COMPUTATION OF SHOCKWAVE STRUCTURES IN WEAKLY IONIZED GASES BY SOLVING BURNETT AND MODIFIED RANKINE-HUGONIOT EQUATIONS

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Abstract

The modified Rankine-Hugoniot equations across a standing normal shockwave were discussed and adapted to obtain jump conditions for shockwave structure calculations. Coupling the modified Rankine-Hugoniot equations with the Burnett equations, the shockwave structure in a weakly ionized gas flow was computed and analyzed for a wide range of free-stream Mach numbers ranging from 1.75 to 6.0, with ionization ratio ranges from 0 to 5 parts per million. Results indicated that the modified Rankine-Hugoniot equations for shockwave structures in weakly ionized gas are valid for a small range of ionization fractions at low free-stream Mach numbers. The jump conditions also depend on the value of free-stream pressure, temperature and density. The computed shockwave structure with ionization indicated that by the introduction of the weakly ionized gas particles in the main flow field, shockwave thickness was slightly increased and shockwave strength could be reduced.

Introduction

Over the past two decades, scientists [1], [2] have observed that hypersonic shockwaves could be altered by weakly ionizing the air flows. Shadowgraph experiments conducted at AEDC [3] showed that with a small amount of pre-ionization—about one part per million background ionization fraction—when shockwaves pass through the weakly ionized air, a normal Mach 6 shockwave was transformed into a Mach-3-like shockwave. The shockwave stand-off distance was also increased. Other researches [4], [5] indicated that with small ionization levels, typically at about one part in a million by mass, the strength of the bow shock ahead of the supersonic projectile such as sphere was reduced, the net drag on the sphere was reduced, and the shockwave stand-off distance increased. In addition to the effect of gross shockwave properties, the shock structure was also modified including a weakening shock front and a broadening shockwave thickness [5]. The remaining question was how the weakly ionized gas could achieve drag reduction and how it could alter the shockwave structure.

Shockwaves are regions of gas flow with discontinuities or strong gradients in pressure, temperature, density and velocity. The strength of the shockwave can be evaluated by the pressure ratio, density ratio and temperature ratio across the normal shockwave. The analytical and experimental study of shockwave structures in a neutral gas provides vital information on fluid physics. Shockwave structure determines shock strength, shockwave thickness, shock standoff distance and heat transfer rate. Figure 1 shows the schematic drawing of detailed shockwave structure in a neutral gas.

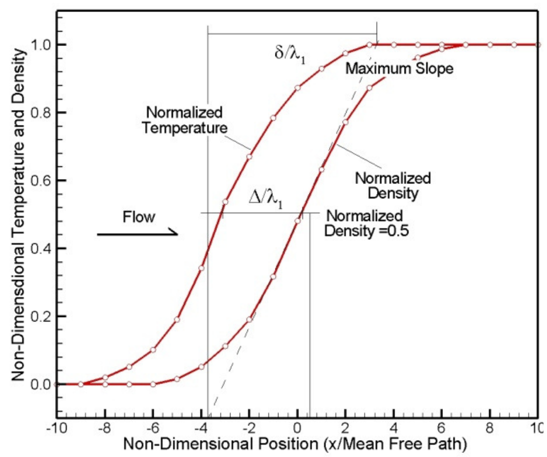

Figure 1. Schematic drawing of shockwave structure in neutral gas

An ideal shockwave has a thickness of zero. However, in reality, with viscous effects, shockwave thickness is usually a few mean-free paths thick. Taking a tangent to the normalized density curve through the shockwave, at the location with maximum slope, the non-dimensional shockwave thickness (δ/λ_1) can be determined as in Figure 1, where λ_1 is the molecular mean-free path of the free-stream gas. The normalized density, ρ_n , and normalized temperature, T_n , inside the shockwave are defined as

$$T_n = \frac{T(x)-T_1}{T_2-T_1}, \qquad \rho_n = \frac{\rho(x)-\rho_1}{\rho_2-\rho_1} \qquad (1)$$

In equation (1), the sub-index *1* denotes conditions ahead of shockwave and *2* denotes those conditions downstream of the shockwave. The separation of the non-dimensional density and temperature profile is called the temperature-density shift, Δ, measured at 50% of the non-dimensional density profile location. Shockwave thickness is given as δ.

