A Generalized Model for Cost of Manufacturing: A Deviation-based Formulation

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ABSTRACT

Realistic modeling of cost of manufacturing is essential both for estimating cost of a product and for optimizing alternate manufacturing strategies. Although there are many factors that affect cost of manufacturing, the single most important parameter for estimating the cost of manufacturing is tolerance (deviation from the nominal size). Since tolerance affects the manufacturing cost directly, in the past most cost formulations were constructed using some form of inverse power function of a single tolerance parameter. However, to meet stringent quality and functional requirements more parts are now being controlled with geometric tolerances on top of size tolerances. In these situations, it is very difficult to establish a single representative tolerance parameter that can be used for estimating cost of manufacturing. A more general formulation is needed to extend the cost functions to deviations of all features of a part and the mating relations between parts of an assembly so that the total effect of the prescribed geometric tolerances could be captured. In this paper, existing single parameter cost formulations have been reviewed and then a generic deviation-based cot of manufacturing formulation has been presented. The constraints and advantages of extending the cost functions to multi-dimensional formulations have also been briefly discussed.

INTRODUCTION

Estimation of cost of manufacturing a part before the actual production (during the design phase) is essential to optimize alternate manufacturing strategies and to control the cost of a product. There are many factors that affect cost of manufacturing. Some of them are directly attributable to the manufacturing process and the desired level of tolerance. From a manufacturing point of view tolerance is the single most important parameter as the tolerance value dictates the cost: tighter the tolerance, higher the cost and vise-versa. This cost-tolerance relation is used in most cost formulations by constructing some form of an inverse power function of a single tolerance parameter.

Prior to introduction of geometric tolerances, only the size tolerance (plus-minus tolerances) used to be specified for manufacturing parts. However, prescribing geometric tolerances on top of traditional size tolerances to meet stringent quality requirements, functional requirements and assemblability of parts has become very common engineering practice. In these situations, it is very difficult (often impossible/impractical) to establish a single representative tolerance parameter that can be used for estimating cost of manufacturing. For example, if we look at the single plus-minus size tolerance

(20±0.5 mm) (Figure 1) for the length of the smaller cylindrical feature and consider only the variation of length (axial deviation) as the parameter, we could use just one tolerance parameter to establish the cost function for machining the circular face. However, the part as given in Figure-1 has a positional tolerance (specified as per ASME Y14.5M [1]), that uses multiple datum references and also it includes maximum material conditions (MMC) both for the geometric tolerance as well as for the datum A. This leads to conditions that would need establishment of a virtual condition boundary (VCB) for the cylindrical feature and bonus tolerances and allowable datum shifts (datum A) corresponding to the actual manufactured envelope (AME) of these features have to be taken into account to find the total tolerance. This is a relatively simple case of a single part with one geometric tolerance; even then, in this case establishing an explicit single tolerance parameter that could represent the behavior of the cylindrical feature is difficult, specifically when we want to study various alternative configurations that this part could be manufactured maintaining assemblability with other parts. In these cases a more general formulation that could take into account variation of all parameters of the geometric tolerances would be desirable. Unfortunately no such universal formulation is available. In order to circumvent these difficulties, an extension of the cost functions from the tolerance domain to a generalized deviations domain is proposed so that the total effect of geometric tolerances prescribed for all features of a part and also the effect of mating relations between parts in assembly could be captured.

