Overview and Design of Near Net-Formed Spherical Involute Straight Bevel Gears

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Abstract

Near net-formed straight bevel gears are used extensively in the automotive industry today. The design is mostly made by using tools developed for cut straight bevel gears. In this paper the forged (near-net formed) and cut straight bevel gears are compared in terms of the design, gear blank form, manufacturing process and durability (strength). In addition, the current design methodology is reviewed, and the necessary steps for the proper design of straight bevel differential gears (cut or net-formed) are proposed. Influence of the number of pinions, loading type (fully released, or fully reversed) and gear blank form on durability of the gears is investigated. Finally, modifications to the existing design approach are proposed to account for the specific shape of the forged gear blank form.

Introduction

Straight bevel gears are used for transmission of power between intersecting shafts positioned at, or close to 90º angles. They are mostly used in the relatively low-speed applications. Their most important application is in the field of automotive differentials.

Unlike other types of gears, straight bevel gears can be forged to finished tooth shape. The forged gears have several distinct advantages over their counterparts produced by cutting. Such gears have considerably higher material properties due to the undisturbed grain flow and additional support (webbing), they are much cheaper (in production environment) and faster to manufacture, they can be of higher quality, and the geometry of their teeth is not limited by the tooling or existing tooth generation methods.

Forging of straight bevel gears starts with the blank design, followed by the surface geometry (and micro-geometry) definition. In the next step the die is defined as the negative of the gear surface. Finally, the hot or warm billets are forged into the gears. Process chart of the straight bevel gear forging is shown in Figure 1.

This process has several challenges. First, the geometry of teeth must be accurately defined, because no additional machining of teeth will be performed after forging. Next, the coordinates of the gear surface must be modified to account for thermal distortion, elastic spring-back of material and pattern ease-off. The coordinates are then exported in the

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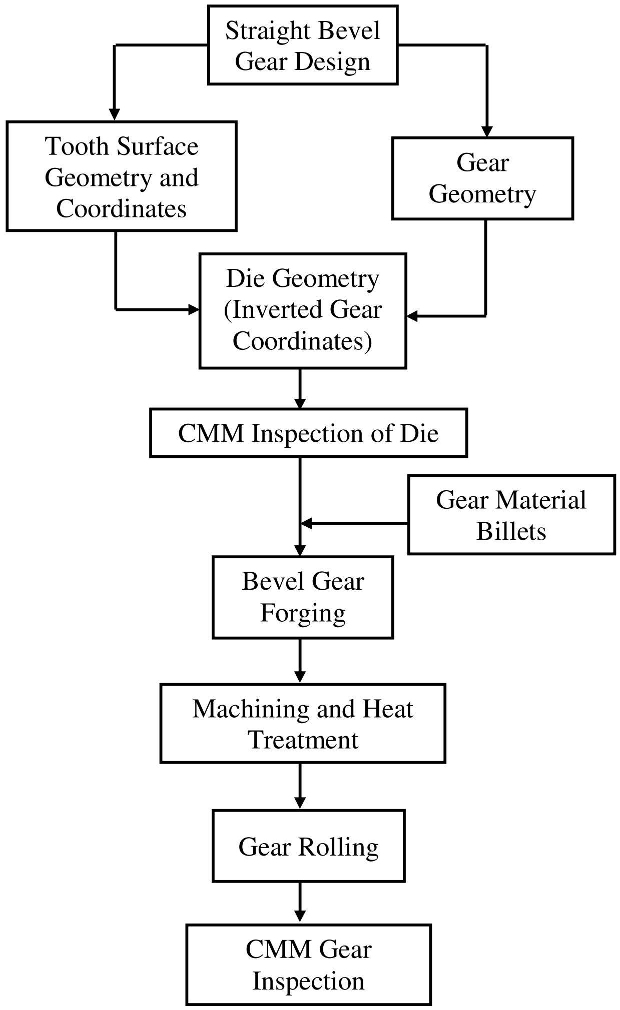

Figure 1: Process chart of straight bevel gear forging.

appropriate form to the CNC machines used in the manufacturing of dies, or electrodes used in die production. Finally, due to the risk of thermal distortion and die wear, the theoretical and actual gear tooth geometry must be frequently compared.