Mathematically, the Boltzmann equation is the true governing equation for kinetic fluid dynamics for all flow regimes. Unfortunately, due to the complexity of solving the Boltzmann equation directly, the continuum Euler and Navier-Stokes equations were used to solve the fluid dynamics problems. The Euler, Navier-Stokes and Burnett equations were derived from the Hilbert-Chapman Enskog expansion of the Boltzmann equation [6-10]. Based on this expansion, the velocity distribution function f was expressed in the series expansion of Knudsen number as

$$f = f^{(0)} + f^{(1)} + f^{(2)} + f^{(3)} + \cdots,$$
$$f = f^{(0)}(1 + a_1 K_n + a_2 K_n^2 + a_3 K_n^3 + \cdots)$$

which is the perturbation expansion of the velocity distribution function about the Maxwellian distribution, where $f^{(0)}$ is the Maxwellian distribution function, a_i, (i=1,2,3,…) are functions of density, molecular velocity, and temperature, The Knudsen number, K_n , is defined as the ratio of the mean free path of a particle (λ) divided by the characteristic distance (L) over which the macroscopic variables change appreciably, $K_n = \lambda/L$. In this expansion, the Knudsen number, K_n, a small perturbation parameter, must be less than 1.0. Flow regimes can be defined by the Knudsen number. If K_n is much smaller than one, then the Navier-Stokes equations are valid. If K_n is in the range of 0.01-1.0, the flow is typically in the transitional regime [11-13]. In general, the convergence of this expansion is asymptotic as the Knudsen number goes to zero. Substituting this expansion into the Boltzmann equation, taking moments of the Boltzmann equation yields the continuum equations of fluid mechanics, a set of conservation equations describing global conservation of density, momentum, and energy. To close this system of equations requires constitutive equations, which express viscous stress and heat flux in terms of the distribution function rather than macroscopic gradients. The conventional Euler equations are the zeroth-order approximation, the Navier-Stokes equations are the first-order approximation and the Burnett equations are the second-order approximation. As the Knudsen number increases, the Navier-Stokes equations gradually deteriorate because the transitional nonequilibrium effect prevails [11], [12]. It is natural to consider the Boltzmann equation as the governing equation for the transitional flow problems. However, the full Boltzmann equation is very difficult to solve because the collision term is very complex physically as well as numerically.

The flow across the shockwave is in a highly transitional regime where the Knudsen number is relatively large. Typically, shockwave thickness is on the order of a few mean-free path. If the flow characteristic length, L, is 20 times the mean-free path, λ, then the Knudsen number will be in the range of 0.05. It is necessary to consider the high Knudsen-number effects when solving flow characteristics through shockwave. It was shown that Burnett equations are higher-order in accuracy in the transitional regime compared to the conventional Navier-Stokes equations, and they provide better accuracy in shock-structure prediction [11-13]. The numerical procedure [12], [13] to solve the Burnett equations is explicit. The Burnett terms and boundary conditions were calculated using the Navier-Stokes solution as an initial value. The Burnett stress and heat-flux terms were then treated as source terms and added to the Navier-Stokes equations in order to finally solve them and obtain the Burnett solution. The Burnett solution is a perturbation solution of the Navier-Stokes equations.

In the shockwave-structure prediction analysis, the classical Rankine-Hugoniot equations [14] were applied to describe discontinuous conditions across the shockwave. The classical Rankine-Hugoniot equations were derived from the laws of conservation of mass, momentum and energy for a control volume involving a standing normal shockwave. It relates the densities and pressures for the perfect gas in front of and in back of the normal shockwave.