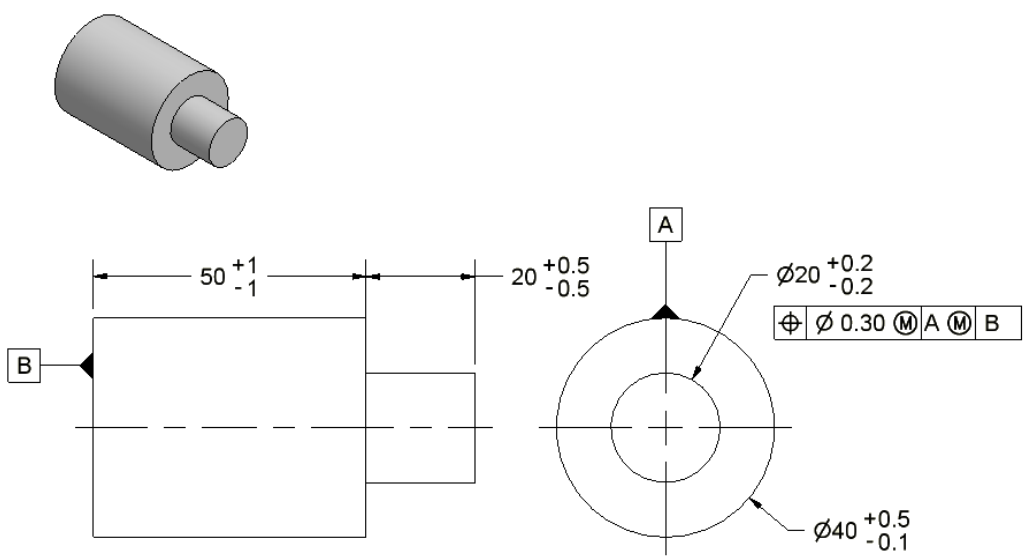


Figure -1: Geometric tolerance specified using feature control frame (FCF)

In this paper, existing single parameter cost formulations are reviewed and then an extension of cost formulations to a generic deviation-based formulation is presented. A brief discussion on how the deviation-based formulations could be used and what are the advantages and disadvantages of extending the function to such multi-dimensional formulations are presented. First, the basic factors that affect cost of manufacturing are considered.

Costs associated with manufacturing a part are dependent on several parameters including tolerance specification, material, dimensions, geometric shape, sequence of manufacturing operations, and the process capabilities. Many researchers have analyzed various issues associated with the cost of manufacturing in relation to tolerances. There are various methods to formulate these cost function [2]. Essentially all cost of manufacturing formulations are monotonically decreasing functions of a single tolerance parameter. Effects of various functional forms like inverse power law, exponential decay, etc., and the effect of process capabilities have been studied by many researchers [3].

Since in general, more than one operation is required to transform the raw blank into the final finished part, the cost of manufacturing is a function of the process sequence and how much accuracy is achieved in each stages of operation. This cost is also affected by the setup error in each machining process. The total cost of production thus becomes a sum of the costs associated with each process [4].

While none of the methods mentioned above could claim to be universal, there are several limitations with the one-parameter (single tolerance) cost of manufacturing formulation. In reality, a manufactured surface would rarely have a single tolerance value. Apart from a size tolerance, there could be more than one geometric tolerances (form, positional, and orientation tolerances, etc) prescribed for a feature and it would be difficult to formulate a single parameter representing all these tolerances that could effectively be used for representing the cost of manufacturing. The proposed deviation-based formulation could eliminate these problems and effectively model the cost in a generic manner.

REVIEW OF SINGLE-PARAMETER COST FORMULATIONS

Traditional one dimensional cost-tolerance models have typical shapes as shown in Figure-2. Most of these formulations are some form of power law, some are exponential decay functions and there are some formulations with piece-wise continuous curves (Table-1) [3].

In table-1, the equations represent cost per part. **T** is the tolerance parameter and symbols **A, B, m, k**, are constants. The A term represents fixed costs, such as tooling, setup, etc. and the B term express cost of producing single component dimension for the specified tolerance T. Complete details of most of these formulations and comparison of these models are found in [5].

Each of the above formulations has been used by the authors for optimal tolerance allocation using suitable optimization methods as indicated in the table. These methods have their limitations and suitability for specific applications.

