defined as 0.6, what means that during the gearing the addition of the times used for the transition between the levels of medium, maximum and minimum rigidity will be, once more, 40% of the contact period of each tooth. In the figure 12 the variation of the rigidity according to this scheme is presented. The simulated signal and its spectrum of frequencies are presented, respectively, in figures 13 and 14.

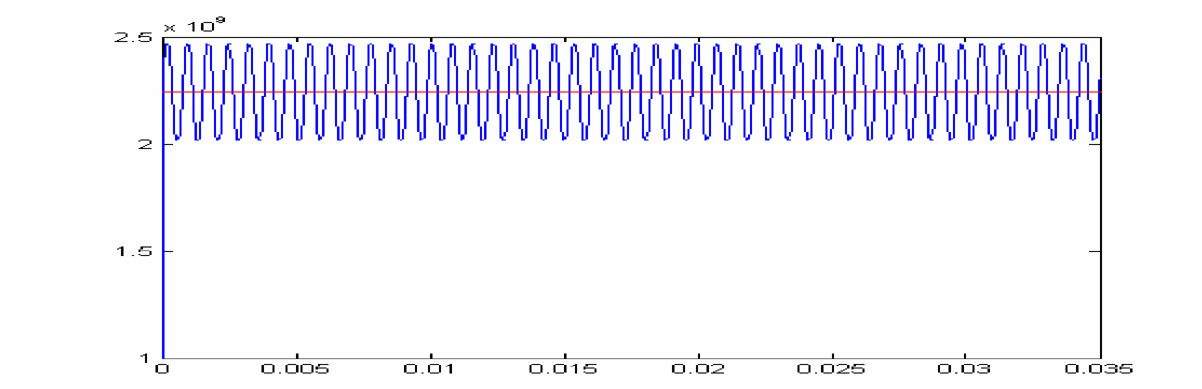

Figure 12. Variation of *k(t)* using cycloids.

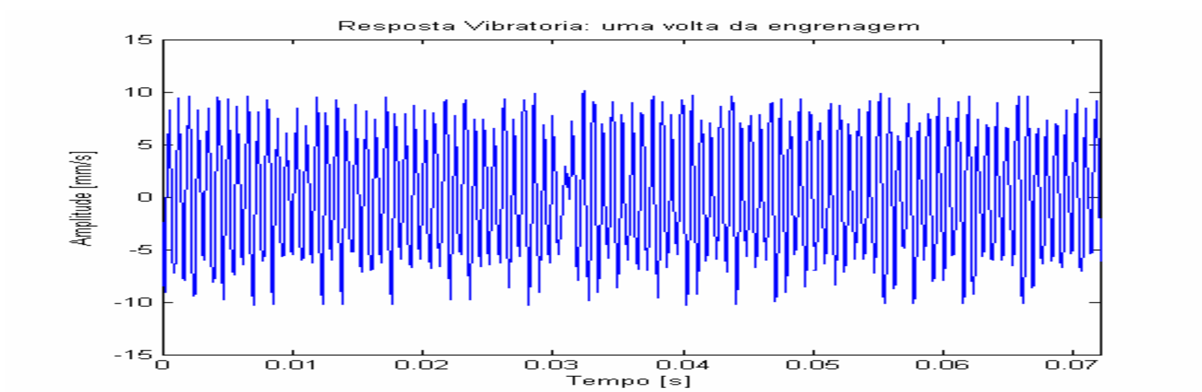

Figure 13. Vibratory signal simulated for one rotation on the crown.

In figure 12 it can be seen that, in spite of the minimum and maximum values of the rigidity being represented by parallel lines to the horizontal axle, these values are achieved in a continuous way, without any transition “ripple” between the increasing and decreasing paths and the respective minimum and maximum values.

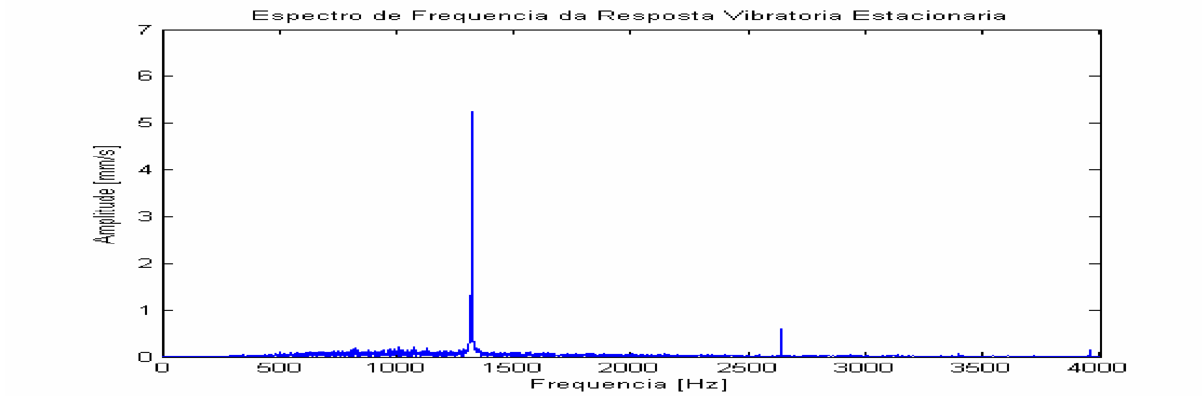

Figure 14. Frequency spectrum of the signal in figure 12.

