Mode-Frequency Analysis of Laminated Spherical Shell

Umut Topal
Department of Civil Engineering
Karadeniz Technical University
61080, Trabzon, Turkey
umut@ktu.edu.tr

Abstract

This paper deals with mode-frequency analysis of a simply-supported equal-sided sector of a laminated spherical shell. The problem is modelled using finite element package program ANSYS. The formulation based on first-order shear deformation theory. Four elements are chosen along each edge of the sector. The reduced method of eigenvalue solution is chosen for the undamped mode-frequency analysis. The first five modes are extracted to obtain the fundamental frequency (first mode natural frequency). The numerical studies are conducted to determine the effects of width-to-thickness ratio (b/h), degree of orthotropy (E_1/E_2), fiber orientations (θ) on the the non-dimensional fundamental frequency. The results are given in graphical form and the obtained results are compared.

Introduction

Fibre reinforced laminated composite materials are being increasingly used in aerospace and other applications due to their high specific strength, high specific stiffness and low specific density. Composites, in the form of shells, find application in aerospace and other industries. Spherical shells are used for many structures such as aerospace vehicles, roof domes, pressure vessels and submarines. Thus, the free vibration of composite spherical shell is an important problem to be investigated. The free vibration of laminated shell of revolution has been considered in literatures [1-17].

This paper deals with mode-frequency analysis of simply supported laminated spherical shell using first-order shear deformation theory (FSDT). The reduced method of eigenvalue solution is chosen for the analysis. The first five modes are extracted to obtain the fundamental frequency (first mode natural frequency).The numerical studies are conducted to determine the effects of width-to-thickness ratio (b/h), degree of orthotropy (E_1/E_2), fiber orientations (θ) on the non-dimensional fundamental frequency. The results are given in graphical form and the obtained results are compared.

Statement of the problem

Figure 1 shows a laminated doubly curved panel (open shell) of rectangular planform, of total thickness h. x_1 and x_2 represent the directions of the lines of curvature of the middle surface, while the x_3-axis is a straight line perpendicular to the middle surface. R_i (i=1, 2) denotes the principal radii of curvature of the middle surface. The thickness of the *k*th layer is denoted by $h^{(k)} = x_3^{(k)} - x_3^{(k-1)}$, in which $x_3^{(k)}$ and $x_3^{(k-1)}$, k=1,…..,N are the distances from the reference surface to the outer (top) and inner (bottom) faces, respectively, of the *k*th lamina, with N being the total number of layers.

The displacement field, based on first-order shear deformation theory, is given by

$$\bar{u}_i = (1 + x_3/R_i)u_i + x_3\phi_i, \quad i=1, 2, \quad \bar{u}_3 = u_3 \tag{1}$$

in which $\bar{u}_i$ (i=1, 2, 3) represents the components of displacement at a point x_i (i=1, 2, 3), while u_i denotes the same for the corresponding point at the mid-surface. Assumptions of shallowness ($\xi_3 \langle\langle R_i$), vanishing geodesic curvatures, transverse inextensibility and the first-order shear deformation theory, corresponding kinematic relations of a doubly curved shell are given:

$$\begin{aligned} &\varepsilon_1 = \varepsilon_1^o + x_3\kappa_1,\ \varepsilon_2 = \varepsilon_2^o + x_3\kappa_2,\ \varepsilon_4 = \varepsilon_4^o, \\ &\varepsilon_5 = \varepsilon_5^o,\ \varepsilon_6 = \varepsilon_6^o + x_3\kappa_6 \end{aligned} \tag{2}$$

where

$$\begin{aligned} &\varepsilon_1^o = u_{1,1} + \frac{u_3}{R_1},\ \varepsilon_2^o = u_{2,2} + \frac{u_3}{R_2},\ \varepsilon_4^o = u_{3,2} + \phi_2 - \frac{u_2}{R_2}, \\ &\varepsilon_5^o = u_{3,1} + \phi_1 - \frac{u_1}{R_1},\ \varepsilon_6^o = u_{2,1} + u_{1,2},\ \kappa_1 = \phi_{1,1}, \\ &\kappa_{21} = \phi_{2,2},\ \kappa_6 = \phi_{2,1} + \phi_{1,2} - \frac{1}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\left(u_{2,1} - u_{1,2}\right) \end{aligned} \tag{3}$$