Comparison of Cut and Near Net-Formed (Forged) Straight Bevel Gears

Comparison of Design Processes

Both of the processes follow the same preliminary design, originally intended for cut gears. The process will be described in more detail in the next section. The design stage of the cut gears finishes with the *design summary* which also contains cutting machine setup. Tooth form of the forged gears is obtained in an additional step in which a specialized software is used to provide the coordinates of the surface points. The coordinates are used to design solid models of the gears, to perform contact analysis, and finally to design and manufacture forging dies.

Comparison of Production Processes

Cutting of straight bevel gears is a natural extension of the process used in manufacturing of other types of gears. The material from blank is generally removed by using circular cutters [1]. While the gear teeth in parallel axes applications (spur and helical) have involute tooth forms, such a tooth form is not easily applicable to production of straight bevel gears. The cutters for involute straight bevel gear applications would have to have slightly curved blades, instead of straight ones used in production of spur and helical gears. For that reason the straight bevel gears are usually manufactured by using different tooth forms, most often ones being Coniflex® (with octoidal tooth form) [1-3], and Revacycle® (with circular tooth form) [1-3]. Cutting of straight bevel gears in production requires specialized machines and cutters, and blanks either forged or cut from solid. When a small number of gears are required (prototypes), it is possible to use general milling machines for production of cut bevel gear prototypes [4].

Dies used in straight bevel gear forging are generally manufactured by using two methods, graphite electrode electro-discharge machining and direct milling. In EDM machining, graphite electrode (shaped as a gear) is fed into the solid block of material to produce a cavity of the die. Direct milling uses end-milling on CNC milling machines. Direct milling is regarded as a more accurate process of production [5] due to the fact that it requires smaller number of steps for gear forging. Finally, forged bevel gears are produced by hot or cold forming of cylindrical billets in two-part die. Cold forming yields better accuracy, but requires much larger forging presses and it is usually used for smaller size gears. For improvement of accuracy of hot forming, an additional cold forming (only minor deformation) can be added as a finishing step. Both of the above mentioned processes (cutting and forging) are followed by additional machining that excludes toothed portion of the gears (spline broaching or rolling, pinion bore machining and sizing, back face machining), and by heat treatment.

Production cycle time for manufacture of a straight bevel gear is much shorter when forging process is used. Generally, cutting would require 4 or more seconds per tooth [6] (yielding at least 40 seconds per gear in addition to mounting and dismounting time), while a forging requires approximately the same time for the manufacture of the whole gear. This advantage, in addition to the strength improvements to be mentioned later, makes the forging process a clear choice for straight bevel gear manufacture in production environment. On the other hand, bevel gear prototypes are more efficiently produced by gear cutting, where small changes are easily implemented. Forging process often requires more than one iteration (more that one die) to properly account for heat distortion and desired contact pattern.

Comparison of Durability

Increase in strength is the largest advantage of forged over cut gears. The bending fatigue strength (durability) benefit of gear forging was observed throughout the history, with test results ranging widely. The main two reasons for the improvement of durability come from the favorable microstructure of the forged material, and possibility to add webbing (reinforcement) to the forged gears.

Grain structure of the material remains mostly randomly oriented during cutting process. Flow of the material (grain flow) during forging creates a favorable grain structure that is capable of resisting higher loads. The estimates of bending fatigue life benefits of forged spur gears range widely – from 30% [7], over two times [8] to more than seven times longer life [9]. The contact fatigue improvement is also expected due to the compressive residual stress at the surface of the teeth.

The shape of the gear blanks produced by these two processes is rather different. This can be observed in Figures 2 and 3, showing cut and forged gear pair. While the tooth surfaces of both gear pairs must remain the same (to properly transfer power), the forged gear pair has additional material (*web*) added to its back (*heel*) and front (*toe*) portion. This additional material lowers the bending stresses further by 8 to 10% (according to FEA analyses of gears with and without webbing), which translates to up to 2 times longer bending fatigue life. Cut gears can not have such form, because the reinforcement (webbing) would be removed during cutting.