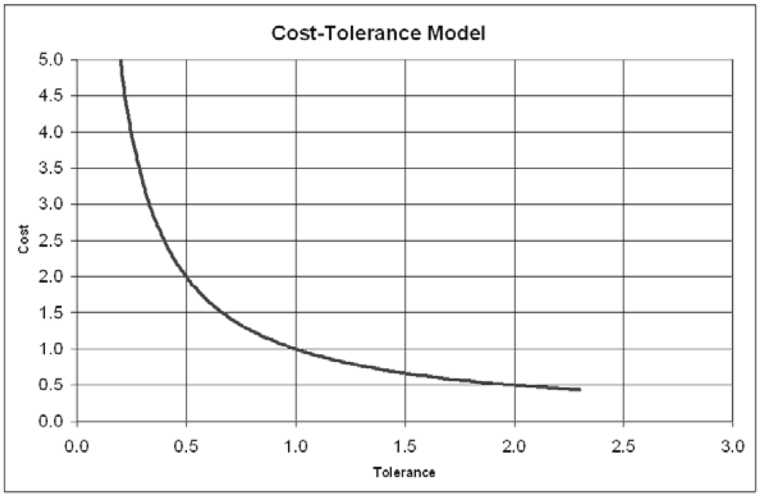


Figure-2: Typical Shape of One-dimensional Cost-Tolerance Function

In case of multi-process operations, cost functions for each process would be different. Also if there are alternate processes available for producing a part, cost tolerance models for each process needs to be considered separately. In these cases, cost models for each process could be imposed to define the operating zones for each process (Figure-3). Procedure for tolerance allocation calculations using these functions has been detailed in [3].

Table-1: Various Cost Tolerance Formulations from [3]

Model	Equation	Method	Author
Linear	$A - B\,T$	Linear prog	Edel and Auer
Reciprocal	$A + B/T$	Lagrange mult	Chase and Greenwood
Reciprocal Squared	$A + B/T^2$	Lagrange mult	Spotts
Reciprocal Power	$A + B/T^k$	Lagrange mult	Sutherland and Roth
	B/T^{k_i}	Nonlin prog	Lee and Woo
Exponential	$B\,e^{-mT}$	Lagrange mult	Speckhart
		Geom prog	Wilde and Prentice
		Graphical	Peters
Expon/Recip Power	$B\,e^{-mT}/T^k$	Nonlin prog	Michael and Siddall
Piecewise Linear	$A_i - B_i\,T_i$	Linear prog	Bjork, Patel
Empirical Data	**Discrete points**	Zero-one prog	Ostwald and Huang
		Combinatorial	Monte and Datseris
		Branch & Bound	Lee and Woo

DEVIATION-BASED COST OF MANUFACTURING

Before the deviation based cost of manufacturing formulations are introduced, the concept of a) generic deviations of a feature and b) mapping between deviation parameters and tolerance parameters are needed. For brevity, these concepts will be described in brief here and the relations that have already been established in our earlier works [6-10] would be used.

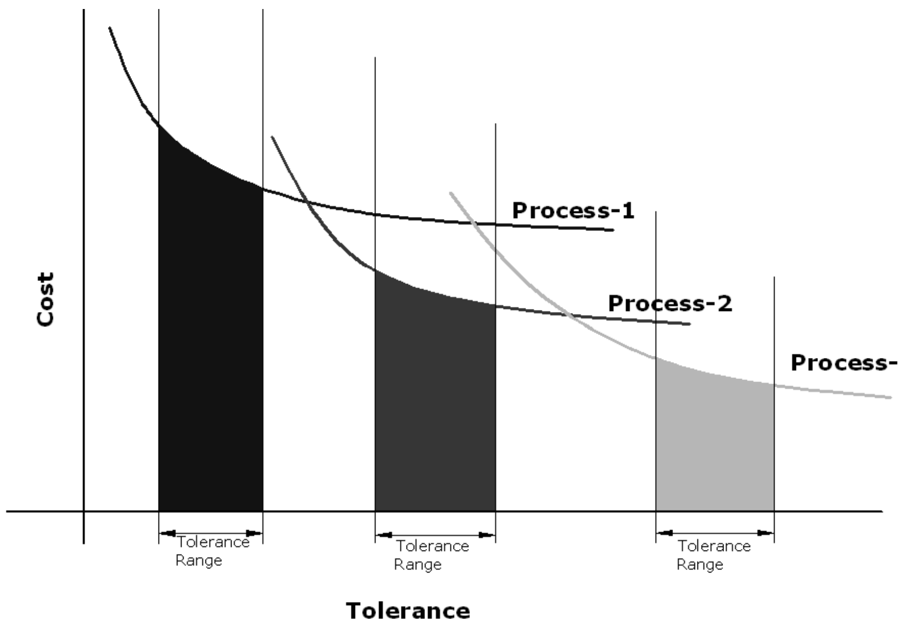
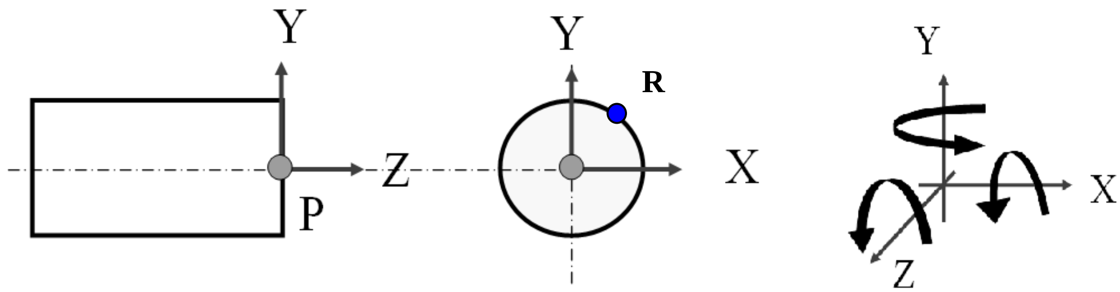