In the spectrum of frequencies we can observe that there are indicating signals of impacts in the gearing, but not as severe as when the square shaped wave is used. This effect represents the admission of the existence of a certain level of shock between the teeth. This is a more realistic condition, once real gear pairs always have some kind of noise from the shock and displacement the gearing process.

Damage simulation

With the purpose of observing the behavior of the software in the simulation of several residual defects that had been taken considered in this work, each one of them will be applied to the engaged pair already modeled. Initially the presence of a crack in the 15th tooth of the pinion will be considered in such way that it will cause a reduction of 10% in the equivalent system rigidity when in gets in contact. Next it will be simulated the presence of a deviation of centers between the pinion and its shaft of 0.1m, m = module of the gears. Finally the overlapping of the two defects will be done. These three cases are applied in the situations where the variation of the rigidity is modeled with the usage of a square shaped wave approximated by Fourier's series, as a co-sine function and with the usage of cycloids curves.

Crack in the tooth

In figures 14, 15 and 16 we can see the simulated vibratory signal from a rotation of the gear, when we consider the presence of a severe crack in the 15^{th} tooth of the pinion, obtained respectively with the usage, to approximate the flotation of the gearing rigidity of the model, of the co-sine function, called case (a.1); Fourier's series, case (b.1); and cycloid curves, case (c.1).

In all cases it's possible to observe clearly the discontinuity of the signal in the instant of the time that corresponds to the beginning of its action on the gearing. In figures 17, 18 and 19 the spectrum of corresponding frequencies are presented, we can observe that since it refers to a transient defect, the effect is the appearing of an indicative structure of resonance, between the frequency of the shafts and of the gearing, as it's expected in these cases. It is known from the analysis of signals that, in this type of resonant response, the lack between its lateral waves is equal to the frequency of shaft rotation where the gear is fixed. This can be if verified in the spectrums of the signals obtained during the simulation.

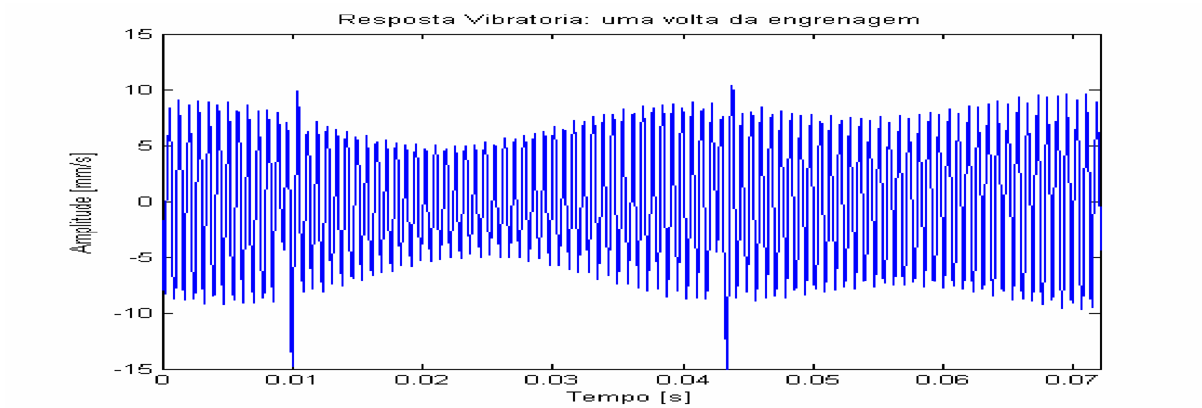

Figure 14. Simulated vibration signal referring to a rotation of the gearing, case (a.1).

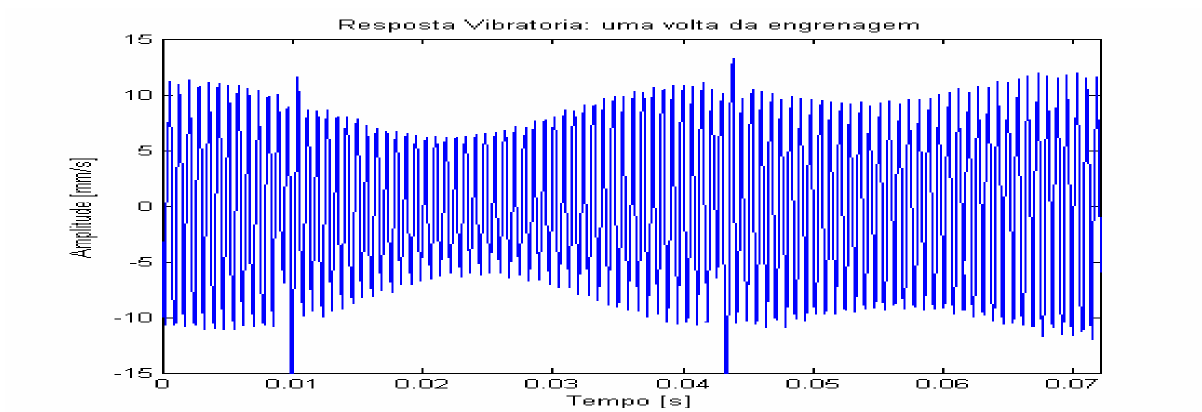

Figure 15. Vibration Signal referring to a rotation of the gearing, case (b.1).