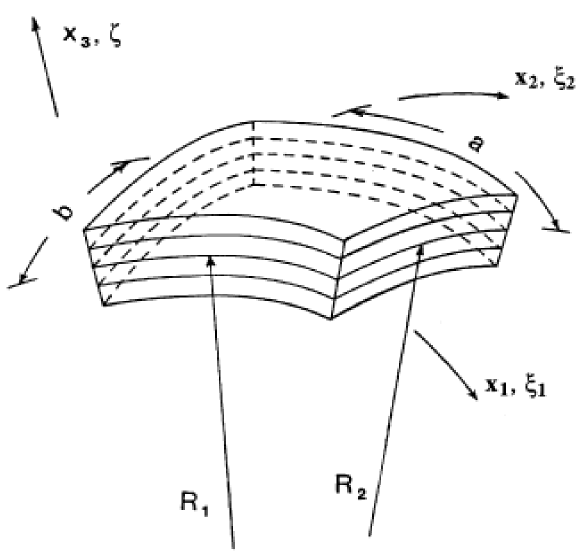

Figure 1. A laminated doubly curved panel of rectangular planform

The equations of motion can be derived by substituting the above kinematic relations in an expression for virtual work, the details of which are omitted in the interest of brevity of presentation. These can be written as follows:

$$N_{1,1} + N_{6,2} + \frac{1}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)M_{6,2} + \frac{Q_1}{R_1} = C_1^{''} \tag{4}$$

$$N_{6,1} + N_{2,2} - \frac{1}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)M_{6,1} + \frac{Q_2}{R_2} = C_2^{''} \tag{5}$$

$$Q_{1,1} + Q_{2,2} - \frac{N_1}{R_1} - \frac{N_2}{R_2} = C_3^{''} \tag{6}$$

$$M_{1,1} + M_{6,2} - Q_1 = C_4^{''} \tag{7}$$

$$M_{6,1} + M_{2,2} - Q_2 = C_5^{''} \tag{8}$$

where

$$C_i^{''} = \left(P_1 + \frac{2P_2}{R_i}\right)u_{i,tt} + \left(P_2 + \frac{P_3}{R_i}\right)\phi_{i,tt} \quad (i=1,2),\; C_3^{''} = P_1 u_{3,tt} \tag{9}$$

$$C_{i+3}^{''} = \left(P_2 + \frac{P_3}{R_i}\right)u_{i,tt} + P_3\phi_{i,tt} \quad (i=1,2), \tag{10}$$

in which surface-parallel and rotatory inertias are included. P_i (i=1, 2, 3) are as presented below

$$(P_1, P_2, P_3) = \sum_{k=1}^{N} \int_{x_3^{(k-1)}}^{x_3^k} \rho^{(k)} \left(1, x_3, x_3^2\right) dx_3 \tag{11}$$

where $\rho^{(k)}$ represents the density of the layer material. N_1, N_2, N_6 are the surface-parallel stress resultants, while M_1, M_2, M_6 are moment resultants (stress couples), and Q_1 and Q_2 are the transverse shear stress resultants, all per unit length. The stress resultants and couples (N_i, M_i, Q_i) are given as follows:

$$N_1 = A_{11}\left(u_{1,1} + \frac{u_3}{R_1}\right) + A_{22}\left(u_{2,2} + \frac{u_3}{R_2}\right) + B_{11}\phi_{1,1} + B_{12}\phi_{2,2}, \tag{!2}$$

$$N_6 = A_{66}\left(u_{2,1} + u_{1,2}\right) + B_{66}\left\{\phi_{2,1} + \phi_{1,2} - \frac{1}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\left(u_{2,1} - u_{1,2}\right)\right\}, \tag{13}$$

$$M_1 = B_{11}\left(u_{1,1} + \frac{u_3}{R_1}\right) + B_{12}\left(u_{2,2} + \frac{u_3}{R_2}\right) + D_{11}\phi_{1,1} + D_{12}\phi_{2,2}, \quad (14)$$