Figure 3 compares the shape of toe and heel portions of the cut and forged gears. Closer examination of the toe portions of pinion and gear reveals another potential problem with cutting of straight bevel gears. Namely, cut parts in the figure have very thin toe portions (~ 0.3 mm), and could not be used in a practical application. The possible ways to increase thickness of a toe (front) portion of the cut gears include reducing face width and reducing contact ratio. Reduction in face width increases stresses, while the decrease in contact ratio leads to the rougher transfer of power and possible premature failures due to dynamic (impact) loading.

In addition, forging offers a larger freedom in choosing the shape of the gear root region, which can lead to the further benefits in terms of strength. It can also prevent (or at least delay) undercut which becomes a limiting factor for gears with long face width.

Figure 3 shows that the length of the gear tooth tip (tip length) on the cut gears is larger than the corresponding length on forged gears. In order to decrease bending stresses, designers are tempted to further decrease the outside diameter of the gears to make more space for the reinforcement (webbing) on the back of the mating gear. This causes higher contact stresses at the tip and root regions of the gears, which can lead to the premature failures due to contact fatigue (pitting and spalling). In such case the tip length should be extended, and heel (back) webbing either reduced, or completely removed.

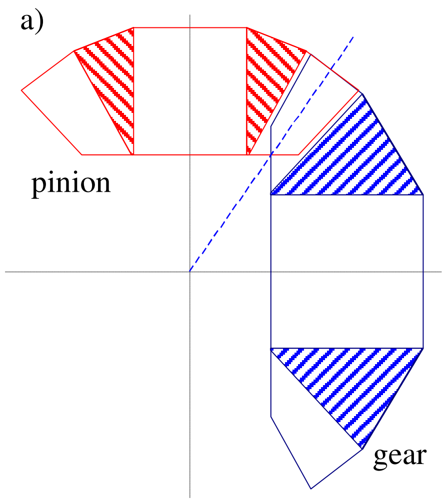

Figure 2: Example of a) cut and b) forged straight bevel gears

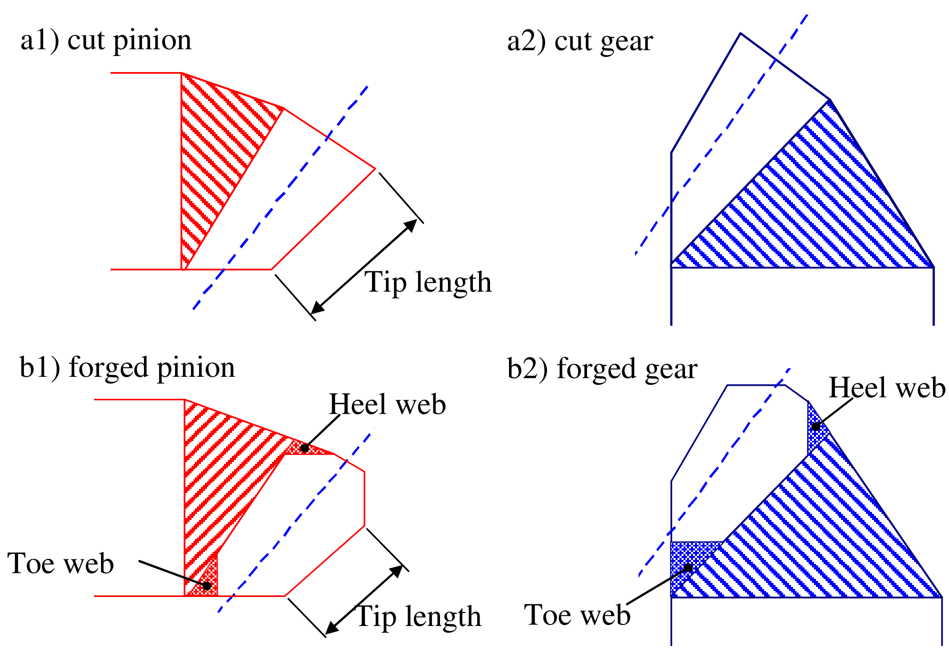

Figure 3: Comparison of toe and heel portions of a) cut straight bevel gears, and b) forged straight bevel gears

As a summary, there are considerable differences between cut and forged gears in terms of design procedure, production procedures, blank shape and performance. Forged gears have considerable benefits in almost all of these categories, and that makes forging a primary production process utilized in industry today. Unfortunately, a considerable portion of the existing design process was developed for cut gears. Changes to the design and development of analysis procedures will be addressed in the following parts of this work.