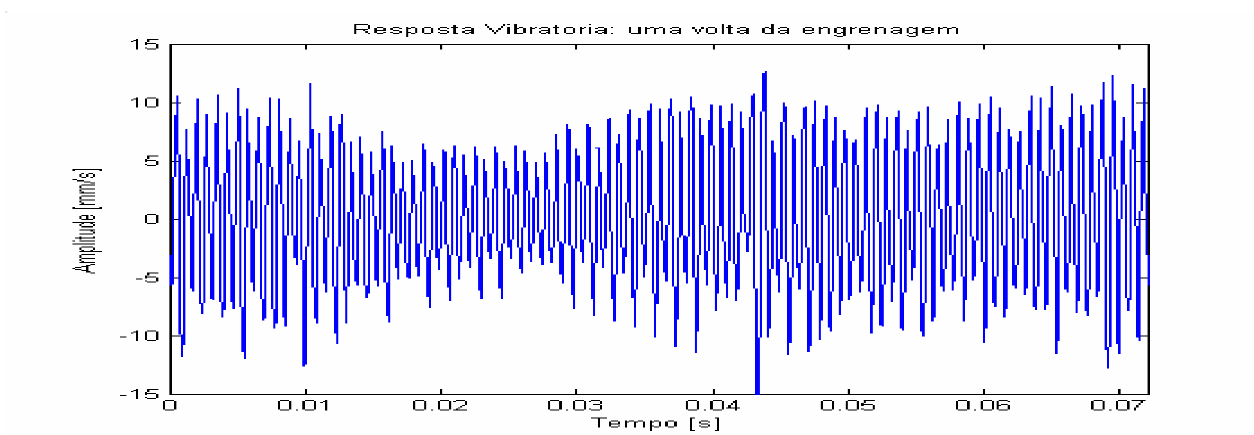

Figure 16. Vibration signal referring to a rotation of the gearing, case (c.1).

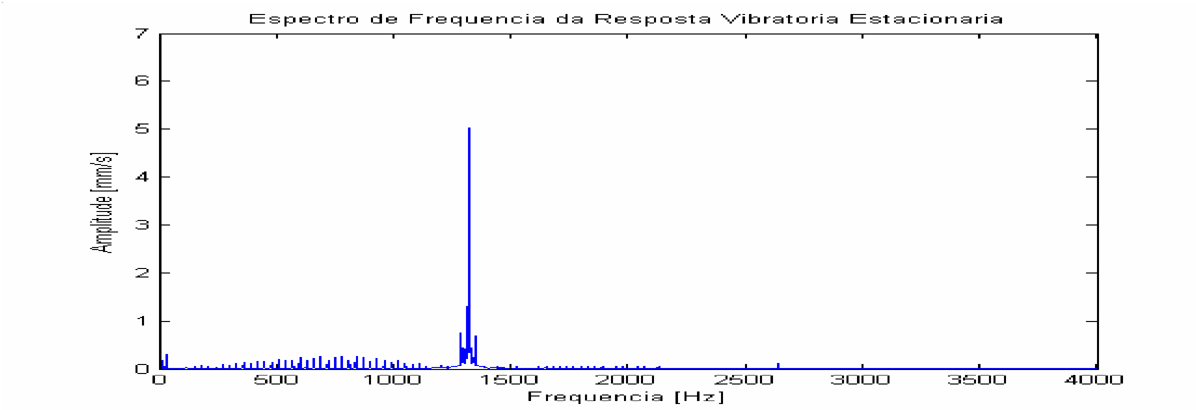

Figure 17. Frequency spectrum of the signal frequency of figure 13, case (a.1).

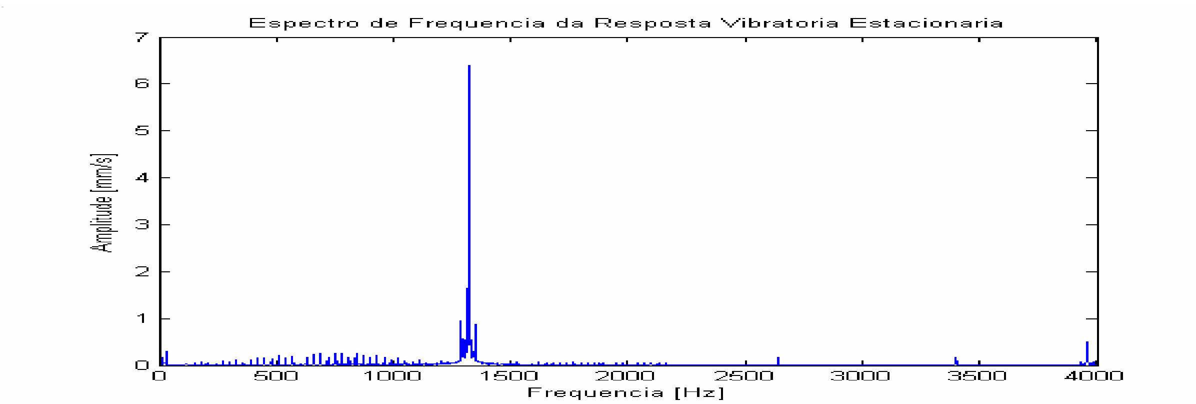

Figure 18. Frequency spectrum of the signal in figure 14. Case (b.1).

