

# MODELING AND VALIDATION OF AUTONOMOUS RENDEZVOUS AND DOCKING OF AIR BEARING VEHICLES

Amir Mobasher, Alabama A&M University; Paul Shiue, Christian Brothers University; Hossein Jamshidi, Alabama A&M University; Zhengtao Deng, Alabama A&M University

## Abstract

A new mathematical model representing the motion and control of an air-bearing vehicle (Chase Vehicle) was developed. Closed form solutions for the motion of the vehicle are presented. Subsequently, equations are utilized to arrive at a simple equation that describes the thruster firing time as a function of the distance traveled and the number of thrusters being fired. A set of experiments was performed in which the two thrusters at the rear of the chase vehicle were fired for durations of 2 and 3 seconds, and the resulting motions were compared with those obtained for theoretical predictions. Results obtained via this analytical technique compared remarkably well with those obtained with the simulations performed via Simulink including the experimental results obtained in the laboratory. The comparison between the simulations and analytical results yielded an error less than one percent for all the cases considered. Both theoretical and numerical predictions were within 1% for all cases studied.

## Introduction

The research for space exploration has renewed interest in developing technologies to include programs in robotic and manned systems. Rendezvous and Docking missions between two spacecraft have been one of the primary topics of interest for NASA since 1960. Recent evolution of these topics provides examples of the Hubble Space Telescope, Space Shuttle, Russian MIR Dockings, and the International Space Station (ISS) construction missions. A common similarity linking the current NASA mission philosophy and the very first Gemini docking mission is that at least one of these spacecraft has always been piloted by astronauts and supported by a virtual army of ground personnel. The advance of Rendezvous and Docking technologies from manual to automated capabilities is the objective of the Autonomous Rendezvous and Docking (AR&D) project. The reduction in the recurring cost of routine docking missions is essential. For the missions requiring operations due to long communication delays, AR&D becomes predominant. The AR&D project objectives are establishing design criteria, test facilities, procedures, simulation techniques, etc. The simulation techniques influence standardization of the AR&D systems. Establishment of the test facilities and pro-

cedures support the development and verification of future systems prior to flight. Demonstrations of most of the objectives were presented through flight experiments, 6 Degrees of Freedom (6-DoF) Hardware-In-The-Loop (HITL) simulations, and digital test.

The AR&D is primarily comprised of the Chase Vehicle (CV), Target Vehicle (TV), and 3-Point Docking Mechanism (TPDM). These are necessary in one or more phases of an automated docking mission. Some of the AR&D system mission scenarios include: (1) autonomous phasing and rendezvous with a target spacecraft after the CV arrival in-orbit, (2) automated approach and departure maneuvers, and (3) automated "soft dock" with the TV. The AR&D system is able to meet all of the requirements without ground intervention while providing real-time monitoring capability. Presently, NASA has several developing missions that will require AR&D capability.

The Chase Vehicle has an on-board computer that performs hardware command, telemetry, guidance, navigation and control, collision avoidance maneuvers (CAMs) and system monitoring functions. A critical element to the system is a Video Guidance System (VGS). This system provides real-time data to the on-board computer determining the relative position and altitude between the active sensor that is mounted on the chase vehicle and the passive target that is mounted on the target vehicle. The VGS uses laser diodes to illuminate retro-reflectors in the target. Then a solid-state camera detects the return from the target, and a frame grabber and digital signal processor convert the video information into the relative positions and altitudes [1-4]. A new generation of video-based sensors called Advanced Video Guidance System (AVGS) was developed with improved performance and additional capability for longer range operations. The new design combines the sensor head and electronics module into one unit [3]. The Target Vehicle is equipped with a set of cones that align with the TPDM latches. A set of passive reflectors serve as the Global Positioning System (GPS) to the CV having a stabilized altitude. This hardware was successfully integrated and tested at NASA-MSFC FRL (Flight Robotics Laboratory) to verify the operational characteristics of the VGS in the low-earth-orbit environment. The 3-Point Docking Mechanism performs the actual physical latching of the two spacecrafts.