Design Procedure

Review of the Currently Used Design Procedures for Straight Bevel Gears

Straight bevel gears are traditionally designed by using either Gleason program package, or AGMA equations. The methods yield almost identical results, with several differences which will be pointed out later in this section.

Gleason, as a main manufacturer of gear cutting equipment and machines, offers a program package that, in addition to gear blank (design geometry) summary, also offers machine setup and cutting summary. The package consists of two main programs, A261 (used for Revacycle® design) and A201 (used for Coniflex® design).

Both of the Gleason programs utilize optimization procedure based on desired bending stress factor and tip thickness ratio in normal plane. Bending stress factor is based on the general Stress-Life (S-N) curve and it is used as a measure of the expected life ratio of the pinion and gear. The factors equal to 0 and 0.18 designate ‘equal stress’ (gear and pinion have equal bending stresses) and ‘equal life’ (gear and pinion have equal fatigue lives) concepts, respectively. In addition, the programs check if the gears are undercut, and it is another limiting factor in the design. The undercut, as defined by Gleason, is a function of both, gear and tool geometry, and for the forged bevel gears (which do not use cutting tool) the tool related undercut do not play any role.

The AGMA procedure [10, 11] for design of straight bevel gears is similar to the Gleason Coniflex® straight bevel gears. The procedures yield very similar results.

Design of Straight Bevel Differential Gears

Several modifications should be made to properly apply the mentioned procedures to the differential gears. They can be divided into two groups – modifications to account for the loading histories, and modifications to account for the specific geometry of the differential gears.

A typical differential consists of two sidegears and at least two pinions, Figure 4. The Gleason and AGMA procedures, being written for a general application, consider different arrangement in which one sidegear and one pinion are in contact. Assuming that the gear ratio (number of sidegear teeth divided by the number of pinion teeth) is equal to 1.5, such arrangement would result in the gear tooth load (stress) history shown in Figure 5a. The stress

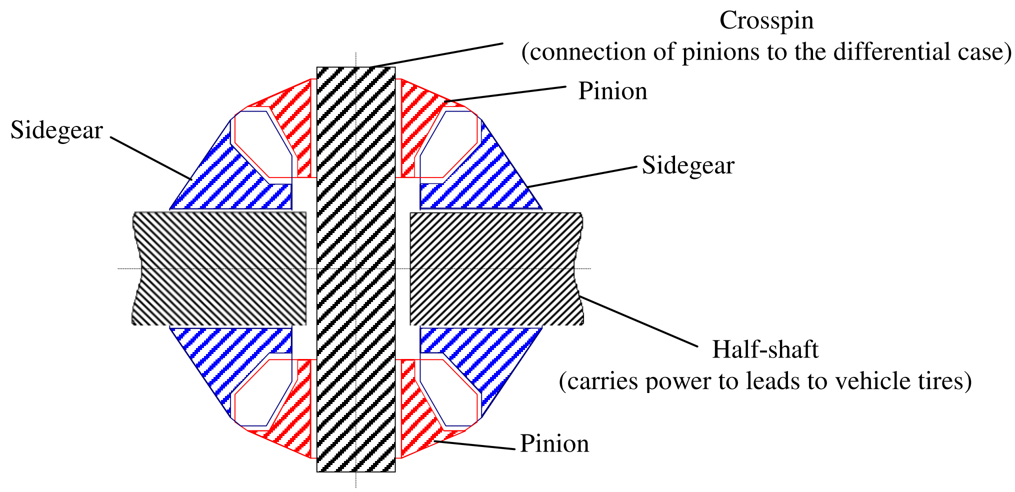

Figure 4: Straight bevel gears in a differential assembly (differential case not shown)

