# APPLICATION OF SIX SIGMA TO GEAR BOX MANUFACTURING

Kambiz Farahmand, North Dakota State University; Jose Victorino Marquez Grajales; Mohsen Hamidi, North Dakota State University


## Abstract

This study applies Six Sigma to optimize the keyway cutting operation on a shaft of a gear box produced by Horton Automatics Company. Since the operation had not been optimized, many problems had been raised in the assembly of keys with shafts and a considerable number of scrap parts had been produced which deteriorated profitability of the company. The main problem was that the keyway width did not allow keys to be assembled into the shaft tightly. To solve the problem and optimize the process, DMAIC (Define, Measure, Analyze, Improve, and Control) methodology of Six Sigma was applied. In this paper, the steps of DMAIC to optimize the operation are presented and illustrated. In the Define phase, the problem was defined and the specific operation of the manufacturing process was recognized by the PFMEA (Process Failure Mode Effects Analysis) technique. In the Measure phase, the operation was studied and the process capability was measured. In the Analyze phase, the factors that affect the keyway width were identified by a cause-and-effect analysis. In the Improve phase, the best combination of the levels of the factors was determined by using the Design of Experiments (DOE) technique and the best combination was applied. Finally, in the Control phase, some recommendations were given in order to keep the process in good condition.


## Introduction

In statistical terms, "reaching Six Sigma" means that the process or product performs with almost no defects, but the real message of Six Sigma goes beyond statistics [1]. Six Sigma is a philosophy of managing that focuses on eliminating defects through practices that emphasize understanding, measuring, and improving processes [2]. The focus of Six Sigma is reducing variability in key product/service quality characteristics to the level at which failure or defects are extremely unlikely [3]. The model of a Six Sigma process assumes that if the process is centered at the target and the nearest specification limit is six standard deviations from the mean, the process will operate at the 3.4 parts-per-million defect level [3]. Six Sigma was heavily inspired by six preceding decades of quality improvement methodologies such as quality control, TQM, and Zero Defects [4], [5]. Motorola first made Six Sigma popular in the 1980s, AlliedSignal embraced it in the early 1990s and then General Electric made it the most popular management philosophy in history [6].

Six Sigma efforts target three main areas of improving customer satisfaction, reducing cycle time, and reducing defects [1]. Improvements in these areas usually represent dramatic cost savings to businesses, as well as opportunities to retain customers, capture new markets, and build a reputation for top-performing products and services [1]. Unlike mindless cost-cutting programs which reduce value and quality, Six Sigma identifies and eliminates costs which provide no value to customers, or waste costs [7]. Basu and Wright [8] listed some real benefits from the adoption of Six Sigma: for example, in 1997 Citibank undertook a Six Sigma initiative and after just three years it was reported that defects had reduced by ten times; General Electric reported that $300 million invested in 1997 in Six Sigma delivered between $400 million and $500 million in savings, with additional incremental margins of $100 to $200 million; and Wipro Corporation in India says that two years after starting in 1999, defects were reduced to such an extent as to realize a gain of eight times over the investment in Six Sigma.

Six Sigma employs a well-structured program methodology; namely, Define, Measure, Analyze, Improve and Control (DMAIC) or Define, Measure, Analyze, Design and Validate/Verify (DMADV) [9]. DMAIC is used for projects aimed at improving an existing business process and DMADV is used for projects aimed at creating new product or process designs [10]. The five steps of DMAIC are as follows [8]. Figure 1 shows a flow diagram of the steps of DMAIC.

1) Define opportunities: This is done through identifying, prioritizing, and selecting the right projects.
2) Measure performance of the projects and process parameters.
3) Analyze opportunities: Opportunities are analyzed by identifying key causes and process determinants.
4) Improve performance: This is achieved by changing the process so as to optimize performance.
5) Control performance: This is essential if gains are to be maintained.

In the following sections, the steps of DMAIC applied to the manufacturing process of a gear box produced by Horton Automatics Company are described. Since 1960, when they developed the first automatic sliding door in America, Horton Automatics Company has been designing and manufacturing automatic systems such as automatic sliding, swinging, folding, and security revolving doors, service windows, presence and motion detection systems, Automated People Mover (APM) transit doors, and vehicle door operators.