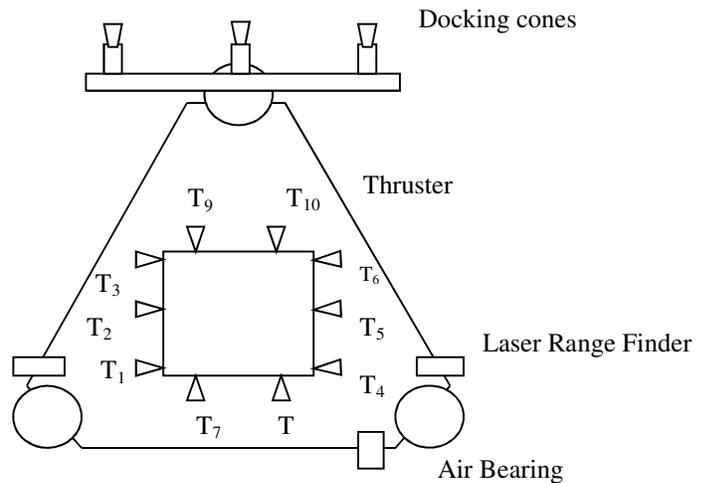
The AR&D system was designed and tested in a 1-G (earth's surface) environment. The operation of the mission is accomplished by stepping through a set of automated maneuvers through which the CV moves towards the TV, leading to an eventual dock [5], [6]. The initial design and integration of the AR&D was completed in 1998 and tested in 1999 at NASA's Marshall Space Flight Center (MSFC). The facility is comprised of a 30 x 10m<sup>2</sup> flat floor which houses an air-sled with 3 degrees of freedom, a 3-DoF Chase Vehicle with associated hardware and software, and a 3-point docking mechanism. The air-sled weighs around 2500 pounds and is equipped with 10 mini thrusters, each capable of generating 4 pounds of thrust and can steer the vehicle in the fore, aft, and yaw orientations. Its purpose was to demonstrate technologies and mission strategies for automated rendezvous and docking of spacecraft in Earth orbit. Some of the latest innovations in Global Positioning System space navigation such as laser-sensor technologies and automated mission-sequencing algorithms were integrated into this system. Performance of the ground-based AR&D system was exceptional during the tests performed over a period of 6 months [6].

In this study, a new mathematical model for the motion of the 3-DoF ground-based Chase Vehicle in the Flight Robotics and Contact Dynamics Simulation laboratory at Marshall Space Flight center was developed. This mathematical model describes the motion of the vehicle as a function of time for the fore and aft positions. The model was then used to make a parametric study to correlate the thruster firing time with distance and the number of thrusters fired. As a result of this study, a two-step docking process was proposed. First, an assessment of the orientation of the CV relative to the target was achieved. Then, a prediction on the thruster firing time requirements was made and the chaser vehicle was brought within the vicinity of the three-point docking system. In the second step, on-board closed loop control algorithms were activated to perform the final docking. A set of experiments was performed in which the two thrusters at the rear of the air sled were fired for durations of 2 and 3 seconds after which the resulting motions were compared with those obtained for theoretical predictions as well as those obtained via simulations.

## Methodology and Approach

The air-bearing vehicle (Chase Vehicle) used in the flight Robotics Lab at NASA-MSFC for the simulation of AR&D scenarios consisted of 3 air bearings responsible for lifting the vehicle, as well as ten thrusters, three-point docking cones and a laser range finder with associated software and hardware. Six compressed air tanks, embedded within the vehicle, supplied the energy needed for lifting the vehicle as

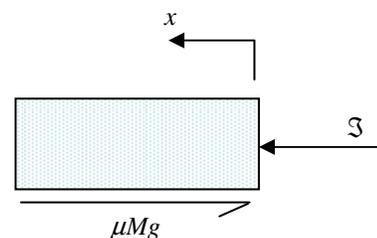
well as generating thrust in the fore and aft directions. Figure 1 depicts a two-dimensional view of the ground-based chaser vehicle with its associated components.