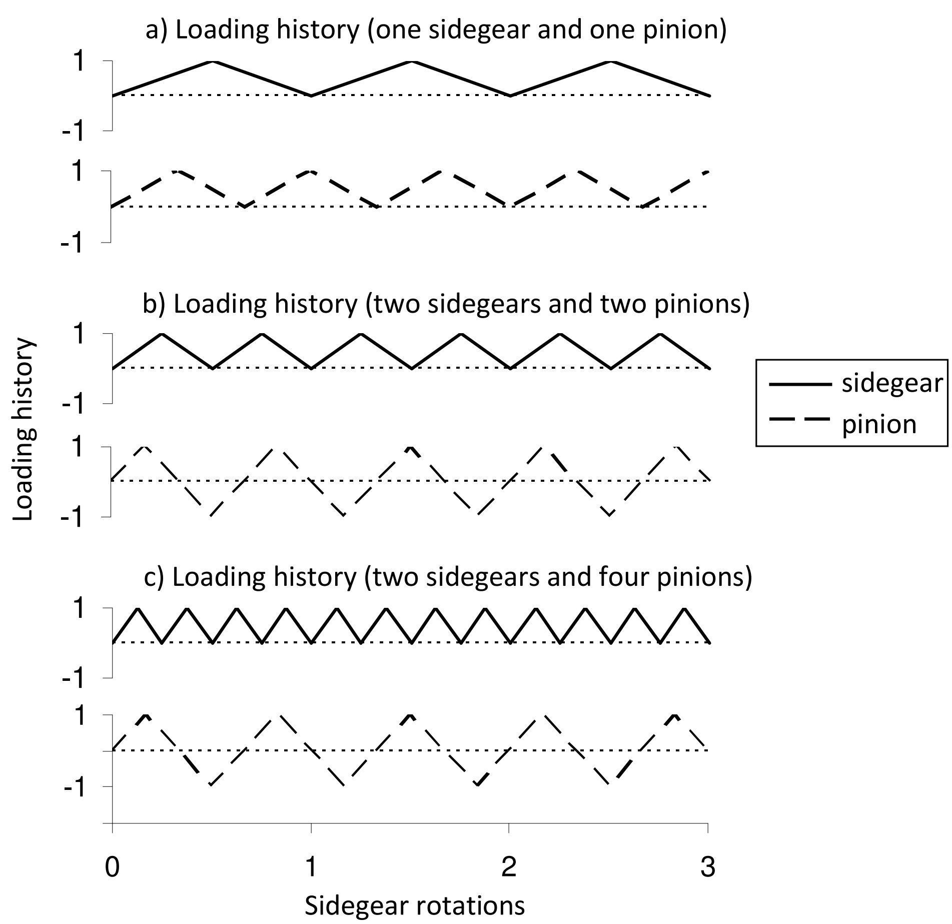

Figure 5: Loading history in the system with a) one sidegear and one pinion, b) two sidegears and two pinions, and c) two sidegears and four pinions

history of the gears in the differential assembly from Figure 4 would be represented by Figure 5b. Comparing loading of the sidegears in Figures 5a and 5b reveals that the sidegear in the differential with two pinions experiences two times larger number of cycles. In the differential with four pinions (Figure 5c), the sidegear would experience four times larger number of cycles. This difference in the cycle count could simply be taken into consideration after obtaining results (stresses) from the design procedures. The modified number of cycles could be then used in the bending and contact fatigue analysis to predict the durability of the differential sidegears.

The Figures 5a and 5b show pinion tooth loading histories with completely different character. The pinion in a differential assembly contacts two sidegears by the opposite sides (flanks) of the teeth. Such arrangement results in the *reversed loading* (Figure 6b) which causes more damage in the root region of the teeth than the loading of the pinion in contact with one sidegear (Figure 5a). Bending fatigue analysis must be evaluated by using a reversed loading cycle for each pinion revolution. Due to the fact that it contacts two sidegears with the opposite flanks, the differential pinion accumulates one contact fatigue cycle per rotation.