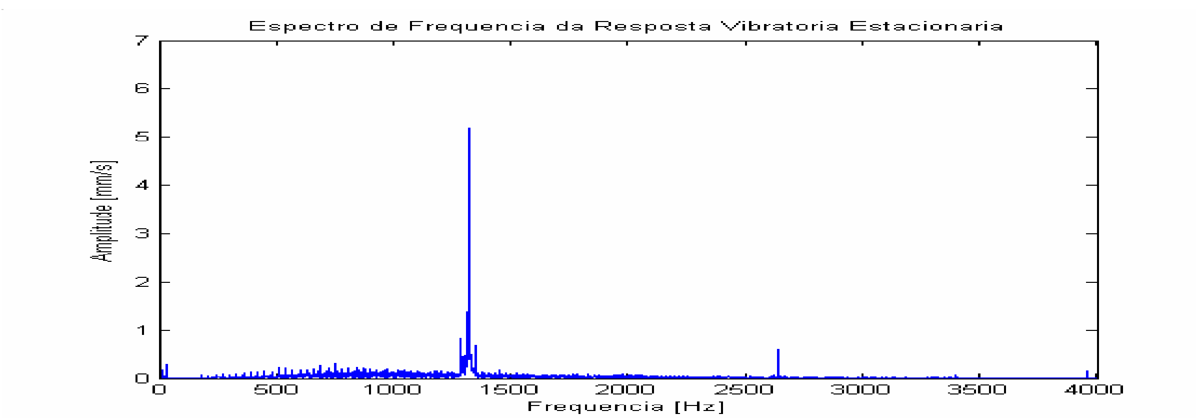

Figure 19. Frequency spectrum of the signal in figure 15. Case (c.1).

A more careful analysis of the frequency spectrums shows that, when the square shaped wave approximated by Fourier's series is used or described with the usage of cycloids, it is possible if to identify the characteristic symptoms of a certain level of noise in the gearing, what describes well the expected response of an actual pair of gears.

Assembly out of center

The simulated vibratory signal referring to one rotation of the gear, when it is considered a deviation between the center of the pinion and of the shaft, used to approximate the rigidity fluctuation of the model to the co-sine function, case (a.2); Fourier's series, case (b.2); and cycloid curves, case (c.2), are presented in figures 20, 21 and 22, respectively. The respective spectrums of frequencies are presented in figures 23, 24 and 25.

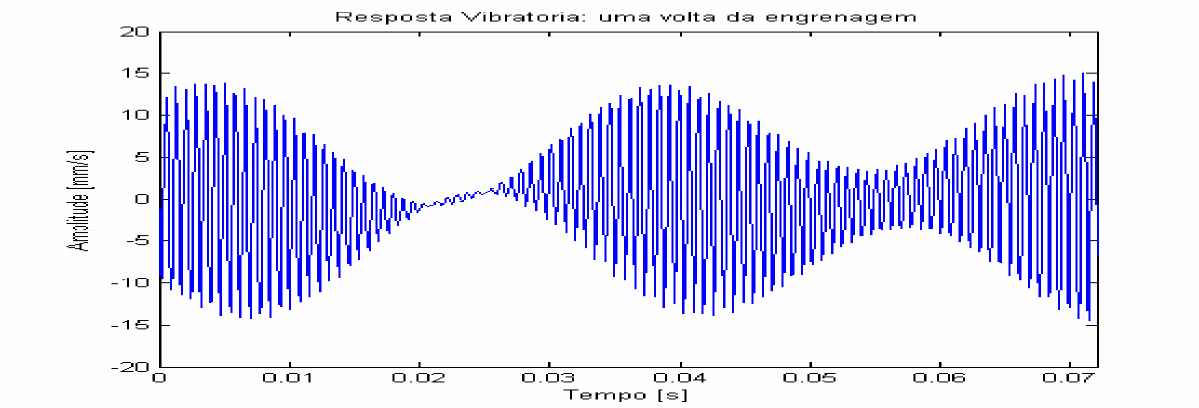

Figure 20. Vibratory signal referring to one turn of the gear, case (a.2).