**Figure 1. Schematic of the air-bearing vehicle (Chase Vehicle) used in the Laboratory setup. The vehicle consists of 3 air bearings responsible for lifting the vehicle which are supplied by three pressurized tanks, ten thrusters, three-point docking cones and laser range finder with associated software and hardware**

The simplified model of the air-bearing vehicle is shown in the form of a free body diagram displayed in Figure 2. It has mass  $M$ , with the coefficient of friction  $\mu$ , and is subjected to the thrust force of  $\mathfrak{F}$ . Writing the equation of motion for this system yields:

$$M\ddot{x} = \mathfrak{F} - \mu Mg \quad (1)$$



**Figure 2. Free body Diagram of the air bearing vehicle with the forces acting on it**

Due to the loss of mass (air) in the system, the mass of the air-bearing vehicle at a given time is given by

$$M = M_0 - \dot{m}t \quad , \quad (2)$$

where  $M_0$  is the initial mass of the air-bearing vehicle inclusive of fuel, and  $\dot{m}t$  is the mass of the fuel lost at a given time,  $t$ . The acceleration of the vehicle at a given time  $t$  is obtained as

$$\ddot{x} = \frac{\mathfrak{S}}{M_0 - \dot{m}t} - \mu g \quad (3)$$

Integrating equation (3) using initial condition  $\dot{x}(0) = u_0$  along with simplification, velocity is obtained

$$\dot{x} = u_0 + \frac{\mathfrak{S}}{\dot{m}} \ln\left(\frac{M_0}{M_0 - \dot{m}t}\right) - \mu g t \quad (4)$$

Further direct integration of equation (4) applying initial condition  $x(0) = x_0$  yields a non-linear expression for the position as:

$$x = \left[ x_0 - \left(\frac{\mathfrak{S}}{\dot{m}}\right)\left(\frac{M_0}{\dot{m}}\right)\ln\left(\frac{M_0}{M_0 - \dot{m}t}\right) \right] + \left[ u_0 + \frac{\mathfrak{S}}{\dot{m}} \ln\left(\frac{M_0}{M_0 - \dot{m}t}\right) + \frac{\mathfrak{S}}{\dot{m}} \right] \cdot t - \frac{1}{2} \mu g t^2 \quad (5)$$

Equations (3), (4), and (5) are valid as long as the thrusters are on. Once the thrusters are shut down (i.e., no external load is applied to the vehicle and  $\dot{m} = 0$ ), the foregoing equations reduce to the following:

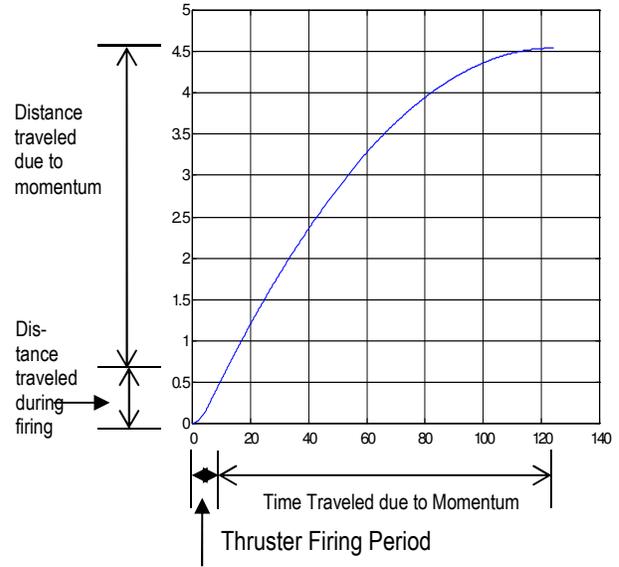
$$\text{acceleration: } \ddot{x} = -\mu g \quad (6)$$

$$\text{velocity: } \dot{x} = u_0 - \mu g t \quad (7)$$

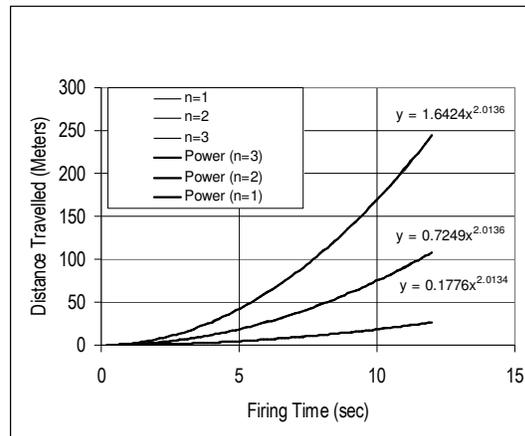
$$\text{position: } x = x_0 + u_0 t - \frac{1}{2} \mu g t^2 \quad (8)$$

Equations (3) through (5) are solved for the case while thrusters are on and equations (6) through (8) are used when the thrusters are turned off. The CV goes to rest via friction. It must be noted that equations (6) through (8) are implicit within equations (1) through (4). A reasonable graphic representation of the position of the CV as a function of time is shown in Figure 3. A typical laboratory mission consists of a thruster firing time in the range 1 to 12 seconds during which the CV is accelerated. After that, all thrusters are shut down and the momentum carries the air-bearing vehicle forward. However, very low friction in the laboratory environment is taken into consideration for the deceleration period of the vehicle.