The above review of the loading histories shows that it is necessary to additionally process the results (stresses) obtained from the currently available design procedures. Traditionally, strength of pinion and sidegear is judged by calculating factor F , proposed by Gleason

$$F = \frac{\log(\frac{\sigma_g}{\sigma_p})}{\log(\frac{N_{tg}}{N_{tp}})} \tag{1}$$

where σ_g and σ_p are bending stresses on sidegear and pinion, and N_{tg} and N_{tp} are their respective number of teeth. It is easy to show that the factor becomes equal to zero when the stresses on pinion and sidegear are equal to each other. Life of a component can be predicted by using Basquin's equation (2)

$$\sigma_{ai} = \sigma_f^{'} (N_i)^b, \tag{2}$$

$$\sigma_a = (\sigma_{\max} - \sigma_{\min})/2 \tag{3}$$

$$\sigma_m = (\sigma_{\max} + \sigma_{\min})/2 \tag{4}$$

where N_i and σ_{ai} and are predicted life (number of cycles) and alternating stress (number of cycles) of the component *i*, $\sigma_f^{'}$ and b and material properties, σ_a is alternating stress and σ_m mean stress component. The equation (2) is created by using *fully reversed loading* in which test specimen is cyclically loaded in tension and compression (Figure 6b,

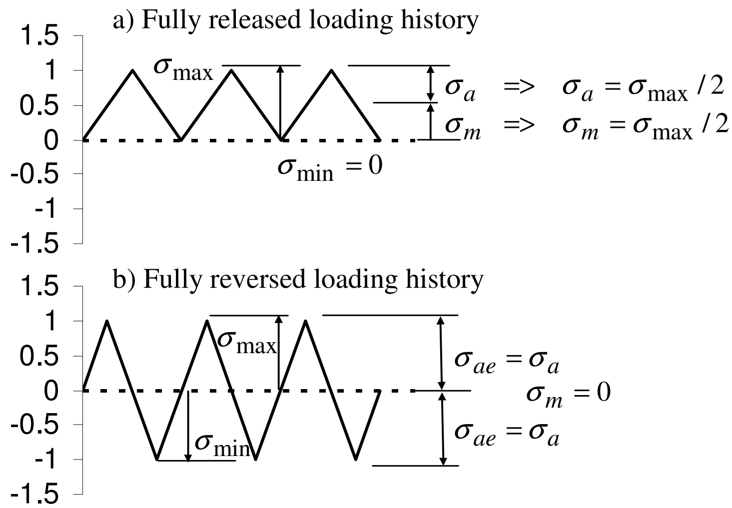

Figure 6: Example of a) fully released loading, and b) fully reversed loading.

$R = \sigma_{min} / \sigma_{max} = -1$, $\sigma_a = \sigma_{max}$ $\sigma_m = 0$), but it can also be created by using load history with any other ratio R. The material constants $\sigma_f^{'}$ and b determined from the tests are valid only for the test stress ratio R. From the previous discussion, it is clear that the same SN curve can not be used for pinions and sidegears (due to the different ratio R of their load histories), and the stresses must be first converted to the *equivalent alternating stress* by using Goddman's equation

$$\sigma_{ae} = \frac{\sigma_a \sigma_u}{\sigma_u - \sigma_m} \tag{5}$$

where σ_u is the *ultimate tensile strength* of material. Now, σ_{ae} can be determined from equation (2) for known (or desired) life N_i. Assuming that the ratio R is known, the maximum stress, which is also the result of the Gleason or AGMA procedures, can be obtained from

$$\sigma_{max} = \frac{\sigma_{ae}\sigma_u}{\left(\frac{1+R}{2}\right)\sigma_{ae} + \left(\frac{1-R}{2}\right)\sigma_u}. \tag{6}$$

Number of pinions in differential assembly, n_p, can be taken into account by using expression

$$N_g = N_p n_p \left(\frac{N_{tp}}{N_{tg}} \right), \tag{7}$$

where N_g is the number of cycles that sidegear teeth experience when the sidegear is rotated while the differential case is held stationary (fixed). If the number of sidegear revolutions with respect to differential case, $N_{g/c}$, is known, then the number of sidegear cycles can be obtained as $N_g = N_{g/c} \cdot n_p$. Finally, equation (7) can be used to calculate number of pinion cycles for the desired number of sidegear revolutions.