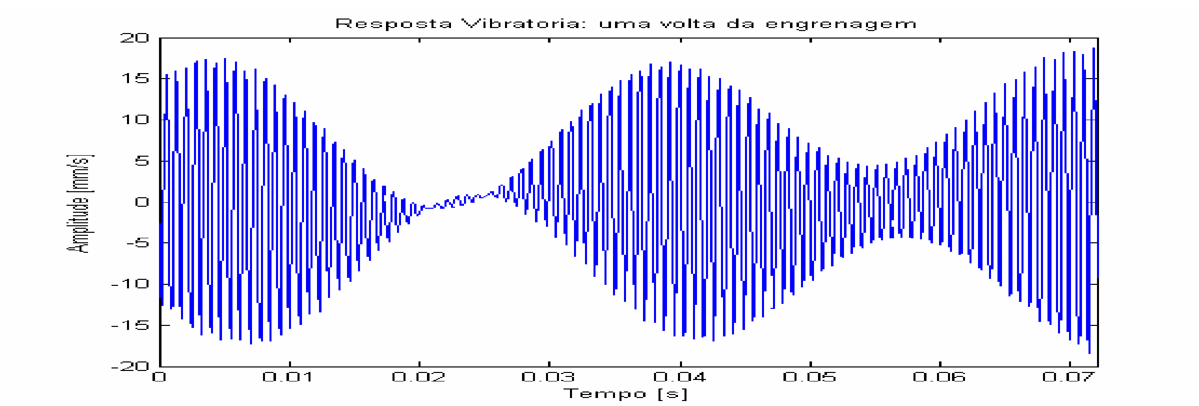

Figure 21. Vibratory signal referring to one turn of the gear, case (b.2).

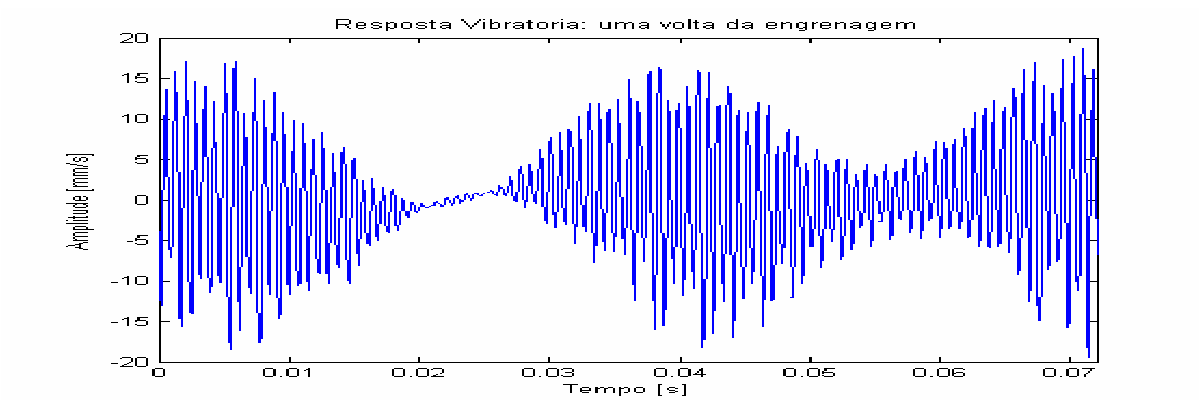

Figure 22. Vibratory signal referring to one turn of the gear, case (c.2).

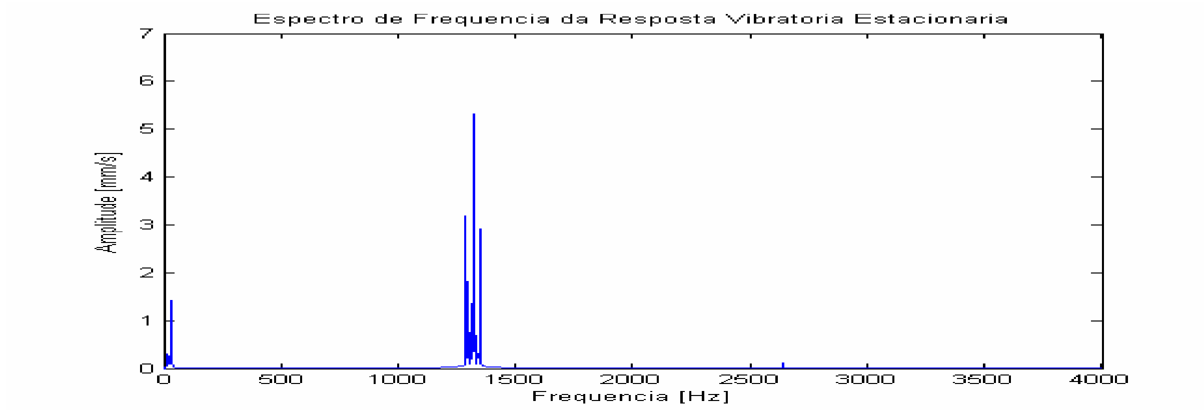

Figure 23. Spectrum of frequency of the signal presented in figure 20, case (a.2).