Figure-3: Multi-process Cost-Tolerance Functions [3]

The concept of representing the variations of a feature in terms of small deviations at strategic points on a feature has been used by many researchers [11-12]. These deviations are called small displacement torsors (SDT). Each SDT has six components (three linear, three rotational) corresponding to the six degrees of freedom (DOF) associated with a feature. Although these SDTs look like 6-component vectors (Figure-4), they are not true vectors as three of the components are linear and the other three are rotational and they follow a different transformation rule [11]. One interesting aspect of these torsors is that even though they have six components, only one to four of the six DOF are required to represent the behavior of standard engineering features. For example, to represent the axial displacement of a cylindrical rod, only one linear component is required and to represent the variation of a planar surface only three of the six parameters are required. In figure-4, a point P is used at the center of the circular face and the torsor D_P is used to represent the behavior of the circular-planar feature. The first D_P shows the general 6-components and the next one shows the three displacement components (one linear displacement along the Z-direction and two rotational components along the X and Y directions) that are relevant for the planar feature. These three parameters could be used to define the displacement of any point R on the circular face.

When the displacements are small, the torsors could effectively represent the behavior of the feature. These SDTs have been used here as the deviation parameters to formulate the cost of manufacturing functions. However, since the deviation parameters are not directly used to prescribe geometric tolerances, a mechanism is needed to transform these deviation parameters into tolerance parameters so that the final results could be converted to tolerance specifications conforming to ASME Y14.5M geometric tolerancing standards.



$$D_P = (X_{rot} \;\; Y_{rot} \;\; Z_{rot} \;\; X_{disp} \;\; Y_{disp} \;\; Z_{disp})^T$$

$$\Downarrow$$

$$D_P = (X_{rot} \;\; Y_{rot} \;\; - \;\; - \;\; - \;\; Z_{disp})^T$$

Figure-4: Deviation Components of a Planar Cylindrical Feature

This transformation from the deviation space to the tolerance parameter space is carried out using suitable mapping schemes as described below.

It has been established [8-10] that the geometric tolerance specified as per ASME Y14.5M could be mapped to the generic deviation parameters through a series of explicit or implicit transformations. These mapping relations become a set of constraints that restrict the domain of the deviation parameters in the deviation space. Cost functions defined in terms of the deviation parameters must remain within these zones. In the remaining part of this section, the development of a cost of manufacturing formulation in terms of the deviation parameters is discussed.

For modeling the cost of manufacturing, some suitable form of generic function of the deviation parameters could be used and the mapping between the deviation parameters and the tolerance parameters could then be used to link the cost function to the tolerance specification. The notation (θx, θy, θz, δx, δy, δz) is used here to represent the six components of a SDT. Thus, the cost function $C(\delta)$ defined as a function of some tolerance parameter δ, would become a function of the six deviation parameters $C(\delta) = C(\delta(\theta x, \theta y, \theta z, \delta x, \delta y, \delta z))$.

There could be various forms/structures for these functions depending on specific surface features and manufacturing processes and experimental results would be needed to establish typical functions for domain-specific applications. In this work, the cost of manufacturing a part is shown as an explicit product of six functions of the six deviation parameters in the form:
$C(d) = C_1(d1)*C_2(d2)*\ldots.C_6(d6)$ where $d = (d1, d2, d3, d4, d5, d6) = (\theta x , \theta y , \theta z , \delta x , \delta y , \delta z)$ is the deviation parameters characteristic of the feature.

Depending on the nature/type of the feature, some of the functions will be constants (invariants) and could be eliminated, for example, for some j, we could use $C(dj) \equiv 1$. This will correspond to the deviation parameters that are invariants of the feature.