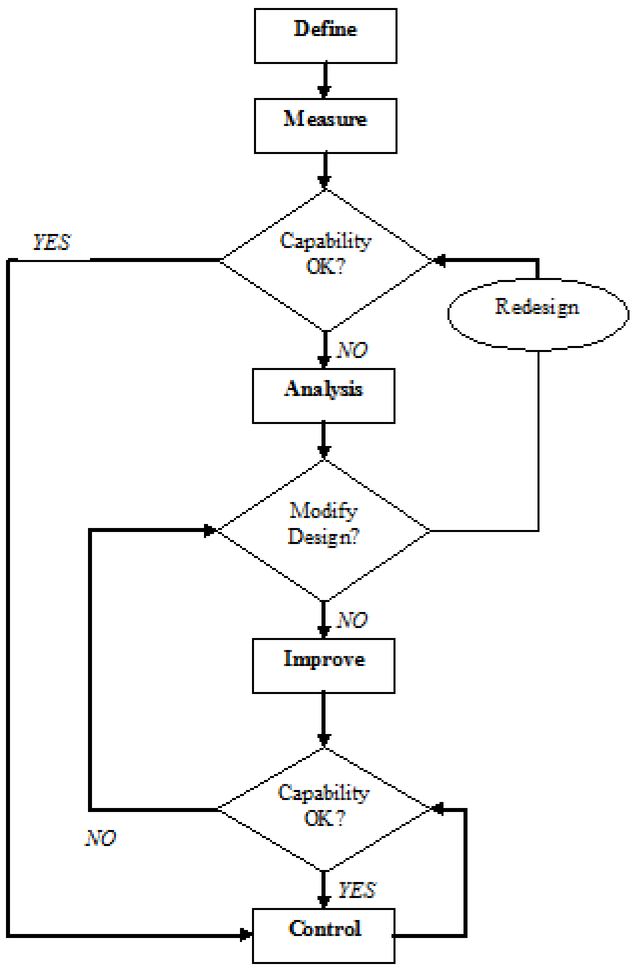


**Figure 1. Flow Diagram of DMAIC**

## Define Phase

The automatic swinging door uses the 7000 gear box in order to open and close the door. The gear box includes three shafts and three gears. Each gear is attached to a shaft by the employment of a key. The most common type of keys is flat key, as shown in Figure 2. On the shaft, the keyway is classified according to the process by which it is made. Figure 3 shows three of the most common keyways. The main function of the key is to transmit torque from a component to a shaft (see Figure 4).

In one part of the production line, gears and shafts are manufactured and in the other part, gears, shafts, keys, and other components, such as casings and switches, are assembled together. One of the main subassemblies of the product is the C7113 subassembly, which includes a C7034 shaft and a C7017 gear. There were many failures occurring in the C7113 subassembly. The main problem was that the key did not fit in the keyway and this made a considerable number of scrap parts which had to be reworked or discarded.