$$M_6 = B_{66}\left(u_{2,1} + u_{1,2}\right) + D_{66}\left\{\phi_{2,1} + \phi_{1,2} - \frac{1}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\left(u_{2,1} - u_{1,2}\right)\right\} \quad (15)$$

$$Q_1 = A_{55}\left(u_{3,1} + \phi_1 - \frac{u_1}{R_1}\right)K_1^2 \quad (16)$$

where A_{ij}, B_{ij} and D_{ij} (i, j=1, 2, 6) are extensional, coupling, and bending rigidities, respectively, while A_{ij} (i, j=4, 5) denotes transverse shear rigidities. N_2, M_2 and Q_2 can be obtained from the expressions for N_1, M_1 and Q_1, respectively, by replacing subscript 1 by 2, 5 by 4, $\frac{1}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ by $-\frac{1}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$, and vice versa. K_1^2 and K_2^2 are shear correction factors.

For the finite element analysis, if the damping is neglected, the equation of motion of the structure for free vibration can be written as

$$[M]\{\ddot{D}\} + [K]\{D\} = \{0\} \quad (17)$$

where $\{D\}$ is a vector containing the unrestrained nodal degrees of freedoms, $[M]$ is a structural mass matrix, $[K]$ is a structural stiffness matrix. Since $\{D\}$ undergoes harmonic motion, the vectors $\{D\}$ and $\{\ddot{D}\}$ become

$$\{D\} = \{\overline{D}\}\sin\omega t, \qquad \{\ddot{D}\} = -\omega^2\{\overline{D}\}\sin\omega t \quad (18)$$

where $\{\overline{D}\}$ vector contains the amplitudes of $\{D\}$ vector and ω is the frequency. Therefore, Eq. (17) can be written in as

$$([K] - \lambda[M])\{\overline{D}\} = 0 \quad (19)$$

where $\lambda = \omega^2$ is the eigenvalue and $\{\overline{D}\}$ becomes the eigenvector

Numerical Results and Discussions

For this study, a square plate with the following dimensions and mechanical properties is selected:

$$a{=}b{=}100\text{mm}, R_1 = R_2 = 300\text{mm}, h = 1\text{mm}, E_1 = 25\text{x}10^6\ \text{Pa}, E_2 = 1\text{x}10^6\ \text{Pa},$$
$$G_{12} = G_{13} = 0{,}5\text{x}10^6\ \text{Pa}, G_{23} = 2\text{x}10^6\ \text{Pa},$$
$$\nu_{21} = 0{,}25,\ \rho = 1\text{gm}/\text{mm}^3$$

Effect of Material Anisotropy

Three-layer cross-ply (0/90/0) and angle-ply (45/-45/45) laminates with b/h=100 are analysed to study the effect of material anisotropy. An increase of E_1 / E_2 ratio, keeping E_2 same, leads to an increase in the dimensionless fundamental frequency in the case of both cross-ply and angle-ply laminate (Figure 2). Also as seen, the fundamental frequencies in the case of angle-ply laminate are higher than those in the case of both cross-ply laminate.

