Reliable and Robust H_∞ Control for a Class of Uncertain Descriptor Systems

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Abstract

This paper discusses the reliable and robust H_∞ control problem for a class of descriptor systems with parametric uncertainties and external disturbances in the case of actuator failures. By solving a set of matrix inequalities, a sufficient condition and a controller are obtained for the reliable and robust H_∞ control problem of the descriptor systems.

Introduction

Descriptor systems are generalized and natural representations for many practical control plants such as robots, electric circuits, complex chemical engineering process, economical systems and so on. Compared with the standard state-space models, the form of descriptor system model can indicate the equality constraint conditions about dynamic expressions. In recent years, control problems of descriptor system have attracted much attention [1-4].

Robust control mainly focus on controller design to deal with uncertainties which are widely existed in practical control systems, for example, model parametric uncertainty and external disturbance. As an important approach of robust control, H_∞ control for uncertain systems has been intensively investigated in the last two decades [1, 2, 4, 5].

A control system designed to tolerate failures of sensors or actuators, while maintaining an acceptable level of the closed-loop system stability and performance, is called a reliable control system. For safety-critical systems, such as air vehicles, chemical plants and nuclear power plants, safety and reliability are even more important than good performance. So the reliability should be an important consideration when designing the controller. Recently, reliable control problem has also received considerable attention and some research results have been obtained for the reliable control in the case of sensor and actuator failures [6-10].

This paper deals with the theoretical aspects of the H_∞ control problem for a class of descriptor systems with parametric uncertainties and external disturbances in the case of actuator failures. Based on a set of matrix inequalities, a sufficient condition is obtained to solve the reliable and robust H_∞ control problem. By solving the matrix inequalities, the controller can be obtained.

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Problem Statement

Consider a class of descriptor systems with parametric uncertainties and external disturbances described by

$$E\dot{x}(t) = [A + \Delta A]x(t) + Bu + Gw \qquad (1a)$$

$$y = Cx \qquad (1b)$$

where $x \in X \subseteq R^n$, $t \in R$, $u \in R^m$, $y \in R^l$ and w denote the descriptor state, time, control input, measurement output and external disturbance, respectively. *A, B, C* and *G* are smooth matrix-functions, $A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{n\times l}$. *m, n* and *l* are positive integers. ΔA is the parametric uncertainty, and satisfies the assumption A1:

$$\text{A1: } \Delta A = D\Delta F \qquad (2)$$

$D \in R^{n\times p}$, $\Delta \in R^{p\times q}$, $F \in R^{q\times n}$, *E* and *F* are the known matrix-functions, Δ is known matrix-function, and satisfies :

$$\Delta^T \Delta \le I \qquad (3)$$

$I \in R^{q\times q}$ is an identity matrix.

The uncertain descriptor system under consideration is depicted in Figure 1.

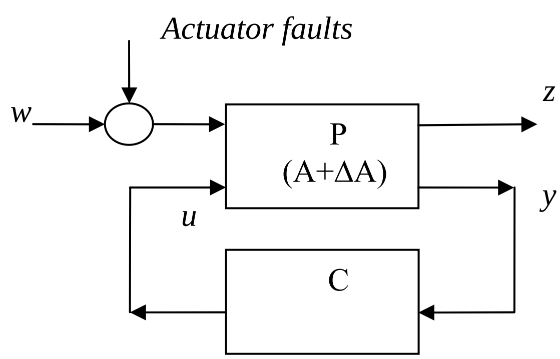

Figure 1 Descriptor system block diagram

In the following, the definition of the reliable and robust H_∞ control problem to be discussed is addressed.

Definition 1 [5]**:** System (1) is said to be tolerant and have H_∞ performance γ, if for given positive number γ, the system (1) is canonical, stable and pulseless, and the L_2-gain from the disturbance w to the control output z is finite, i.e.,

$$\int_0^\infty \|z\|_2^2 dt \le \gamma^2 \int_0^\infty \|w\|_2^2$$

The parameter γ is said to be the H_∞-norm bound for the reliable H_∞ state feed back control.

Hence, to system (1), the robust and reliable H_∞ control problem is to construct the controller *u*, such that the closed-loop system (1) is tolerant and have H_∞ performance γ in the case of existing parameters uncertainties, external disturbances and actuator failures.

Actuators of a given system are classified into two sets. One set is susceptible to failures, denoted by $\Omega \subseteq \{1,2,\cdots,m\}$ hereafter. The other set is robust to failures, denoted by $\overline{\Omega} \subseteq \{1,2,\cdots,m\} - \Omega$. For simplicity, assume that actuators in this set never fail. Introduce a decomposition:

$$B = B_\Omega + B_{\overline{\Omega}}$$

where B_Ω and $B_{\overline{\Omega}}$ are formed from *B* by zeroing out columns corresponding to Ω and $\overline{\Omega}$, respectively.