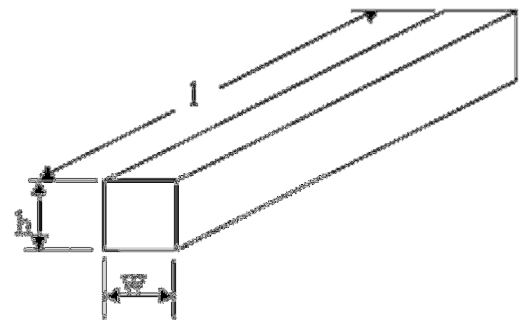


**Figure 2. A Flat Key (w: Width, h: Height, l: Length) [11]**

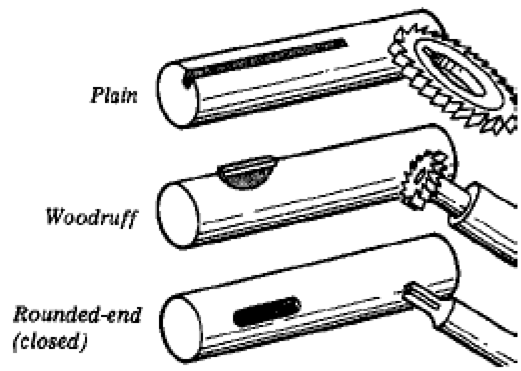


**Figure 3. Three Types of Keyways [11]**

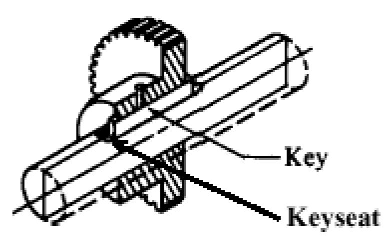


**Figure 4. Keys Role for Transmitting Torque to a Shaft [11]**

The primary purpose of the Define phase is to ensure that the team focuses on the right things [9]. In order to identify all of the problems in the manufacturing processes of the shaft, PFMEA (Process Failure Mode Effects Analysis) was applied. The main processes on the shaft are a hobbing process, performed by a hobbing machine, to cut teeth on the shaft and a cutting process, performed by a milling machine called "key machine", to cut the keyway on the shaft. The PFMEA for these two processes is shown in Figure 5. Through PFMEA, the following items were identified or determined for each process:

- Potential failure modes.

- Potential effects of failure and severity (SEV) rankings of the consequences of failure; the severity ranking is based on a relative scale ranging from 1 to 10 [12].
- Potential causes of failure and occurrence (OCC) rankings based on how frequently the cause of the failure is likely to occur; the occurrence ranking scale is based on a relative scale from 1 to 10 [12].
- Process controls for detecting the failure and detection (DET) rankings based on the chances the failure will be detected prior to the customer finding it; the detection ranking scale is on a relative scale from 1 to 10 [12].
- The overall risk of each failure which is called Risk Priority Number (RPN): RPN = SEV × OCC × DET; the RPN (ranging from 1 to 1000) is used to prioritize all potential failures to decide upon actions leading to risk reduction.
- Recommended actions to reduce the risk (RPN).

The PFMEA in Figure 5 indicates that the key machine process, which is performed by a milling machine, has the highest RPN. Based on the observations in this study, it was found that the method of cutting the keyway on the shaft had not been studied and validated by the plant and there was much confusion in the way to cut the keyway on the shaft. A vertical milling machine was used to cut the keyway. The operator frequently changed the machine setup and generated different dimensions in the width of the keyway; consequently, many keys could not be assembled to the shafts tightly and properly due to incorrect size of the keyway.

## Measure Phase

In this phase, a process capability analysis was performed. A sample of 30 pieces was taken and Minitab software was used to do the analysis. $C_{pk}$ index is a process capability index. Equation (1) shows the formula used for calculating $C_{pk}$. Usually, $C_{pk}$ should be above 1.33

$$C_{pk} = \min\left[\frac{USL - Mean}{3\sigma}, \frac{Mean - LSL}{3\sigma}\right] \qquad (1)$$

| Line No. | Process Function / Requirements | Potential Failure Mode | Potential Effect(s) of Failure | SEV | Class | Potential Cause(s) / Mechanism(s) of Failure | OCC | Current Process Controls Prevention (P) Detection (D) | DET | RPN | Recommended Action (s) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 15 | | | | | | | | | | | |
| 16 | Hobbing machine | Incorrect angle | Interference between gears | 8 | | Error Operator | 4 | Red Line machine | 3 | 96 | |
| 17 | | | | | | | | | | | |
| 18 | | Run Out | Egg Shape | 8 | | Arbor vibration | 4 | Dial indicator | 9 | 288 | |
| 19 | | | | | | | | | | | |
| 20 | | sharpening to much > 1 inch | Broke the hobb | 7 | | Do not check the width of the teeth | 9 | Visual | 9 | 567 | Implement a gage to measure the width of the teeth of the hobb. |
| 21 | | | | | | | | | | | |
| 22 | | Sharpening at wrong angle | Interference between gears | 8 | | Error Operator | 4 | Red line machine | 3 | 96 | |
| 23 | | | | | | | | | | | |
| 24 | | | | | | | | | | | |
| 25 | | Incorrect handle | Hobb Broken | 8 | | No trained | 7 | NONE | 9 | 504 | |
| 26 | | | | | | | | | | | |
| 27 | | | | | | | | | | | |
| 28 | key machine | 4 flu | To loose ( 2 times) some curve ( 1 time) | 8 | | Many flu take off too much material | 10 | Caliper | 9 | 720 | Identify which are the critical variables and put it in control |
| 29 | | | | | | | | | | | |
| 30 | | 2 flu | 2 times ok 0.187 | 8 | | Don't take enough material | 10 | Caliper | 9 | 720 | Identify which are the critical variables and put it in control |
| 31 | | | | | | | | | | | |
| 32 | | move dimension | deeply way | 6 | | wear out | 7 | caliper | 9 | 378 | |
| 33 | | | | | | | | | | | |
| 34 | | forget key way on the part | already welded | 7 | | Error Operator | 2 | NONE | 9 | 126 | |