Equation (5) provides a relationship for the total distance traveled,  $d$ , as a function of thruster firing time,  $t$ , and the number of thrusters fired,  $n$ ; that is,  $d = d(t, n)$ . However, in a practical sense, the interest lies in finding an expression that describes a relationship between the firing time of the thrusters as a function of the total distance that the CV travels and the number of thrusters fired, that is  $t = t(d, n)$ .



**Figure 3. Typical motion of an air-bearing vehicle once the thrusters are fired for a certain period of time. The first-stage vehicle accelerates as long as thrusters are active. Once thrusters are shut down, the vehicle travels a certain distance and finally comes to rest due to friction**



**Figure 4. Family of curves for distance traveled as a function of the number of thrusters fired**

To accomplish  $t = t(d, n)$  a test matrix varying thruster firing time and number of thrusters ( $n$ ) was established. In the test matrix, the thruster firing time ranges between 0.2 to 12 seconds and the number of thrusters varies from 1 to 3. Simulations were performed for each case, and the distance corresponding to the thruster firing time and number of thrusters was recorded. To establish the relationship for  $d = d(t)$ , a least square curve was passed through the resulting data points (distances). To find a correlation for the number of thrusters, a least square curve was again passed through the coefficients of the individual set of equations, which are shown in Figure 4. The resulting equation for the individual

set of equations was represented by  $d = d(n,t)$ . The form of this resulting equation was obtained as

$$d = 0.183n^2t^2 \quad (9)$$

which, upon simplification, yields

$$t = 2.337 \frac{\sqrt{d}}{n} \quad (10)$$

Equation (10) emphasizes the time required for firing  $n$  thrusters to carry the CV a total distance  $d$  either in the  $x$  or  $y$  direction. In practice, this equation may be employed to position the CV within the vicinity of the target area and then use the control algorithms for docking the CV.

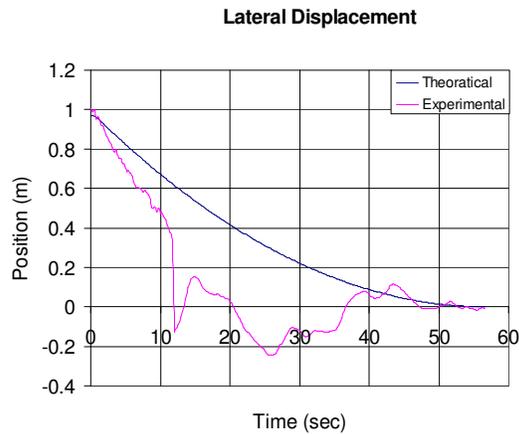
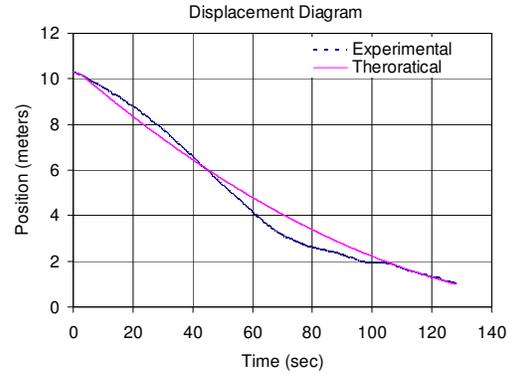
## Results and Discussions

In order to compare the validity of equation (10) Simulink was used to simulate the physical model and then employed to run a set of parametric tests for a varying number of thrusters from  $n = 1, 2,$  and  $3$  for firing duration  $t = 1$  through  $8$  seconds. The outcome of the test cases for the analytical results versus the simulations is presented in Table 1.