In a weakly ionized gas flow, pre-ionized charged particles existed in addition to the natural particles present in the natural gas. The high-temperature electrons may excite the vibration modes of molecules, leading to a vibration temperature much higher than the neutral gas temperature [15]. These charged particles can affect the neutral gas-flow characteristics. Shockwaves in weakly ionized gas should exhibit different features from shocks in neutral gases. When a shockwave is present in the weakly ionized gas, large gradients exist in both charged particles (ions and electrons) and neutral gas particles for density. Since electrons move much faster than ions or heavy neutral particles, they will diffuse much faster than ions or neutral particles. Ions will remain near the shock, but electrons will diffuse away from the shock. Also, ions and electrons will be separated and form an electrical double layer across the shockwave [16]. This layer produces an electrostatic body force at the shock directed against the flow as schematically indicated in Figure 2. As a result, the thickness of the shockwave in weakly ionized gas is much wider than the shockwave thickness in neutral gas. The pressure jump across the shockwave in weakly ionized gas is reduced when compared to the jump in neutral gas [16].

To understand the effects associated with the interaction of electromagnetic forces and electrically conducting fluid flow requires integration of several disciplines such as fluid dynamics, electrostatics, chemical kinetics, and atomic physics. Extensive experimental data and simulations are essential for resolving controversial issues. Research attention has been focused on the numerical simulation model development [15-17]. The development of an accurate and robust numerical scheme for weakly ionized gas-flow problems faces challenges in the coupling of the Maxwell equations with the Navier-Stokes equations through the interaction of electromagnetic fields with the momentum and energy equations.

Shockwave-structure research in weakly ionized flow has mainly been conducted by solving the coupling of the plasma and fluid dynamics Euler equations. The Euler equations were solved for shock structure [16]. A simple model for shockwave jump conditions was proposed and the classical Rankine-Hugoniot relations across the shockwave were modified to include electrostatic force produced by ions and electrons. The effect of weakly ionized gas on shockwaves was analyzed using an electrostatic body force derived from Coulomb's law across the shockwave. The shockwave thickness was approximated by a hypergolic tangent function. It was assumed that shockwave thickness was about one mean-free path if the free-stream Mach number was above 5. However, in reality, shockwave thickness is unknown prior to the calculation, and Euler equations cannot provide an accurate shockwave structure even without the presence of weakly ionized gas. Shockwave structures cannot be accurately solved using the technique proposed by Seak and Mankowski [16] if shockwave thickness was not given.

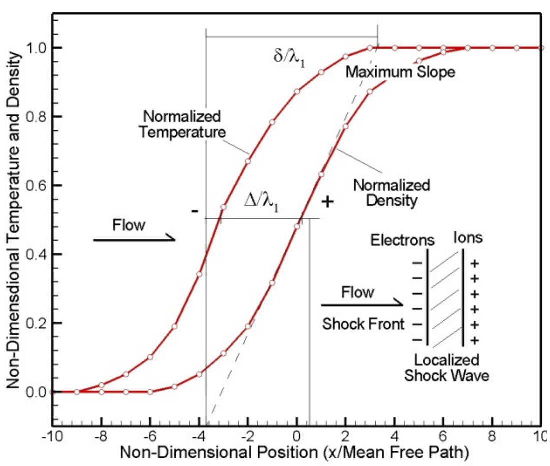

Figure 2. Schematic shockwave structure in weakly ionized gas. Electrostatic body force exists

In this study, the shockwave structure in the weakly ionized gas flow was studied. The objective of this study was to develop a way to numerically investigate the shockwave structure in a weakly ionized gas flow by solving Burnett equations coupled with modified Rankine-Hugoniot relations.

Mathematical Model Description

The shockwave structure in weakly ionized gas is computed by solving the Burnett equations with the addition of weak ionization effects. It was assumed that the interaction of the weakly ionized gas with fluid could be represented by an electrostatic force acting against the flow direction. It was also assumed that the electrostatic body force could be modified to predict the jump conditions across the shockwave for shockwaves in a weakly ionized gas [16]. Combining the modified Rankine-Hugoniot relations with the Burnett equations, the shockwave structure can be solved in three steps: 1) apply modified Rankine-Hugoniot relations to compute jump conditions (pressure, temperature, velocity and density) across a normal shockwave, where the jump conditions depend on the ionization fraction; 2) determine the magnitude of the electrostatic force acting on the shockwave region against the flow direction, based on the ionization ratio and jump conditions. Since the coupling of the electrostatic field with shock discontinuity is relatively weak for a low-ionization fraction, it was assumed that an averaged electrostatic body force within 80 mean free paths in the direction against the flow inside the shockwave could be used. And, 3) modify the Burnett equations with an electrostatic body force as a source term in order to predict shockwave structures in a weakly ionized gas.