**Figure 5. PFMEA Chart**

In Figure 6, the results of the analysis are shown. The width of the keyway should be 0.188 (-0.000, +0.002) inches; LSL (Lower Spec Limit) is 0.188 inches and USL (Upper Spec Limit) is 0.190 inches. $C_{pk}$ of the process was 0.31, indicating that the process was in a bad condition. As seen in Figure 6, the sample mean was larger than the expected mean (0.189), which means that the keyway width was usually larger than the width that is ideal for assembling the key onto the shaft. Also, some widths were outside the spec range, less than LSL or more than USL, and as a result many keys could not be assembled onto the shafts tightly and properly due to the incorrect width of the keyway.

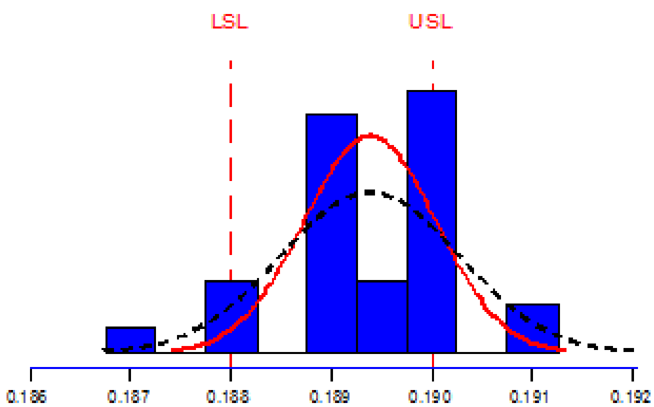


**Figure 6. Process Capability Analysis**

# Analysis Phase

In order to find out the possible causes that affect the keyway cutting process, a fish-bone diagram was developed, as shown in Figure 7. The possible causes are divided in setup, operator, end milling, and machine. For setup, the variables to control are the center of cutter and the feed rate. The center of the cutter does not affect the keyway width, but the effect of the feed rate was unknown and the operator was confused about the suitable feed rate for the operation. The effect of the feed rate, then, had to be investigated. For operator, this is a factor that can be controlled with operator training. For end milling, there are three variables to control: wear out, the material of the cutter, and the number of flutes in the cutter. Wear out can cause imperfections in the keyway and there was no estimated time to change the cutter; it was only changed when it was broken. To control this cause, the supervisor was told to monitor the start date of a new cutter and how many pieces could be made with the same cutter. The types of cutters used were either carbide or high-speed steel, which are quite similar and do not affect the keyway width. Basically, the selection of the cutting tool (carbide or high-speed steel) was determined not to be a contributing factor to the incorrect dimension in the keyway. Also, there were two types of cutters: 2-flute cutters and 4-flute cutters. The effect of the number of flutes was unknown and had to be investigated. For the machine factor, the variables to control are: the holder of the shaft, machine cleaning, and the head machine. The holder of the shaft should be locked but it is dependent on the operator. Cleaning is done by the operator but there was not a maintenance program. The head machine is an important factor but it is dependent on the operator; as long as it is locked and 90 degrees to the arm, it does not create any problems.

Therefore, the factors for which the effects were unknown were identified to be the number of flutes (2 or 4) in the cutter and the feed rate. The feed rate was determined by the RPM (Revolutions per Minute) of the milling machine. The rotational speed of the machine can be 1750 RPM or 2720 RPM. Basically, to improve the process, the effects of the number of flutes of the cutter and the speed of the machine had to be investigated.