**Table 1. Comparison of the distance traveled obtained via analytical and simulation results performed by Simulink for the thruster firing times of 1 through 8 seconds and number of thrusters varying from 1 to 3**

Firing Duration (sec)	Number of thrusters Fired		
	$n = 1$	$n = 2$	$n = 3$
1	0.18 m (analytical)	0.73 m	1.64 m
	0.18 m (numerical)	0.74 m	1.60 m
2	0.73 m	2.93 m	6.59 m
	0.71 m	2.92 m	6.58 m
3	1.64 m	6.59 m	14.83 m
	1.60 m	6.58 m	15.01 m
8	11.72 m	46.71 m	-
	11.53 m	46.63 m	-

In all cases, the analytical results obtained via equation (10) compared remarkably well, within 1% error limit, with those obtained via the Simulink simulations. Figure 5 compares the results of the theoretical equations for displace-



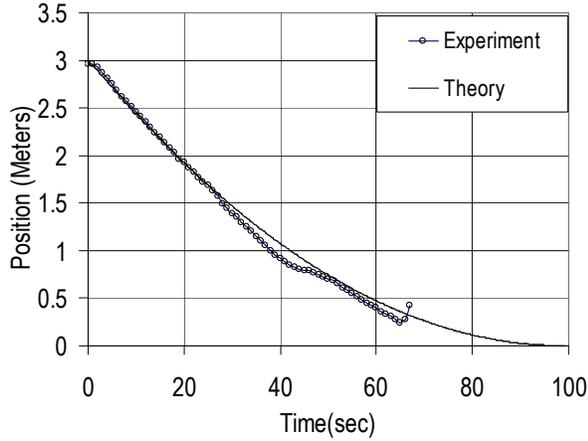
ment given by equation (5) with those of the experimental results for the fore and aft displacements.

**Figure 5. Comparison of theoretical and experimental results for (a) forward, and (b) lateral displacements as a function of time of the CV**

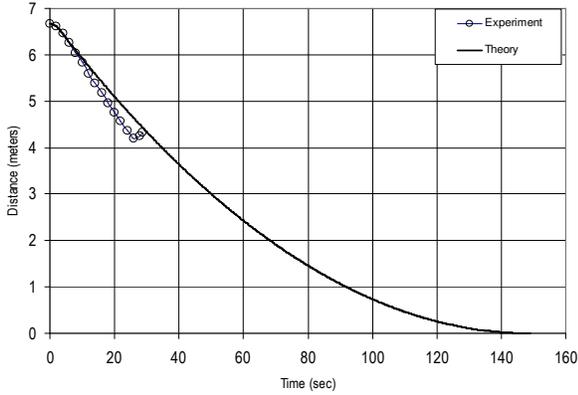
The control algorithm was utilized for zeroing in the air-bearing vehicle to the one-meter range. The experimental results for the fore displacement compared relatively well with the theoretical prediction. However, for the lateral (aft) position, experimental results deviated significantly from the theoretical prediction. To validate equations (5) and (10), a set of experiments was performed in which two thrusters at the rear of the air sled were fired for durations of 2 and 3 seconds. The resulting motion was compared with those obtained for the theoretical model.

Further examination of the theory and experiments was conducted for firing durations of 2 and 3 seconds. The plots are displayed in Figures 6(a) and 6(b), respectively. For the case of a 2-second firing time, the maximum distance traveled was predicted as 3m. The experiment and theory matched fairly well. It appears that for both cases, where thruster firing period increases, the theory and experiment compared somewhat better than those of smaller thruster firing periods. Each experiment was repeated three times;

however, due to the imperfections in the floor and other noises within



(a)



(b)

**Figure 6. Comparison of path prediction between experimental and theoretical results for 2 seconds firing duration (a) and 3 second firing time (b)**

the laboratory environment such as air movement caused from air conditioning ducts, it was difficult to draw meaningful average data from the three tests. Experimental data in this study proved to be highly sensitive to the environment. Other sources of error may be due to the friction coefficient used for the simulation and time delay used in the software program in firing the thrusters.

Further, to examine the experimental results (Figure 6-a) obtained in the laboratory with the analytical results obtained in theory, given by equation (5), the hypothesis statistical test was conducted.