Define the set of actual actuator failures of the given system as ω, which is a subset of Ω, that is, $\omega \subseteq \Omega$. Introduce a decomposition $B = B_\omega + B_{\overline{\omega}}$ which is similar to $B = B_\Omega + B_{\overline{\Omega}}$, in the case of existing actuator failures, the system (1) may be represented in the form:

$$\begin{aligned} E\dot{x}(t) &= [A + \Delta A]x(t) + B_{\overline{\omega}}u + Gw \\ y &= Cx \end{aligned} \tag{4}$$

and satisfies [1]:

$$B_\Omega B_\Omega^T = B_\omega B_\omega^T + B_{\Omega-\omega} B_{\Omega-\omega}^T \tag{5}$$

$$B_{\overline{\Omega}} B_{\overline{\Omega}}^T = B_{\overline{\omega}} B_{\overline{\omega}}^T - B_{\Omega-\omega} B_{\Omega-\omega}^T \tag{6}$$

For the description system, there is the following conclusion.

Lemma 1[3]**:** For the descriptor system

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + Bw \\ y &= Cx \end{aligned}$$

to guarantee that the system tolerant and have H_∞ performance γ, the necessary and sufficient conditions to be satisfied are:

(1) There exists $X \in R^{n\times n}$ such that

$$E^T X = X^T E \geq \mathbf{0}$$

$$X^T A + A^T X + C^T C - \frac{\mathbf{1}}{\gamma^2} XBB^T X < \mathbf{0} \tag{7}$$

or

(2) There exists $Y \in R^{n \times n}$ such that

$$YE^T = EY^T \geq \mathbf{0}$$

$$YA^T + AY^T + BB^T - \frac{\mathbf{1}}{\gamma^2} YC^T CY < \mathbf{0} \tag{8}$$

Main Results

For the uncertain descriptor system (1), there is the following conclusion:

Theorem 1: For given uncertain descriptor system (1), if there exist $\varepsilon > \mathbf{0}$, $\delta > \mathbf{0}$ and $X \in R^{n \times n}$ that satisfy

$$E^T X = X^T E \geq 0 \tag{9}$$

$$X^T A + A^T X + C^T C + \delta X^T D D^T X + \frac{1}{\delta} F^T F - \frac{1}{\varepsilon} X B_{\bar{\Omega}} B_{\bar{\Omega}}^T X < 0 \tag{10}$$

Hence, the controller

$$u = Kx(t) \tag{11}$$

$$K = -\frac{\mathbf{1}}{\mathbf{2}\varepsilon} B^T X \tag{12}$$

can make the closed-loop system to be tolerant and have H_∞ performance γ.

Proof: With the controller

$$u = -\frac{\mathbf{1}}{\mathbf{2}\varepsilon} B^T X x(t)$$

closed-loop system (1) is

$$E\dot{x}(t) = [A + D\Delta F]x(t) + B_{\bar{\varpi}} u + Gw$$

$$= [A + D\Delta F]x(t) - B_{\bar{\varpi}} \frac{1}{2\varepsilon} B^T X x + Gw$$

$$y = Cx$$

As

$$\left[\sqrt{\delta} X^T D - \frac{1}{\sqrt{\delta}} F^T \Delta^T \right] \cdot \left[\sqrt{\delta} D^T X - \frac{1}{\sqrt{\delta}} \Delta F \right] \geq 0$$

and also according to assumption A1: $\Delta^T \Delta \leq I$, there is the following inequality:

$$X^T D\Delta F + F^T \Delta^T D^T X \le \delta X^T DD^T X - \frac{\mathbf{1}}{\delta} F^T \Delta^T \Delta F$$
$$\le \delta X^T DD^T X - \frac{\mathbf{1}}{\delta} F^T F$$

We have

$$X^T [A + D\Delta F - B_{\varpi} \cdot \frac{1}{2\varepsilon} B^T X] + [A + D\Delta F - B_{\varpi} \cdot \frac{1}{2\varepsilon} B^T X]^T X + C^T C - \frac{1}{\gamma^2} XGG^T X$$
$$= X^T A + X^T D\Delta F - X^T B_{\varpi} \cdot \frac{1}{2\varepsilon} B^T X + A^T X + F^T \Delta^T D^T X - X^T B_{\varpi} \frac{1}{2\varepsilon} \cdot B^T X + C^T C - \frac{1}{\gamma^2} XGG^T X$$
$$= X^T A + A^T X + X^T D\Delta F + F^T \Delta^T D^T X - \frac{1}{\varepsilon} XB_{\varpi} B_{\varpi}^T X + C^T C - \frac{1}{\gamma^2} XGG^T X$$
$$\le X^T A + A^T X + \delta X^T DD^T X + \frac{1}{\delta} F^T F - \frac{1}{\varepsilon} XB_{\varpi} B_{\varpi}^T X + C^T C - \frac{1}{\gamma^2} XGG^T X$$

From (10), we can get

$$X^T [A + D\Delta F - B_{\varpi} \cdot \frac{1}{2\varepsilon} B^T X] + [A + D\Delta F - B_{\varpi} \cdot \frac{1}{2\varepsilon} B^T X]^T + C^T C - \frac{1}{\gamma^2} XGG^T X < 0$$

Considering (9), according to Lemma 1, the condition (7) is satisfied .We can draw the conclusion that closed-loop system (1) is tolerant and have the H_∞ performance γ.

Conclusion

This paper considers the reliable and robust H_∞ control problem of a class of descriptor systems with parametric uncertainties and external disturbances in the case of actuator failures. By solving the matrix inequalities, the controller can be obtained. In the case of existing parametric uncertainties and external disturbances, even if a part of the actuators fails, the system will still be tolerant and have the given H_∞ performance. This paper provides a controller design method for some control engineering systems with complex dynamics to achieve good stability.

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Biography

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