As for an example, as was mentioned earlier in this section, for a planar surface there are only three independent parameters represented by $d = (0, \theta y, \theta z, dx, 0, 0)$ that affect the deviation of the surface from its nominal shape. The cost function for such a planar feature can then be represented as: $C(d) = C_x(dx)*C_\theta(\theta y)*C_\theta(\theta z)$

Also, in this case, the form of the two functions for rotational components along the y and z-axes could be of same structure.

For example, for a rectangular planar section with cross-section (2a x 2b), (Figure-5) the mapping relations (from [10]) could be written as:

$$T_{SL} \leq \min (dX+a*\theta y + b* \theta z, dX + a*\theta y -b* \theta z, dX- a*\theta y + b* \theta z, dX- a*\theta y - b* \theta z)$$
$$T_{SU} \geq \max (dX+a*\theta y + b* \theta z, dX + a*\theta y- b* \theta z, dX- a*\theta y+ b* \theta z , dX-a*\theta y - b* \theta z)$$

where (T_{SL}, T_{SU}) are the lower & upper values of the tolerance parameter for the planar surface.

Above relationship could be considered as constraints (restrictions) on the parameters $(dx, \theta y, \theta z)$. The cost function will be valid within the restricted zone.

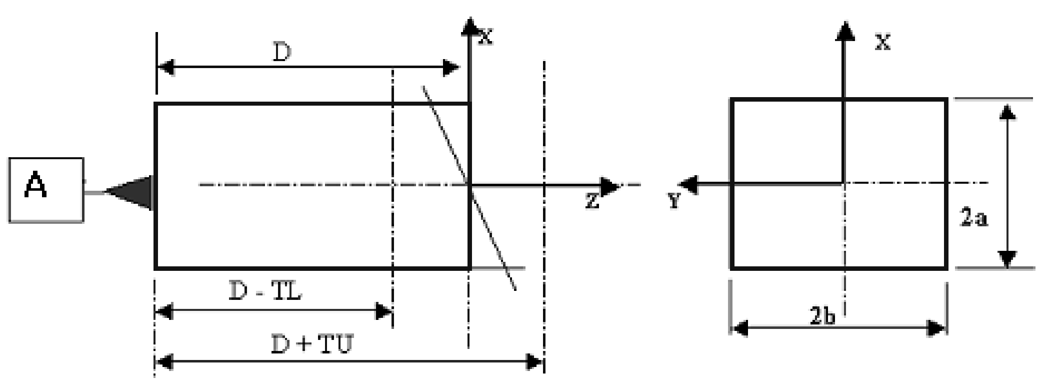


Figure-5: A rectangular-planar feature with a plus-minus size tolerance

To illustrate the cost function, a generic function of the form: $C(x) = f_1 + f_2/(\lambda+|x|)$ is considered, where $\lambda > 0$ is a small constant used to avoid singularity at origin $x = 0$, x is the deviation parameter and $f_1 > 0$ and $f_2 > 0$ are cost constants. Thus, the cost function becomes

$C(d)= Cx(dx)*C\theta(\theta y)*C\theta(\theta z) = (f_{11}+f_{12}/(\lambda+|dx|)) * (f_{21}+f_{22}/(\lambda+|\theta y|)) * (f_{31}+ f_{32} / (\lambda+|\theta z|))$.

For a visual representation of this function as a surface in 3D, removing the θz term, assuming a= b=1, and using $dx=d$, $\theta y=\theta$, the constraints become: $-T_{SL} \leq (d + \theta) \leq T_{SU}$ and $-T_{SL} \leq (d - \theta) \leq T_{SU}$ and the cost function is: $C(d)= (f_{11}+f_{12}/(\lambda+|d|)) * (f_{21}+f_{22}/(\lambda+|\theta|))$

In the d-θ plane, this would look like a tent bounded by four vertical planes defined by the four limits from the tolerance specification (Figure-6) and constant cost contours would look as shown in Figure-7 and the area enclosed by the four constraint lines would be the mapped zone within which the cost function would be valid.