In the case under investigation, the experimental results pertain to measurements of distance traveled by the sled for

a firing duration of 2 seconds and analytical results implies the distance traveled by the sled for the aforementioned firing duration and calculated by equation (5). The hypothesis testing enables one to make a comparison between the performances of the means of experimental with analytical results and determine if the differences in the means are statistically significant. The paired t-test is the appropriate statistical test since the two populations, experimental and analytical, have the same related characteristic of interest which is time. Therefore, the following hypothesis was conducted:

- $H_0$ : There is no difference in the means of two related populations of experimental and analytical data
- $H_1$ : There is a difference

The above hypothesis can be represented as:

- $H_0$ :  $\mu_D = 0$  (where  $\mu_D = \mu_{\text{experimental}} - \mu_{\text{analytical}}$ )
- $H_1$ :  $\mu_D \neq 0$

where  $\mu_D$  is the mean difference between the experimental and analytical results. The pair t-test was calculated as:

$$t = \frac{\bar{D} - \mu_D}{\frac{S_D}{\sqrt{n}}} \quad (11)$$

where  $\bar{D}$  is the average difference between the experimental and analytical results given by:

$$\bar{D} = \frac{\sum_{i=1}^n D_i}{n}$$

and

$$S_D = \sqrt{\frac{\sum_{i=1}^{94} (D_i - \bar{D})^2}{n - 1}}$$

where  $S_D$  is the standard deviation of the differences. For a two-tailed test of the hypothesis with a given level of significance,  $\alpha$ , you reject the null hypothesis if the computed t-value obtained in equation (11) is greater than the upper critical value ( $t_{n-1}$ ) or if it is less than the lower critical value ( $-t_{n-1}$ ). Based on the sample of 94 observations, taken within the same time frame for both experimental and analytical results, using the standard SPSS statistical package, Table 2 was obtained.

**Table 2. Results of Pair t-test for Experimental Data Versus Analytical Data Using SPSS Statistical Package**

Paired Samples Statistics

		Mean	N	Std. Deviation
Pair 1	EXPERIMENTAL	2.028060	94	0.486873006
	THEORETICAL	2.025049	94	0.476461643

#### Paired Samples Correlations

		N	Correlation	Sig
Pair 1	EXPERIMENTAL & THEORETICAL	94	1.000	.000

		Pair 1		
		EXPERIMENTAL & THEORETICAL		
Paired Difference	Mean	0.00301		
	St. Dev	0.0131		
	St Err Mean	N/A		
	95 % Confidence Interval of the difference	Lower	0.00033	
		Upper	0.0057	
	t	2.228		
	df	93		
	Sig(2-tailed)	0.028		

The results in Table 2 reveal that the calculated t-value from equation (11) was  $t = 2.228$ . Based on the  $\alpha$  value (level of significance), the following analysis was done:

$\alpha$	Upper critical value	Lower critical value
0.05	1.9853	-1.9853
0.01	2.3662	-2.3662
0.005	2.6286	-2.6286

The null hypothesis was accepted (there is no difference between the mean performance of the experimental versus analytical data) as long as the t-value is between the upper and lower critical values. For  $\alpha = 0.05$  the null hypothesis was rejected and it was concluded that the mean performance of the experimental results was significantly different than the theoretical results. However, for  $\alpha = 0.01$  or smaller the null hypothesis was accepted since  $-t_{n-1} = -2.3662 < t = 2.228 < t_{n-1} = 2.3662$  and concluded that the mean performances of the experimental results is the same as the theoretical results. In addition, the RMS-error which measures the performance of experimental data was obtained as follow:

$$RMS = \sqrt{\frac{\sum (X_e - X_t)^2}{\sum (X_t)^2}} = \sqrt{\frac{0.566}{415.324}} = 0.0369$$

The result of RMS revealed the percentage of error or deviation of the experimental results from the theoretical results to be as low as  $RMS = .0369$ .

## Conclusions

Analytical solutions obtained describe the position of the CV as a function of time and number of thrusters fired. A working equation was developed that defined the firing time requirement for the distance traveled. This equation compared well with the simulation results obtained via Simulink. The experimental data obtained for the 2- and 3-second firing times and 2 thrusters compared relatively well with the numerical predictions. The statistical analysis performed using SPSS software reinforced the validity of the theoretical equation versus the experimental at the  $\alpha$ -less-than-0.01 level of significance.