One-Dimensional Burnett Equations:

In conservation form, the one-dimensional Burnett equations with body force per unit mass, *f*, in Cartesian coordinates can be written as [13], [18]

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0 \tag{2}$$

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial}{\partial x}(\rho u^2 + p - \sigma) = \rho f \tag{3}$$

$$\frac{\partial(\rho e_t)}{\partial t} + \frac{\partial}{\partial x}(\rho u e_t + pu - \sigma u + q) = \rho f u \tag{4}$$

where,

$$p = (\gamma - 1)\rho(e_t - \frac{u^2}{2}) \tag{5}$$

and where ρ is density, u is the velocity component in the x direction, p is the static pressure, σ is the viscous stress component, q is the heat flux component, and e_t is the specific total energy. The Burnett stress tensor and the heat flux vector component are

$$\sigma = -\frac{4}{3}\mu\frac{\partial u}{\partial x} + \sigma_{Burnett} \tag{6}$$

$$q = -k\frac{\partial T}{\partial x} + q_{Burnett} \tag{7}$$

where

$$\sigma_{Burnett} = \frac{\mu^2}{p}(\alpha_1(\frac{\partial u}{\partial x})^2 + \alpha_2 R\frac{\partial^2 T}{\partial x^2} + \alpha_3\frac{RT}{\rho}\frac{\partial^2 \rho}{\partial x^2}$$

$$+\alpha_4\frac{RT}{\rho^2}(\frac{\partial \rho}{\partial x})^2 + \alpha_5\frac{R}{\rho}\left(\frac{\partial \rho}{\partial x}\frac{\partial T}{\partial x}\right) + \alpha_6\frac{R}{T}(\frac{\partial T}{\partial x})^2 \tag{8}$$

$$q_{Burnett} = \frac{\mu^2}{p}\left(\frac{\gamma_1}{T}\frac{\partial u}{\partial x}\frac{\partial T}{\partial x} + \gamma_2\frac{\partial^2 u}{\partial x^2} + \frac{\gamma_3}{\rho}\frac{\partial u}{\partial x}\frac{\partial \rho}{\partial x}\right) \quad (9)$$

and where k is the thermal conductivity, R is the gas constant, T is temperature, and μ is kinetic viscosity of the gas. The expressions for coefficients for α and γ in equations (8) and (9) can be expressed as [13]

$$\alpha_1 = \frac{2}{3}\omega_1 - \frac{14}{9}\omega_2 + \frac{8}{27}\omega_6$$

$$\alpha_2 = -\frac{2}{3}\omega_2 + \frac{2}{3}\omega_3, \qquad \alpha_3 = -\frac{2}{3}\omega_2, \qquad \alpha_4 = \frac{2}{3}\omega_2$$

$$\alpha_5 = -\frac{2}{3}\omega_2 + \frac{2}{3}\omega_4, \qquad \alpha_6 = \frac{2}{3}\omega_5 + \frac{2}{3}\omega_4$$

$$\gamma_1 = \theta_1 + \frac{8}{3}\theta_2 + \frac{2}{3}\theta_5$$

$$\gamma_2 = \frac{2}{3}\theta_2 + \frac{2}{3}\theta_4, \quad \gamma_3 = \frac{2}{3}\theta_3 \quad (10)$$

In this study, Maxwellian gases were assumed to obtain these coefficients. The values of ω and θ in equation (10) can be obtained [13] as shown here:

$\omega_1 = \frac{10}{3}$	$\theta_1 = \frac{75}{8}$
$\omega_2 = 2$	$\theta_2 = -\frac{45}{8}$
$\omega_3 = 3$	$\theta_3 = -3$
$\omega_4 = 0$	$\theta_4 = 3$
$\omega_5 = 3$	$\theta_5 = \frac{117}{4}$
$\omega_6 = 8$	

Modified Rankine-Hugoniot Equations:

Across the normal shockwave, the modified Rankine-Hugoniot equations for a normal shockwave can be written as [16]

Continuity:

$$\rho_1 u_1 = \rho_2 u_2 \quad (11)$$

Momentum:

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 - \rho_2 f \delta_s \quad (12)$$

Energy:

$$C_{p1}T_1 + \frac{1}{2}u_1^2 = C_{p2}T_2 + \frac{1}{2}u_2^2 - f\delta_s \quad (13)$$

where f is the averaged electrostatic body force per unit mass on shockwave, and δ_s is the shockwave thickness, or the distance required for 99% of jump across the shockwave. The negative body force was used because the coulomb force on the shockwave due to the ion and electron-space charge in and behind the shock was in a direction opposite to the flow. The sub-indexes *1* and *2* denote conditions before and after the shockwave. The electrostatic body force term can be approximated by multiplying the Coulomb force by ion density at the inner surface of the shock front [16],

$$\rho_2 f \delta_s = ke^2 G_{sc}\left(\frac{\varphi\rho_2}{m}\right)^{\frac{5}{3}} \quad (14)$$

where k is the Boltzmann constant, e is the charge of an electron, G_{sc} is a constant incorporating the size and shape of the space charge region, the non-uniformity of the space charge distribution and the inverse of distance square effect, φ is the ionization fraction, which is the ratio of ion number density to the neutral gas number density, and m is the mass of the atom. To determine the electrostatic force, it was necessary to know the shockwave thickness, δ_s. However, the shockwave thickness is unknown prior to the shockwave structure solution. The selection of δ_s becomes critical for the solution accuracy of the electrostatic force. In neutral gases, shockwave thickness is usually in the range of a few molecular mean free paths [13]. In weakly ionized gas, however, the distance between ions and electrons may be many times the shockwave thickness. A complicated full plasma-dynamics equation has to be solved iteratively to compute the electrostatic force. It was shown [16] that under non-equilibrium conditions, the distance between ions and electrons can be 80 or higher of the mean-free paths. Implicit In this study, as the first step in solving shockwave structures in weakly ionized gas using this modified Rankine-Hugoniot model, the distance between the ions and electron layers across the shockwave was assumed to be 80 mean free paths of the free-stream gas, and the averaged electrostatic body force f in equation (14) can be written as

$$f = \frac{ke^2 G_{sc}}{80\lambda_1}\left(\frac{\varphi\rho_2}{m}\right)^{\frac{5}{3}} \quad (15)$$

where $\lambda_1$is the mean free path before the shockwave, which can be determined as

$$\lambda_1 = \frac{16\mu_1}{5\rho_1\sqrt{2\pi R T_1}} \quad (16)$$

In equation (16), μ_1 is the free-stream gas viscosity and R is the gas constant.

Numerical Procedures

The solution technique for the Burnett equations was accomplished through the explicit four-stage Runge-Kutta time integration in time [13]. The convection terms in governing equations (2-4) are discretized using second-order upwind flux differencing with Roe's nonlinear flux limiter. The first-order Navier-Stokes viscous terms, the second-order Burnett viscous terms and the electrostatic body forces are treated as source terms to the discretized equations and are discretized by using a central differencing technique. The jump conditions across shockwaves in argon gas with weak ionization, such as pressure ratio, temperature ratio and density ratio were obtained by solving the modified Rankine-Hugoniot equations (11-13). This set of non-linear equations was solved using the Newton Raphson's iteration technique. The solution of the modified Rankine-Hugoniot equations provides boundary conditions to the Burnett equations.

Results and Discussion

Modified Rankine-Hugoniot Equations:
The jump conditions were obtained across normal shockwaves by solving the Modified Rankine-Hugoniot equations. Note that the jump conditions depend on the free-stream pressure, temperature and density. In this study, free-stream pressure, temperature and density were selected based on the standard air table at 10,000meter altitudes. Argon gas was used for the calculation. Table 1 shows the computed pressure ratio, temperature ratio and Mach number after the shockwave in weakly ionized argon gas with a free-stream Mach number of 6. In these results, weakly ionized effects were introduced through the volume-weighted ionization fraction,

$$\beta = G_{sc}^{\frac{3}{5}}\varphi \qquad (17)$$

Notice that when $\beta = 0$, the gas is neutral and no ionization effects exist. Results are the same as standard normal shock table values.