Expressions (2), (5), (6) and (7) can be used to calculate the desired factor F by using expression (1) and maximum stresses for sidegear and pinion in place of σ_g and σ_p. As an example, let us use material 8620H (case) with properties $\sigma_f' = 1790$ MPa, $\sigma_u = 1600$ MPa, and $b = -0.109$ [12]. These material properties were not correlated to the program and they are used here just for the illustration of the procedure. In general, the test results of already existing gears, preferably in differential assembly should be used to obtain the Stress-Life curve. The system in this example have sidegear number of teeth $N_{tg} = 15$, pinion number of teeth $N_{tp} = 10$ and number of pinions $n_p = 2$. For desired life of 10×10^3 sidegear revolutions with respect to case, the number of sidegear cycles is 20×10^3, and number of pinion cycles is 15×10^3 (equation (7)). Using equation (2), the equivalent alternating stress for pinion and sidegear becomes 608 MPa and 627 MPa, respectively. From equation (6) , and with ratio $R = -1$, the maximum stresses for sidegear and pinion are found to be 881 MPa, and 627 MPa, respectively. Finally, the factor F for equal life of the gears is calculated to be 0.84. The value is rather high, and it could be considerably different with actual values for material (experimentally determined), and ratio R.

As shown in Figure 5b and 5c, the number of pinions have influence on the load history of the sidegear, and accordingly on the factor F. If the number of pinions is increased to $n_p = 3$ and $n_p = 4$, the F factor becomes 0.76 and 0.70, respectively. It should be mentioned that the case with 4 pinions would not be used in this case because it would not be possible to place them equally (at 90° angle from each other). Generally, the possibility of equal spacing of pinions can be checked by using the following rule

$$\frac{2N_{tg}}{n_p} = \text{integer} . \tag{8}$$

Final remark regarding the application of the currently available design procedures comes from the root (fillet) radius of the near-net formed gears. These gears do not depend on the cutting tools and can have considerably different fillet radius from the cut gears. It would be desirable to use the actual fillet radius, which can be done by modifying AGMA procedure [10, 11]

Contact stress on the differential gears is another important parameter in the differential gear design. These gears are heavily loaded and they rotate slowly, preventing proper elastohydrodynamic film formation. In such case the metal-to-metal contact leads to high contact stresses and failure due to spalling. Generally, contact stress depends on three parameters: contact line length, relative curvature (used in Hertzian contact pressure calculation) and applied load. Differential gears usually have reduced outside diameter to reduce the size requirement of the differential case window through which the gears are placed into the case. In addition, as shown in Figure 3, the tip length of the forged gears is generally shorter than the length of their cut counterparts (to make space for webbing reinforcement). Neither Gleason nor AGMA procedures takes into account the influence of shorter tip (i.e. contact line) length on the contact stresses. Both of the procedures calculate the stress factor (Gleason), or stress (AGMA) at the *Lowest Point of Single Tooth Contact* (LPSTC) or *Highest Point of Single Tooth Contact* (HPSTC). The applied load above LPSTC, or below HPSTC is carried by a single tooth. Two teeth share load outside these borders, so it is expected that the contact stress would get lower. At the same time the radius of curvature of a tooth surface gets progressively smaller when moving from tip towards the root. With the low tip length and small curvature radius, it is possible to obtain very high contact stress at the place where the tip of one gear touches the mating gear (*Start of Active Profile* or SAP). The contact stresses (of unmodified surfaces) can be calculated based on the profiles of *equivalent spur gears*, Figure 7. The curvatures, as well as sliding and rolling velocities of the *equivalent spur gear* teeth surfaces can be calculated at several points along the profile [15, 16, 17] and used to understand the influence of tooth truncation and geometry on the contact stresses. Table 1 shows that the contact line (close to pinion root) is reduced, and contact stresses are increased when the sidegear outside diameter is reduced from 104.8 to 93 mm.

Conclusion

Straight bevel gears produced by using two methods, cutting and forging, were compared in this paper. Special attention was given to straight bevel differential gears. Current design procedure was reviewed and modifications were recommended to account for the factors that influence durability of the differential gears: number of planets, blank shape and root geometry. Modification of the contact analysis procedure was proposed to account for modified blank geometry (reduced outside diameter of the gears).