# Improve Phase

Design of Experiments (DOE) is a technique for examining controlled changes of input factors and the observation of resulting changes in outputs, i.e., the response to input changes [8]. DOE was applied in order to determine which factor(s) have major effects on the width of the keyway and what combination of factors' levels gives the best result. As mentioned earlier, there are two factors, number of flutes and RPM, each having two levels. Thus, the experimental design is called a factorial design. In statistics, a factorial experiment is an experiment whose design consists of two or more factors, each with discrete possible values or levels, and whose experimental units take on all possible combinations of these levels across all such factors. Such an experiment allows one to study the effect of each factor on the response variable, as well as the effects of interactions between factors on the response variable. For the vast majority of factorial experiments, each factor has only two levels. Table 1 shows the data gathered for the analysis. In each combination of the levels of the factors, there are ten replications.

The statistical hypothesis is as follows:

$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$

$H_1 : \mu_i \neq \mu_j$ *; For at least one pair of i and j (i, j=1,2,3,4)*

where:

$\mu_1$ : Width mean with 2 flutes and low RPM (1750 rpm)

$\mu_2$ : Width mean with 4 flutes and low RPM (1750 rpm)

$\mu_3$ : Width mean with 2 flutes and high RPM (2720 rpm)

$\mu_4$ : Width mean with 4 flutes and high RPM (2720 rpm)