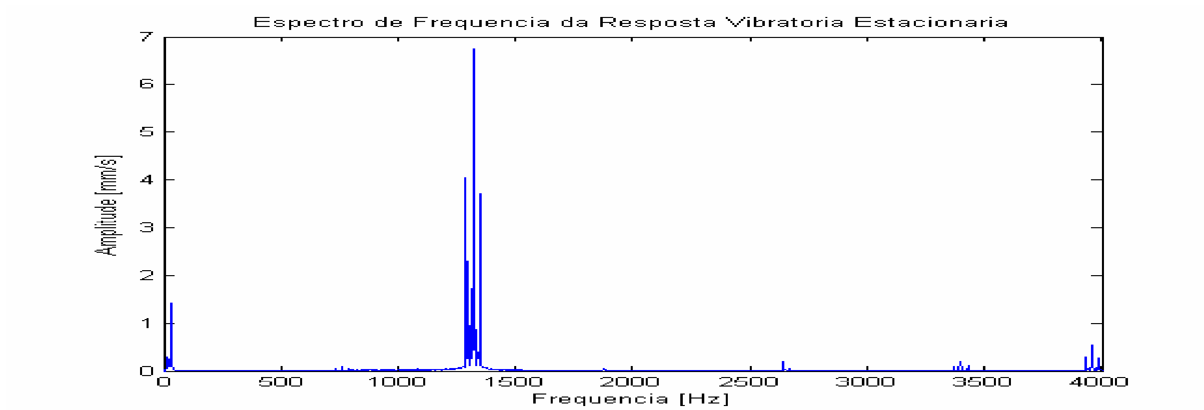

Figure 24. Frequency spectrum of the signal presented in figure 21, case (b.2).

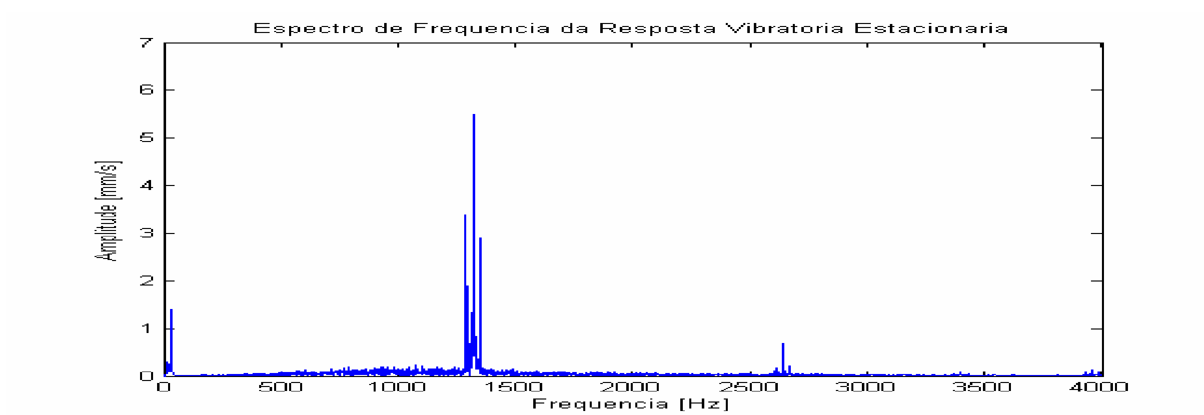

Figure 25. Frequency spectrum of the signal presented in figure 22, case (b.3).

Observing the obtained simulated signals, figures 20, 21 e 22, it can be verified that in all cases the clear modulation due to the turn of the pinion out of the center. On the frequency spectrums, it can be observed the gearing frequency is followed of side bands spaced of the frequency of the pinion shaft turn, 30 Hz. That is exactly the effect expected to be found. In addition, it is possible to observe the frequency of the shaft turn in the left corner. Such frequency appears because that residual defect provokes an exciting force which frequency is identical to the shaft turn frequency.

It is important to highlight that the only considered external excitation was the shaft turn that provokes a small radial displacement which maximum amplitude is the difference between the centers, equation (7). The side bands on the gearing frequency were due to the dynamic response of the model to that excitation.

As previously observed, also in this case the modelation of the fluctuation of the gearing rigidity using cycloids resulted on the appearing of a residual noise that is normal on the actual systems.

Overlapping of the tooth crack with center deviation

Applying equation (10), it was simulated the vibration signals of the system, considering the presence of the two defects already studied. Similarly to the previous items, it was named case (a.3), case (b.3) and case (c.3), the simulations where it was used the formulation of the co-sine function, approximated square wave by the Fourier series and cycloid curves to model the gearing fluctuation rigidity. The results are presented in figures 26, 27 e 28 bellow. In figures 29, 30 e 31 it is presented the frequency spectrums of those signals.