Table 2 shows the computed results for a shockwave Mach number of 3. In both cases, as β increases, the pressure ratio and temperature ratio decrease for a given free-stream Mach number, M_1. As indicated in Table 1, as β increases to 5.0, the Mach number after the shockwave increases to 0.983, which is close to 1.0 for a free-stream Mach number of 3. However, as seen in Table 2, the Mach number after the shockwave was higher than 1.0 when β was larger than 2.25E-06, which was clearly physically impossible.

Normal shockwave theory requires that an incoming free-stream Mach number has to be bigger than 1.0 and a Mach number after a shockwave should be less than 1, in order not to violate the second law of thermodynamics. These results indicated that the simplified Modified Rankine-Hugoniot equations would only be valid for a small value of ionization fraction for moderate free-stream Mach numbers (around 3.0-5.0).

Table 1. Computed jump conditions in weakly ionized argon gas (free-stream Mach number M_1= 6)

$\beta \times 10^6$	P_2/P_1	T_2/T_1	M_2
0.00	44.750	12.120	0.467
0.25	44.401	12.082	0.470
0.50	43.657	12.000	0.476
0.75	42.641	11.885	0.485
1.00	41.422	11.744	0.496
1.25	40.048	11.580	0.510
1.50	38.560	11.395	0.525
1.75	36.993	11.193	0.543
2.00	35.372	10.975	0.562
2.25	33.720	10.742	0.583
2.75	30.393	10.238	0.632
3.00	28.743	9.968	0.659
3.25	27.113	9.689	0.689
3.50	25.511	9.399	0.721
3.75	23.940	9.099	0.756
4.00	22.404	8.790	0.794
4.25	20.904	8.471	0.835
4.50	19.440	8.142	0.881
4.75	18.011	7.803	0.931
5.00	16.616	7.452	0.986

Table 3 shows the pressure, temperature and Mach number after shockwave for free-stream Mach numbers between 1.75 and 6.0, for ionization fractions of 0.75 parts per million. Figure 3 shows the complete set of computed pressure and temperature ratios across shockwave-versus-ionization fractions at free-stream Mach numbers between 1.75 and 6.0. Notice that when the free-stream Mach number was below 1.75, the modified Rankine-Hugoniot equations failed to accurately predict jump conditions across the shockwave if the ionization ratio was larger than 0.75E-06. This result shows another accuracy problem associated with the modified- Rankine-Hugoniot equations.

Table 2. Computed jump conditions shockwave in weakly ionized argon gas (shockwave free-stream Mach number is 3)

$\beta \times 10^6$	P_2/P_1	T_2/T_1	M_2
0.00	11.000	3.667	0.522
0.25	10.737	3.631	0.532
0.50	10.197	3.555	0.555
0.75	9.499	3.451	0.587
1.00	8.712	3.324	0.628
1.25	7.881	3.178	0.679
1.50	7.039	3.015	0.740
1.75	6.200	2.835	0.815
2.00	5.373	2.637	0.907
2.25	4.552	2.415	1.024
2.50	3.708	2.156	1.188

Table 3. Computed jump conditions shockwave in weakly ionized argon gas at different Mach numbers, β=0.75E-06

M_1	P_2/P_1	T_2/T_1	M_2
6.0	42.641	11.885	0.485
5.0	29.012	8.449	0.501
4.5	23.158	6.966	0.513
4.0	17.950	5.639	0.529
3.5	13.395	4.467	0.552
3.0	9.499	3.451	0.587
2.5	6.266	2.588	0.642
2.0	3.682	1.868	0.742
1.75	2.607	1.550	0.836