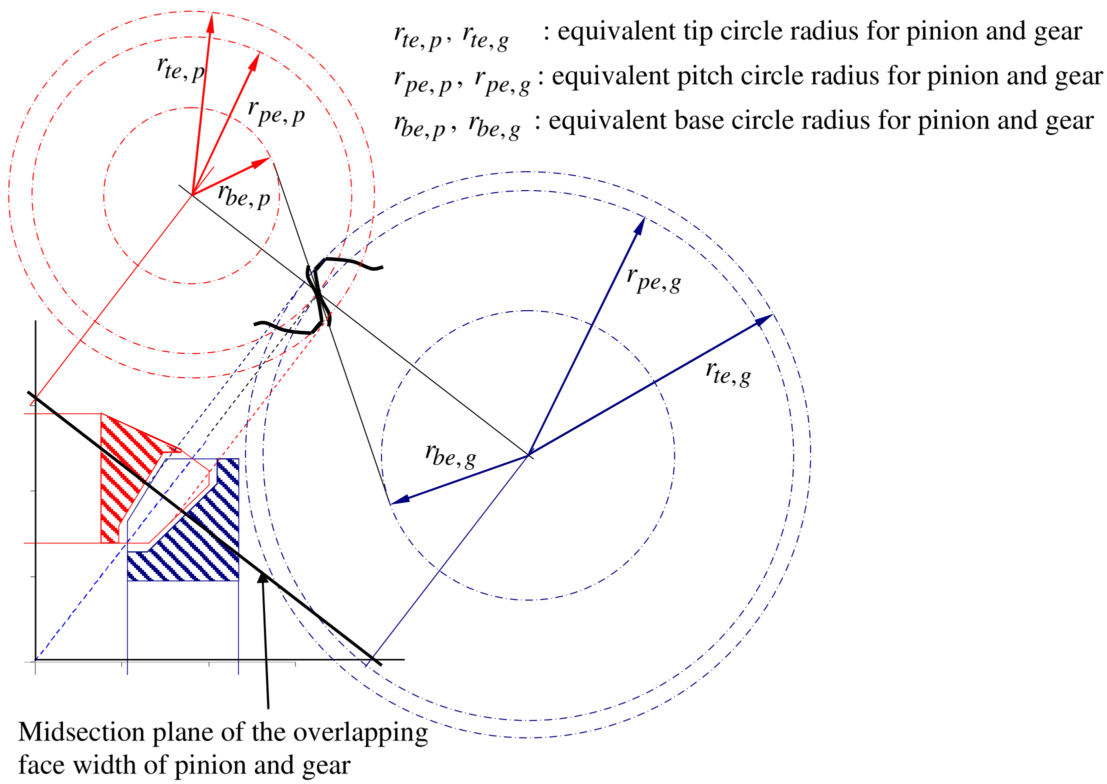

Figure 7: Definition of the equivalent spur gear at the midsection of the overlapping face width of pinion and gear

Table 1: Influence of the sidegear outside diameter on the contact stresses

	Sidegear outside diameter [mm]					
	104.8		100		93	
	Contact stress, [GPa]	Tip length [mm]	Contact stress, [GPa]	Tip length [mm]	Contact stress, [GPa]	Tip length [mm]
Pinion SAP	2.58	15	2.85	12.3	3.47	8.3
Pinion LPSTC	2.72	18.7	3.01	17.9	3.66	13.8
Pitch line	1.5	21.9	1.5	21.9	1.64	18.1
Pinion HPSTC	2.17	17.8	2.17	17.8	2.17	17.8
Pinion EAP	1.69	15.3	1.69	15.3	1.69	15.3

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Biography

DR. HARIS LIGATA earned his PhD degree in 2007 from Ohio State University (Gear and Power Transmission Research Laboratory) for his work on influence of manufacturing errors on performance of planetary gear sets and transmissions. The same year he joined American Axle & Manufacturing in Detroit to work on design and development of parallel axis and bevel gears. In 2008 he was sponsored by the same company to conduct research project at

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