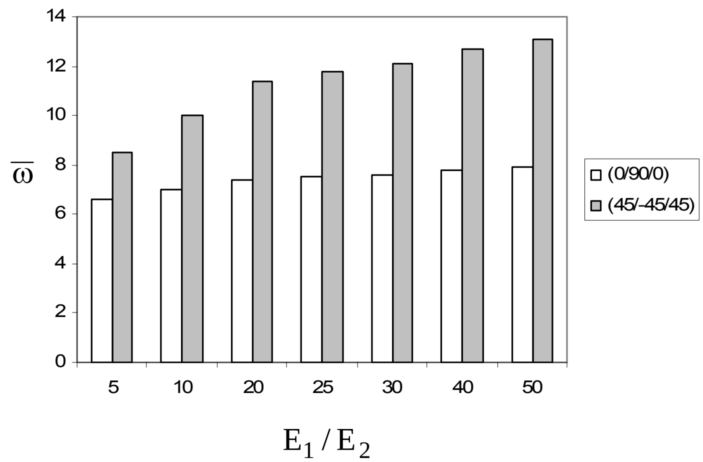

Figure 2. Variation of fundamental frequency with material anisotropy

Effect of Fibre Orientation

Four-layer symmetric $(\alpha/-\alpha/-\alpha/\alpha)$ and anti-symmetric $(\alpha/-\alpha/\alpha/-\alpha)$ laminates with the angle of fibre orientation varying from 0^{o} to 45^{o} with b/h=100 are analysed. As can be seen from Figure 3, a change in fibre orientation angle from 0^{o} to 45^{o} leads to an increase in the fundamental frequency of vibration. Also as seen, the fundamental frequencies in the case of symmetric layup are higher those in the case anti-symmetric layup for 15^{o} and 30^{o}, but it is reverse for 45^{o}.

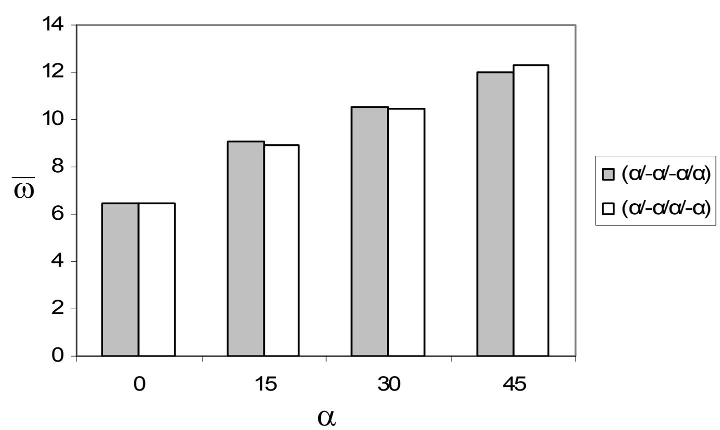

Figure 3. Variation of fundamental frequency with fibre orientation angle

Effect of Width-to-thickness Ratio

Four-layer cross-ply and angle-ply laminates with symmetric and anti-symmetric arrangement of layers and having different width-to-thickness ratios are analysed. As can be seen from Figure 4, as a/h increases, the dimensionless frequency increases. Symmetric layup has higher frequency as compared to anti-symmetric layup in the case of cross-ply laminates whereas the reverse is the case with angle-ply laminates.

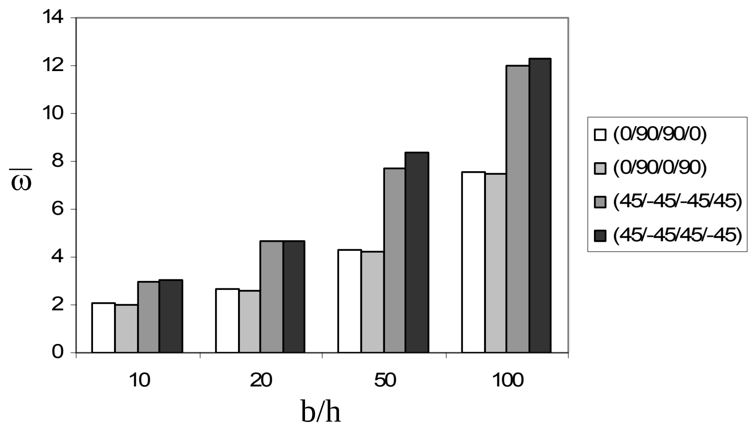

Figure 4.Variation of fundamental frequency with a/h ratio

Conclusions

In this paper, mode-frequency analysis of laminated spherical shell using a finit element model, based on first-order shear deformation theory is presented. The non-dimensional fundamental frequency of vibration is found to increase with increase in width-to-thickness ratio, material anisotropy and angle of fibre orientation. The fundamental frequencies in the case of angle-ply laminate are higher than those in the case of both cross-ply laminate for the effect of the material anisotropy. The fundamental frequencies in the case of symmetric layup are higher those in the case anti-symmetric layup for 15^o and 30^o, but it is reverse for 45^o. Symmetric layup has higher frequency as compared to anti-symmetric layup in the case of cross-ply laminates whereas the reverse is the case with angle-ply laminates for the effect of the width-t0-thickness ratio. This paper can be investigated for different finite element formulations, boundary conditions, aspect ratios and number of layers.