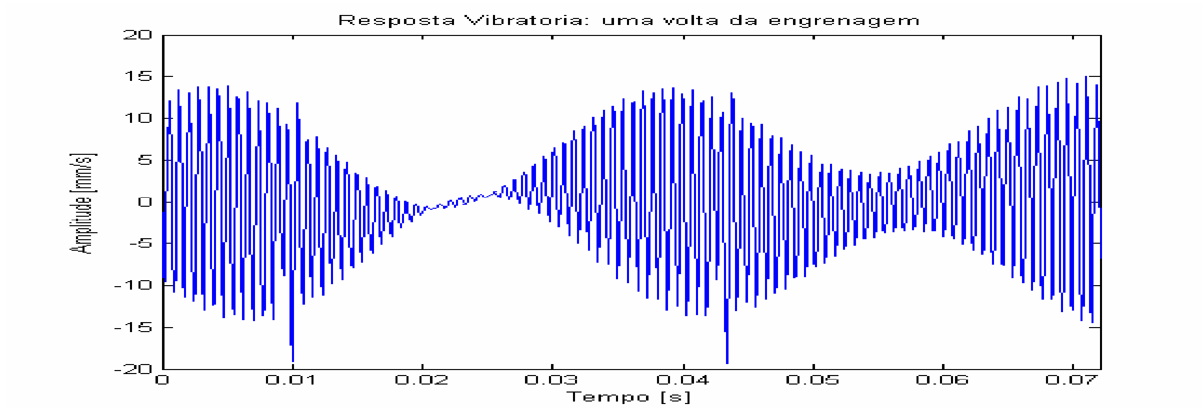

Figure 26. Vibration Signal referring to one turn of the gear, case (a.3).

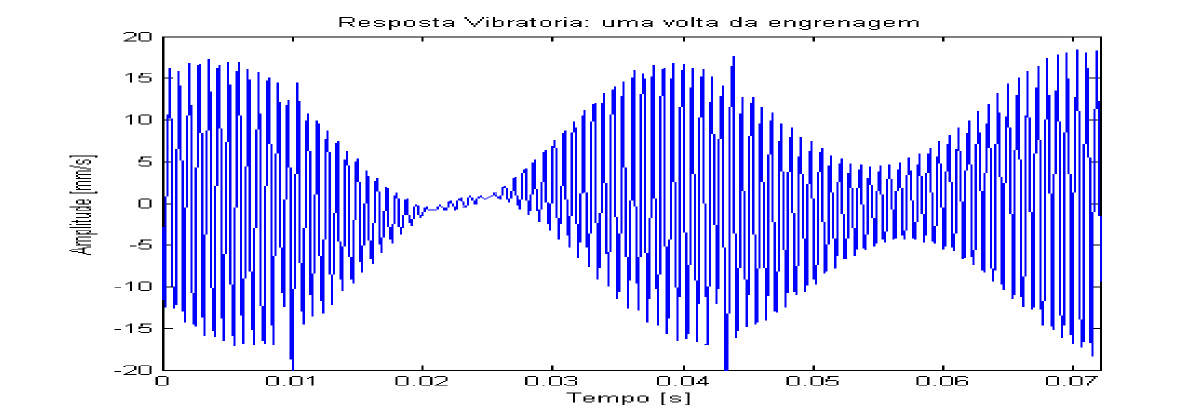

Figure 27. Signal Vibration Signal referring to one turn of the gear, case (b.3).

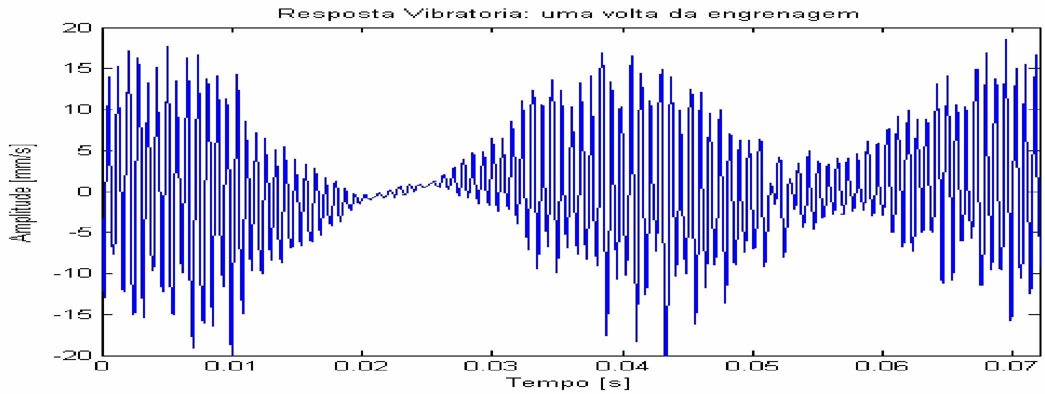

Figure 28. Vibration Signal referring to one turn of the gear, case (c.3).