## Acknowledgements

The authors would like to acknowledge the contribution of the entire AR&D group at NASA Marshall Space Flight Center for this study. Thanks are due to Dr. Mike Freeman and Dr. Gerald Karr for their coordination and support of the Faculty Summer Internship Program leading to this study. Also thanks are due to Dr. M. A. Alim for useful discussion.

## References

- [1] Howard, R. T., M. L. Book and T. C. Bryan, "Video-based sensor for tracking 3-dimensional targets," Proceedings of Spice Laser Radar technology and Applications, 4167, 242-251, 2000.
- [2] Howard, R. T., T. C. Bryan, M. L. Book and J. Jackson, "Active sensor system for automatic rendezvous and docking," Proceedings of Spice Laser Radar technology and Applications II, 106-115, 1997
- [3] Roe, F. D. and R. T. Howard, "The successful development of an automated rendezvous and capture (AR&C) system for the National Aeronautics and Space Administration," Proceedings of the 26th Annual American Astronautical Society Guidance and Control Conference, 2003
- [4] Tanner, E., S. Granade and C. A. Whitehead, "Autonomous rendezvous and docking sensor suite," Proceedings of Spice Laser Radar Technology and Applications VIII, 5086, 329-339, 2003
- [5] Cruzen, C. A. and J. J. Lomas, "Design of the automated rendezvous and capture docking system," Pro-

ceedings AIAA International Space Station Service Vehicles Conference, 2000.

- [6] Cruzen, C. A., J. J. Lomas and R. W. Dabney, "Test results for the automated rendezvous and capture system," Proceedings of 23<sup>rd</sup> annual American Astronautical Society Guidance and Control Conference, 2000

## Biographies

**AMIR MOBASHER** is an Associate Professor in the department of Mechanical Engineering at Alabama A&M University. He received his PhD from University of Alabama-Huntsville. Currently, he is a NASA Administrator's Fellow as part of the United Negro College Funds Special Programs (UNCFSP) working at NASA Marshall Space Flight center. His research spans the areas of Computational Fluid Dynamics, Biodynamics, Manufacturing Systems, Instrumentation and Control Systems and Autonomous Systems for Lunar Rovers. He may be reached at [amir.mobasher@aamu.edu](mailto:amir.mobasher@aamu.edu).

**PAUL SHIUE** is a Professor and Chair of the Mechanical Engineering Department at CBU. He received his B.S. from Tatung University in Taiwan and his M.S. and Ph.D. degrees from the University of Memphis. He is an associate member of the American Society of Mechanical Engineers and a professional member of the American Society for Engineering Education. Dr. Shiue is also a member of editorial advisory board of the International Journal of Engineering Education (IJEE), and he served as guest editor of a special issue in manufacturing engineering education (V20, N4, 5). He was five times a NASA/ASEE Summer Faculty Fellow at Marshall Space Flight Center. Currently, he is focusing on concurrent engineering and design through manufacturing and product realization processes. Dr. Shiue specializes in mechanical systems design and manufacturing, kinematics and dynamics, vibrations, and material testing. He was recognized as the Professor of the Year by Tau Beta Pi, Delta of Tennessee in the 1997-98 academic year. He also received the 1997 Featured Engineer for Excellence in Educating Engineers, Memphis Joint Engineering Council Achievement Award by CBU. He has been recognized in Who's Who Among America's Teachers in 1996, 2002, and 2005. He may be reached at [pshiue@cbu.edu](mailto:pshiue@cbu.edu).

**HOSSEIN JAMSHIDI** is a full professor at Alabama A&M University, Normal, AL. He received his Ph.D. from The University of Alabama in Huntsville in Industrial and Systems Engineering. His areas of specialty are production operations, mathematical programming, statistical analysis, and analysis of manufacturing systems. He has published many papers in these areas. He has conducted workshops for major companies such as Rolls-Royce/The Purdy Corp and Boeing/DACA program, Sponsored by U.S. Air Force. He has won many awards at the university, state, and national

level. Dr. Jamshidi may be reached at [hossein.jamshidi@aamu.edu](mailto:hossein.jamshidi@aamu.edu)

**ZHENGTAO DENG** is a professor in the department of Mechanical Engineering at Alabama A&M University. His research is focused on Advanced Space Propulsion and High Performance Computing. Dr. Deng may be reached at [zhengtao.deng@aamu.edu](mailto:zhengtao.deng@aamu.edu).