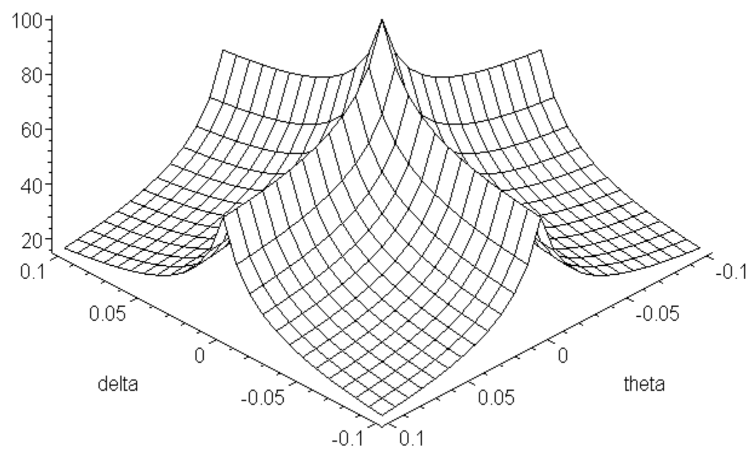


Figure-6: Example of a deviation based cost function

GENERAL OBSERVATIONS

The extension of the cost functions to the deviation-based formulations as shown here are generic and could be used wherever a cost of manufacturing model is required. However, the overhead for using such generalization is two-fold: 1) more parameters are used to represent the function and as a result, computational complexity increases, and 2) suitable mapping relations are required to transform the deviation parameters to the tolerance space. In our earlier work [10] we have developed mapping relations for planar, cylindrical, and spherical features at various geometric tolerances with material conditions (MMC, LMC and RFS).

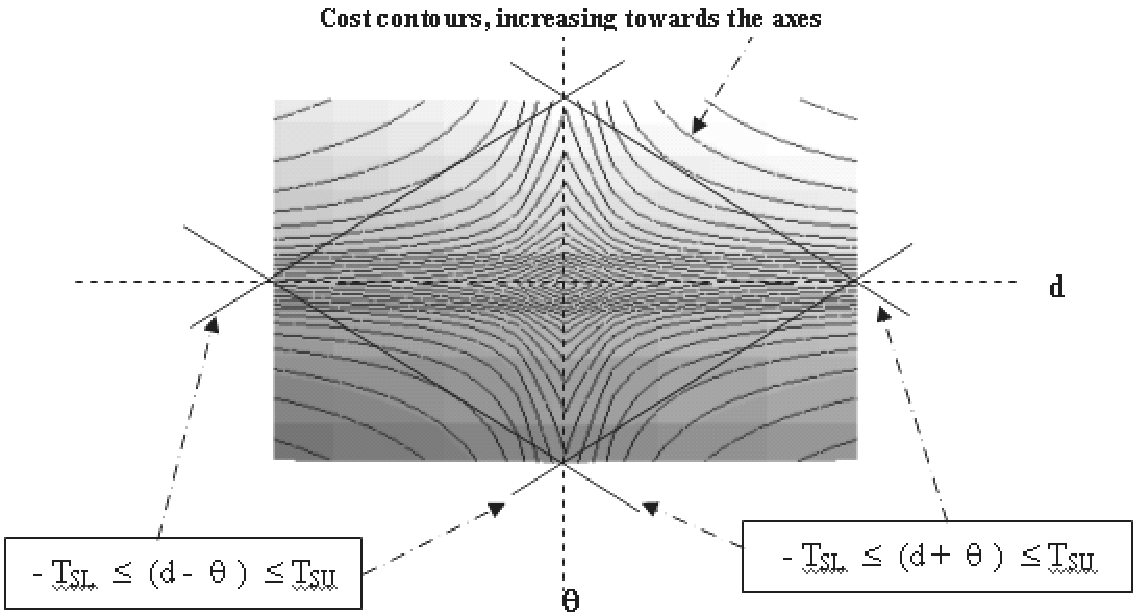


Figure -7: Example cost function contour lines and the four constraints.

CONCLUSION

In this paper we have discussed the limitations of single parameter cost models and presented a procedure for developing a generic deviation based cost of manufacturing formulation. This formulation has been successfully used [9] for synthesis of geometric tolerances of several assembly models including a planetary gearbox. This method could be used for representing the cost of manufacturing for various material removal processes like turning, milling, broaching, etc. However, in order to establish process-specific formulations, experimental data has to be collected and fitted to the proposed model.

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