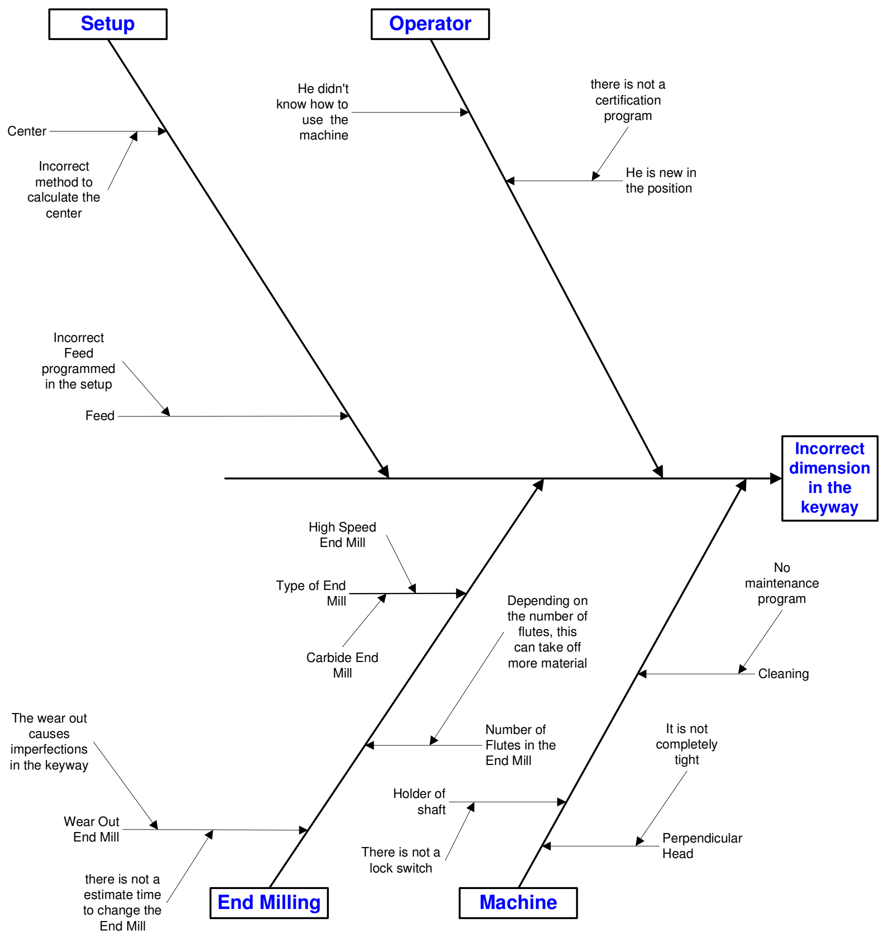


**Figure 7. Fish Bone Diagram**

**Table 1. Width of the Keyway (in inches)**

| Number of Cutter Flutes (Factor A) | RPM (Factor B) | |
|---|---|---|
| | 1750 (Level 1) | 2720 (Level 2) |
| 2 Flute (Level 1) | **0.189, 0.188, 0.196, 0.187, 0.189, 0.188, 0.190, 0.190, 0.188, 0.190** | **0.190, 0.187, 0.187, 0.188, 0.186, 0.188, 0.188, 0.187, 0.187, 0.190** |
| 4 Flute (Level 2) | **0.195, 0.195, 0.192, 0.197, 0.197, 0.196, 0.195, 0.196, 0.195, 0.194** | **0.194, 0.192, 0.194, 0.194, 0.193, 0.194, 0.193, 0.194, 0.193, 0.193** |

The descriptive results of the analysis performed by Minitab software is shown in Appendix 1. As seen in the first table of this appendix, the F test statistic for main effects (factor A: number of flutes and factor B: RPM) is 74.54 and the P value is 0.000. At a significance level of $\alpha = 0.05$, the null hypothesis ($H_0$) could be rejected because $P = 0.000 < \alpha = 0.05$ and $F = 74.54 > F_{0.05,\ 2,\ 36} = 2.84$. The result shows that there is a significant difference between the means of the four possible combinations of the factors' levels, and the effect of the main factors (Flute and RPM) on the keyway width. Since for the 2-way interaction the F statistic was very small and the P value was more than $\alpha$, it was concluded that the factors do not have significant interactions. An interaction between factors occurs when the change in response from the low level to the high level of one factor is not the same as the change in response at the same two levels of a second factor. That is, the effect of one factor is dependent upon a second factor.

In the second table of Appendix 1, the individual effects of the factors were investigated. The P values for Flute and RPM were 0.000 and 0.001, respectively, and were both less than $\alpha = 0.05$. Therefore, it was concluded that both factors have significant effects on the keyway width. In Figure 8, the normal probability plot of the effects is shown. Effects (the related points) that do not fall near the line have a significant impact on the result. The Pareto chart in Figure 9 displays the magnitude and importance of the effects. Any effect that extends past the reference line (the dashed line) is important. As shown in Figures 8 and 9, both factors A and B have significant effects, but factor A (Flute) has the most significant effect. And the interaction effect of AB is not significant.