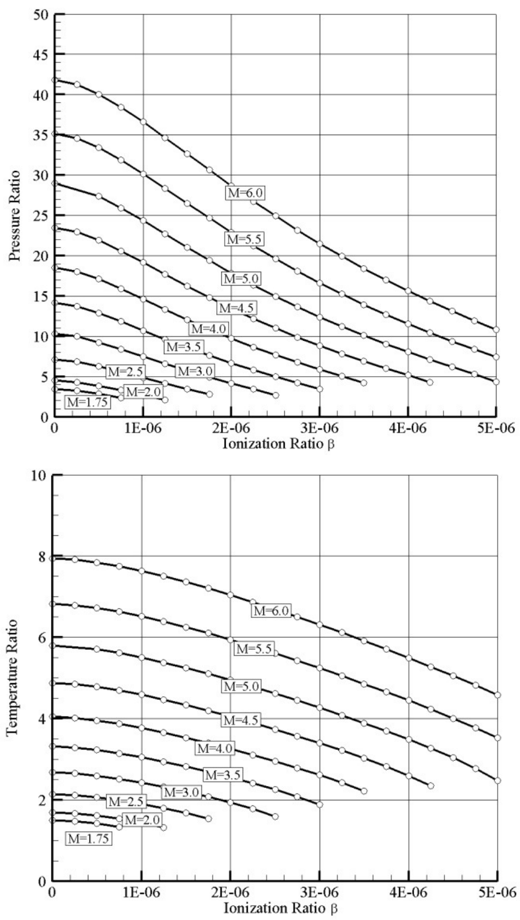

Figure 3. Computed pressure and temperature ratios across normal shock for free-stream Mach numbers (*M=1.75-6.0*) and ionization ratio β

Shockwave structure computation:

The normal shockwave was initially placed in the middle of the computational domain. Over time, the shockwave structure changed. The steady state solution was captured when the four-step Range-Kutta time integration converged. Upstream flow conditions were fixed during the calculation. The downstream pressure condition was fixed and was determined by the solution of the modified Rankine-Hugoniot equations for a given ionization fraction. Other downstream parameters were updated at the downstream exit. The computational domain was 100 times the free-stream molecular mean free path. An averaged electrostatic body force was acting upon this domain for simplicity, as indicated in equation (15). All computations were performed for argon gas. The calculated free-stream Mach numbers ranged from 1.75 to 6.0. Upstream pressure and temperature were chosen from standard air tables at 10,000meter altitudes. Figure 4 shows the normalized temperature and density profile extracted from the shockwave structure solutions using equation (1) inside the shockwave. The free-stream Mach number for this calculation was 5, and the ionization factor, β, was 0.75 parts per million. Physically, the temperature-density shift area was related to entropy change across the normal shock. The horizontal axis was plotted using a non-dimensional scale with reference to free-stream mean-free path.

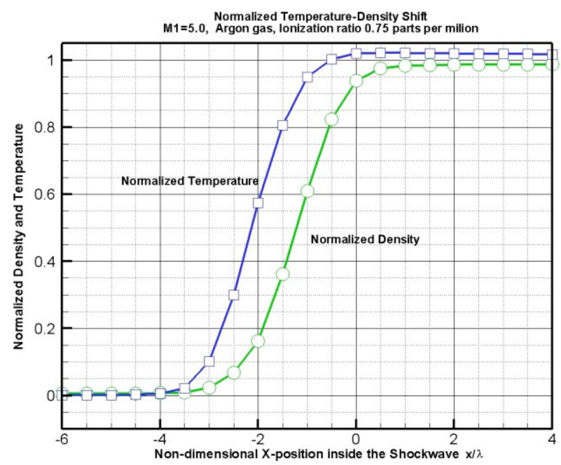

Figure 4. Non-dimensional temperature-density profile for M_1=5.0. argon gas, ionization fraction is 0.75 parts per million

Figure 5 compares the temperature-density shift between ionized argon and neutral argon gas. The ”with ionization” curve represents the solution with ionization of 0.75 parts per million, while the ”without ionization” curve represents the neutral argon gas. A small difference was found at the mid-point of the normalized density profile, where $\rho_n = 0.5$. This indicated that weak ionization does not significantly change the increase in entropy across the shockwave. The entropy change is primarily caused by viscous effects. Figure 6 shows the comparison of the reciprocal shockwave thickness for free-steam Mach numbers between 1.75 and 6.0, between ionized and neutral argon. The solutions were obtained by solving the Burnett equations with and without weak ionization effects. “BUR” represents solution of the Burnett equations without ionization; “ION” represents the solution of Burnett equations with ionization effects. The ionization fraction, β, was 0.75 parts per million for all cases. The reciprocal shockwave thickness (λ_1/δ_s) was measured from the computed, normalized density profile, as in

Figure 4, for Mach 5 and for each Mach number using the normalized density profile in equation (1) and as described in Figure 4.