References

[1] Cugnoni, J., Gmür Th. and Schorderet, A. Identification by Modal Analysis of Composite Structures Modelled with FSDT and HSDT Laminated Shell Finite Elements, Composites: Part A 35, 2004, Vol. 35, pp. 977-987.

[2] Chaudhuri, R. A. and Kabir, H. R. H. Effect of Boundary Constraint on the Frequency Response of Moderately Thick Doubly Curved Cross-Ply Panels Using Mixed Fourier Solution Functions, Journal of Sound and Vibration, 2005, Vol. 283, pp. 263-293.

[3] Kabir, H. R. H. and Chaudhuri, R. A. Free Vibrations of Anti-Symmetric Angle-Ply Finite Doubly Curved Shells, International Journal of Solids and Structures, 1991, Vol. 28, pp. 17–32.

[4] Qatu, M.S. and Leissa, A. W. Free Vibrations of Completely Free Doubly Curved Laminated Composite Shallow Shells, Journal of Sound and Vibration, 1991, Vol. 151, pp. 9-29.

[5] Ding, K. and Tang, L. Exact Solution for Axisymmetric Thick Laminated Shells, Composite Structures, 1999, Vol. 46, pp. 125-129.

[6] Liew, K. M., Peng, L. X. and Ng, T. Y. Three-Dimensional Vibration Analysis of Spherical Shell Panels Subjected to Different Boundary Conditions, International Journal of Mechanical Sciences, 2002, Vol. 44, pp. 2103–2117.

[7] Ram, K. S. S. and Babu, T. S. Free Vibration of Composite Spherical Shell Cap with and without A Cutout, Computers and Structures , 2002, Vol. 80, pp. 1749-1756.

[8] Haojiang, D and Weiqiu, C. Exact Shell Theory Analysis of Free Vibrations of Submerged Thin Spherical Shells, Int. J. Solids Structures, 1998, Vol. 35, No. 33, pp. 4381-4389.

[9] Tan, D. Y. Free Vibration Analysis of Shells of Revolution, Journal of Sound and Vibration, 1998, Vol. 213, No. 1, pp. 15-33.

[10] Young, P. G. A Parametric Study on The Axisymmetric Modes of Vibration of Multi-Layered Spherical Shells with Liquid Cores of Relevance to Head Impact Modelling, Journal of Sound and Vibration, 2002, Vol. 256, No. 4, pp. 665-680.

[11] Lee, J. J., Oh, I. K., Lee, I. and Rhiou J. J. Non-linear Static and Dynamic Instability of Complete Spherical Shells Using Mixed Finite Element Formulation, International Journal of Non-Linear Mechanics, 2003, Vol. 38, pp. 923 – 934.

[12] Engin, A. E. and Liu, Y. K. Axisymmetric Response of A Fluid-Filled Spherical Shell in Free Vibrations, Journal of Biomechanics, 1970, Vol. 3, No. 1, pp. 11-16.

[13] Naghieh M. snd Hayek, S. J. Vibration of Fiber-Reinforced Spherical Shells, Fibre Science and Technology, 1971,Vol. 4, No. 2, pp. 115-137.

[14] De Souza, V. C. M. and Croll, C. G. A. Free Vibrations of Orthotropic Spherical Shells Engineering Structures, 1981, Vol. 3, No. 2, pp. 71-84.

[15] Niordson, F. I., Free Vibrations of Thin Elastic Spherical Shells, International Journal of Solids and Structures, 1984, Vol. 20, No. 7, pp. 667-687.

[16] Evirgen, H and Ertepinar, A. Stability and Vibrations of Layered Spherical Shells Made of Hyperelastic Materials, International Journal of Engineering Science, 1989, Vol. 27, No. 6, pp. 623-632.

[17] Narasimhan, M. C. and Alwar, R. S. Free Vibration Analysis of Laminated Orthotropic Spherical Shells, Journal of Sound and Vibration, 1991, Vol. 154, No. 3, pp. 515-529.

Biographies

TOPAL UMUT is a research assistant at Mechanical Deparment at Karadeniz Technical University in TURKEY. He is a PhD. student. He works on laminated composite structures. For example, vibration, buckling, strength and optimization of laminated composite structures.