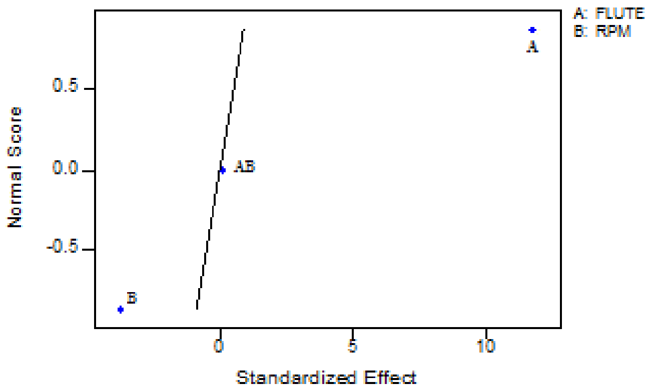

**Figure 8. Normal Probability Plot of Effects**

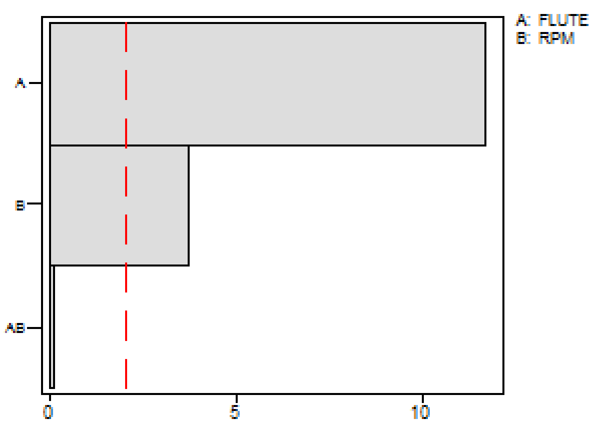

**Figure 9. Pareto Chart of the Effects**

Figure 10 shows the main effect plot. The main effect plot is a plot of the means at each level of a factor. The means for the factors' levels can be seen in the third table of Appendix 1. Since the interaction between Flute factor and RPM factor is not significant, the best combination of these factors can be found by using the main effect plot. As shown in Figure 10, the width increases significantly when the number of flutes changes from 2 to 4. The mean for 2 flutes is 0.1884 and within the spec range (LSL = 0.188 and USL = 0.190), but the mean for 4 flutes is 0.1943 and outside the spec range. The mean for an RPM of 1750 is 0.1923, more than the USL and outside the spec range. But it decreases and falls in the spec range when the RPM is changed to 2720. In conclusion, the best setting for the operation is a 2-flute cutter with an RPM setting of 2720 for the machine.

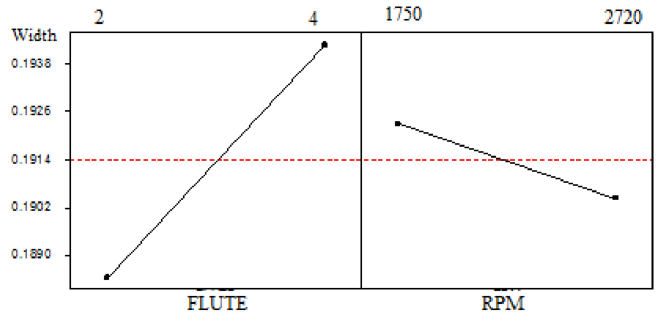

**Figure 10. Main Effect Plot**

In this phase, some parts were operated with the recommended levels of the factors (2-flute cutter, RPM of 2720) and the feed rate of 27.2 in/min. The feed rate was calculated by using equation (2):

$$\text{Feed Rate} = \text{RPM} * \text{Chip Load per Tooth} * \text{No. of Flute} = 2720 * 0.005 * 2 = 27.2 \text{ in/min} \quad (2)$$

For removing some metal particles inside the keyway and having accurate measurements, the edges of the keyway had been polished with sand paper. Since the polish operation on the shaft generated several burrs and spikes around the keyway, which caused the key not to fit within the keyway, a different polishing tool was recommended in order to eliminate all burrs within the keyway.

## Control Phase

To control the dimension of the keyway, the operator should always use a 2-flute cutter and set the RPM of the machine to 2720. Also, it is necessary to have a good 2-flute cutter. If a used cutter is used, problems will be generated in the keyways. The supervisor should monitor the start date of a new cutter and how many pieces are made with the same cutter.

## Conclusions

The problem was that the keyway width in the C7034 shaft of a C7113 subassembly in a 7000 gear box did not fit the width of keys used for its assembly. This problem led to a considerable number of scrap parts. DMAIC methodology of Six Sigma was applied to solve the problem. In the Define phase, PFMEA was applied to distinguish the high-priority area of the process to focus on. The cutting operation performed by a vertical milling machine got the highest RPN and so the study focused on this area. In the Measure phase, the keyway cutting process on the shaft was evaluated and depicted by using a process capability measure of $C_{pk}$. As expected, the mean of the sample was larger than the anticipated mean (according to standard spec). In the Analyze phase, the cause-and-effect analysis was performed to recognize the main causes of the problem and the main factors that affect the keyway width and their levels as well. In the Improve phase, DOE was used to find the best combination of the levels of the factors. A factorial design analysis with two factors (number of flutes of the cutter and RPM of the milling machine) at two levels (2 and 4 flutes, 1750 and 2720 rpm) was performed. The results of the experiment provided by Minitab software showed that there was a significant difference between the means of four possible combinations of the factors' levels. Both factors have significant effects and the most significant factor affecting the keyway width was the number of cutter flutes. The best combination of factor levels is having a 2-flute cutter and an RPM of 2720, which would provide a 27.2 in/min feed rate. Finally, in the Control phase, some recommendations were given in order to keep the process in good condition. The main recommendation is to trace the usage time of cutters because after a certain amount of time, a cutter cannot perform well. During this study, cooperation among engineers and operators and their excellent teamwork in the PFMEA analysis, process capability analysis, cause-and-effect analysis, and DOE were valuable. Actually, Six Sigma methodology produced a team that together would optimize the process and recognize and remove quality problems that had affected customer satisfaction and profitability of the company.