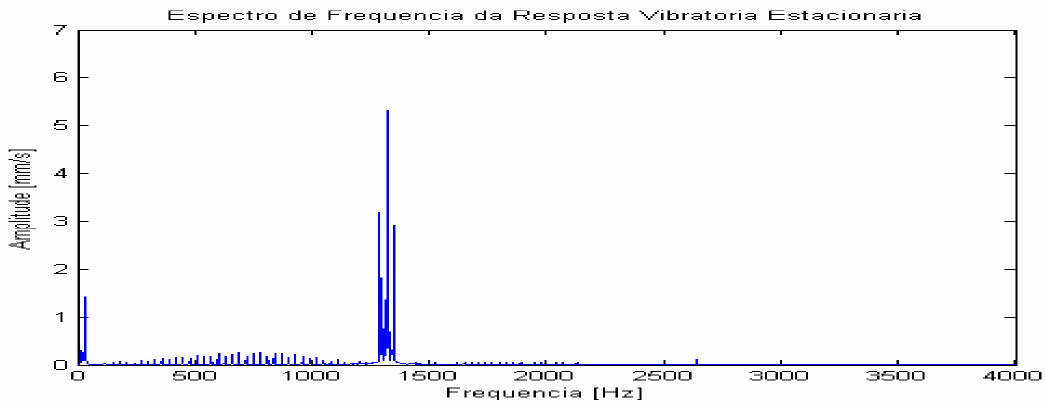

Figure 29. Frequency spectrum of the signal in figure 26, case (a.3).

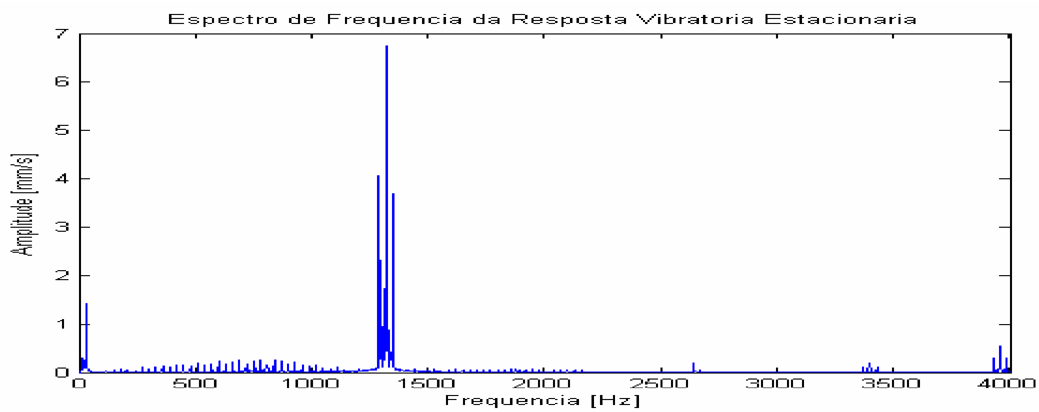

Figure 30. Frequency spectrum of the signal in figure 27, case (b.3).

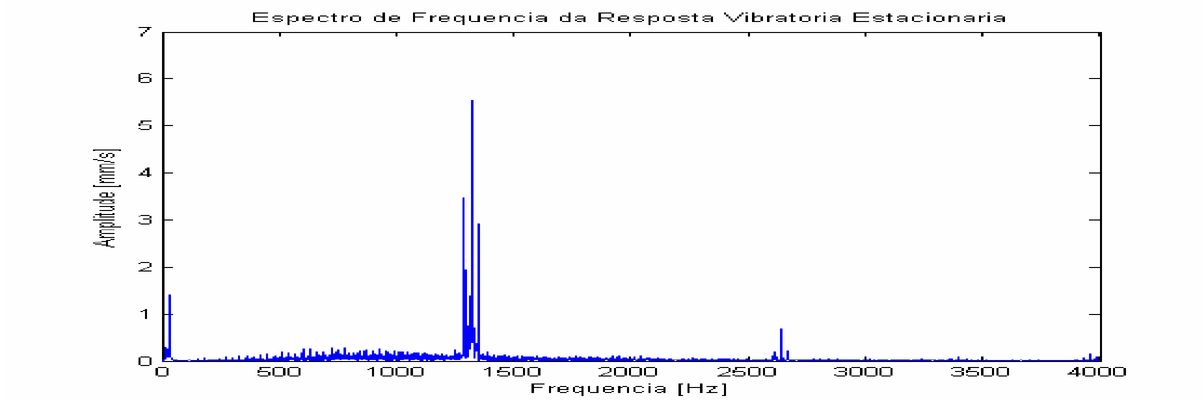

Figure 31. Frequency spectrum of the signal in figure 28, case (c.3).

Analyzing the results, it can be observed that the spectrums are the expected ones and that the overlapping of the effects resulted on a signal closer to the actual, in other words, with the residual assembly defects actuating as a noise on he main defect that is a cracked tooth.

Conclusion

Observing the results it can be verified that the dynamic simulation of a gear pair using a torsional model with one freedom degree, can be very satisfactory, since that it is used a more detailed modeling to describe the equivalent rigidity and in the obtaining of the excitation function to each type of defect to be studied.

Implementing the software from that modeling, is possible to simulate a certain gear pair from its design data and transmitted power, that is an important tool on the development and analysis of the behavior of new or dominated techniques of signal analysis, when it is interested to study its application to gear transmissions.

Observing the several ways to calculate the rigidity, the one that gives a better control in its utilization is the rigidity calculated through cycloids, once it allows to represent the interaction between his gears in a most realistic form, controlling the chock level among them. The rigidity modeled as a co-sine function would be a perfect gear without chock, on the other hand the modeling as a square wave showed to be very severe. That would be the situation which would present chocks with high clearance in teeth.

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