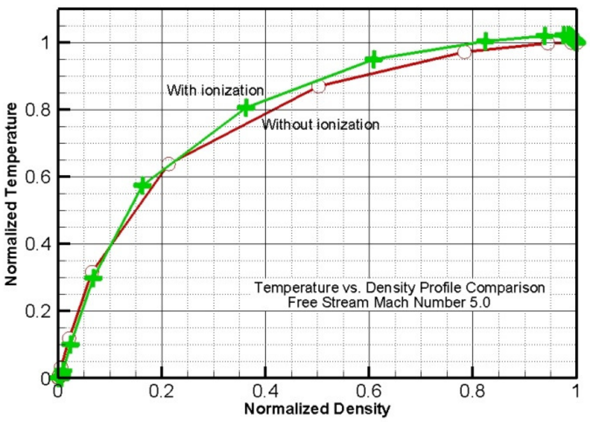

Figure 5. Comparison of the normalized temperature-density shift between weakly ionized gas and neutral gas. Free-stream Mach number is 5, ionization fraction is 0.75 parts per million

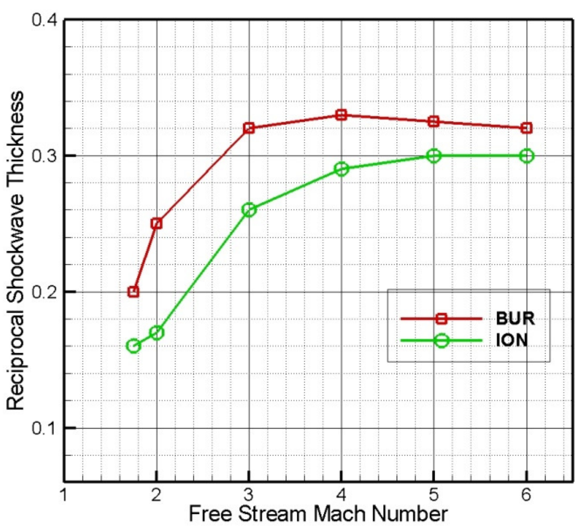

Figure 6. Comparison of reciprocal shockwave thickness (λ_1/δ_s) between ionized argon and neutral argon gas. The ionization fraction is 0.75 parts per million

As indicated in Figure 6, λ_1/δ_s decreases with the existence of weakly ionized gas and, in turn, the shockwave thickness, δ_s, increases with the existence of weakly ionized gas. This demonstrated that the solution of Burnett equation coupling with the modified Rankine-Hugoniot equations can provide a quick and simplified estimate for the reduction of the shockwave strength due to weak gas ionization. This weak ionization shockwave strength reduction was achieved when the ionization ratio, β, was the typical 0.75 parts per million. For low free-stream Mach numbers with high ionization effects, as indicated in Figure 3, the solution of the modified Rankine-Hogoniot equations (11-14) fail to provide valid jump conditions across shockwaves. Extensive numerical analysis and experimental validation is needed in order to fully understand the validity and the limitation of the modified Rankine-Hugoniot equations.

Results and Discussion

The modified Rankine-Hugoniot equations across a standing normal shockwave were discussed and adapted to obtain jump conditions for shockwave structure calculations. Coupling an electrostatic body force to the Burnett equations, the weakly ionized shockwave structure was solved for a wide range of free-stream Mach numbers between 1.75 and 6.0 with modest ionization ratios. Results indicated that the modified Rankine-Hugoniot equations for shockwaves were valid for a small range of ionization fractions. This model failed to accurately predict valid jump conditions for free-stream Mach numbers below 1.75 and ionization ratios above 0.75E-06. As free-stream Mach numbers increase, the modified Rankine-Hugoniot equations can provide valid jump conditions across a normal shockwave for gas with slightly higher ionization ratios. The computed results of shockwave structure with ionization indicated that shockwave strength may be reduced by the existence of weakly ionized gas. However, further numerical simulation and experimental validation using the modified Rankine-Hugoniot equations is needed. Further studies are also needed to analyze the effects of weakly ionized flow electrostatic force on the change of the specific heat ratio.

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