**Appendix 1. Minitab Results of the Factorial Design**

**Analysis of Variance for Width**

| Source | DF | Seq SS | Adj SS | Adj MS | F | P |
|---|---|---|---|---|---|---|
| Main Effects | 2 | 0.00037645 | 0.00037645 | 0.00018823 | **74.54** | **0.000** |
| 2-Way Interactions | 1 | 0.00000003 | 0.00000003 | 0.00000003 | 0.01 | 0.921 |
| Residual Error | 36 | 0.00009090 | 0.00009090 | 0.00000253 | | |
| Pure Error | 36 | 0.00009090 | 0.00009090 | 0.00000253 | | |
| Total | 39 | 0.00046738 | | | | |

**Estimated Effects and Coefficients for Width**

| Term | Effect | Coef | SE Coef | T | P |
|---|---|---|---|---|---|
| Constant | | 0.191375 | 0.000251 | 761.70 | 0.000 |
| FLUTE | 0.005850 | 0.002925 | 0.000251 | 11.64 | **0.000** |
| RPM | -0.001850 | -0.000925 | 0.000251 | -3.68 | **0.001** |
| FLUTE*RPM | 0.000050 | 0.000025 | 0.000251 | 0.10 | 0.921 |

**Least Squares Means for Width**

| | Mean | SE Mean |
|---|---|---|
| FLUTE | | |
| 2 | 0.1884 | 0.000355 |
| 4 | 0.1943 | 0.000355 |
| RPM | | |
| 1750 | 0.1923 | 0.000355 |
| 2720 | 0.1904 | 0.000355 |

# References


[1] Pande, P. S. & Holpp, L. (2002). *What is six sigma?*. McGraw-Hill.

[2] Brue, G. (2002). *Six Sigma for managers*. New York: McGraw-Hill.

[3] Montgomery, D. C. (2010). A modern framework for achieving enterprise excellence. *International Journal of Lean Six Sigma, 1*(1), 56-65.

[4] Stamatis, D. H. (2004). *Six Sigma Fundamentals: A Complete Guide to the System, Methods, and Tools*. New York: Productivity Press.

[5] Montgomery, D. C. (2009). *Statistical Quality Control: A Modern Introduction* (6 ed.). Hoboken, New Jersey: John Wiley & Sons.

[6] Eckes, G. (2003). *Six sigma for everyone*. Hoboken, New Jersey: John Wiley & Sons.

[7] Pyzdek, T. (2003). *The Six Sigma handbook: a complete guide for green belts, black belts, and managers at all levels*. New York: McGraw-Hill.

[8] Basu, R., & Wright, J. N. (2003). *Quality beyond Six Sigma*. Boston: Butterworth-Heinemann.

[9] Ray, S., & Das, P. (2010). Six Sigma project selection methodology. *International Journal of Lean Six Sigma, 1*(4), 293-309.

[10] De Feo, J. A. and Barnard W. W. (2005). *JURAN Institute's Six Sigma Breakthrough and Beyond: Quality Performance Breakthrough Methods.* Tata McGraw-Hill Publishing Company Limited.

[11] Technical drawings of Horton Automatics.

[12] "10 Steps to Conduct a PFMEA". Resource Engineering, Inc. http://www.qualitytrainingportal.com/resources/fmea/fmea_10step_pfmea.htm, retrieved December 3, 2010.


# Biographies


**KAMBIZ FARAHMAND** is currently a professor and department head in the Industrial and Manufacturing Engineering department at North Dakota State University. Dr. Farahmand has more than 30 years of experience as an engineer, manager, and educator. He is a registered Professional Engineer in North Dakota and Texas and past president of IIE Coastal Bend Chapter. Dr. Farahmand may be reached at kambiz.farahmand@ndsu.edu

**MOHSEN HAMIDI** is currently a doctoral student and instructor in the Industrial and Manufacturing Engineering department at North Dakota State University. Mr. Hamidi may be reached at mohsen.hamidi@ndsu.edu