Stochastic Models for Enhancing Agility of Supply Chains

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Abstract

This paper deals with application of the stochastic inventory model to the three-tier supply chain and verifying the values obtained by mathematical model in physical simulation. We investigate three-stage serial supply chain with stochastic demand and fixed replenishment lead-time. Inventory holding costs are charged at each stage, and each stage may incur a consumer backorder penalty cost charged by primary supplier to secondary supplier. The customer-demand follows Poisson distribution. We implement Base Stock model for inventory control at both suppliers. Physical simulation is then designed such a way that it satisfies all the assumptions for mathematical model. Simulation is run to verify the values obtained from mathematical model.

Introduction

Comparison of values obtained by mathematical model of base stock model and simulation values for three-tier supply chain where the demand follows a Poisson distribution is the primary subject of this paper.

We have considered a virtual company with three-tier supply chain. Base Stock Inventory Model is applied at the primary supplier, secondary supplier and at the warehouse. We calculated the fill rate, probability that the order has arrived before demand for each case and calculated reorder points at primary supplier, secondary supplier and warehouse for five replenishment lead times (12,8,6,4 and 2 months)(Table No 2) using this mathematical model.

Simulation was run to confirm that optimum inventory levels i.e. reorder points at warehouse, primary supplier and secondary supplier. Base stock model is applied throughout supply chain.

Background

Inventory management within the supply chain is critical when the demand is not deterministic. Demand variability increases as one moves up the supply chain away from customer and any small changes in customer demand can result in large variation in orders upstream. This phenomenon is known as Bullwhip effect. Thus, it is necessary to study inventory models for uncertain demand. Wilson (1934) [1, 2] has done major work on statistical modeling of production and inventory control. Wilson breaks the inventory control problem into two distinct parts: 1. Determining the order quantity, which is the amount of inventory that will be produced with each replenishment. 2. Determining the reorder point or the inventory level at which replenishment will be triggered. P Zipkin [3] emphasized on backorder policies in multistage supply chain where base stock inventory model is used.

A survey was conducted to identify the key issues related to supply chain facing the ship building industries under a project of NSRP. The key issues are: long lead time, inventory cost, scheduling problem, irregular performance, challenge in synchronizing flow with suppliers, vendors furnishing information late. Wincel and Jeffrey P. [4] introduce lean methodology as the key factor in its supply chain strategies. Issues related to streamlining supply chain are discussed by Copacino, William C. and Cooper [5,6]. Inventory issues in supply chain are explored further by Handfield, Robert B., Nichols, Ayers and James[7,8]

Mathematical Models

Taylor's principles of scientific management [10] were precursor to a host of mathematical models designed to solve the problems associated with manufacturing planning and control. These models formed the foundation for instruction in several operations management (OM) areas like inventory control, scheduling, capacity planning, forecasting and quality control. Of these areas, inventory control saw the development of a variety of mathematical models. These models can be subdivided into two broad areas. Those, that assumed demand to be known and those, which assumed demand to be stochastic in nature.

1. Deterministic Models

One of the earliest deterministic models came out of work of Ford W. Harris [9] (1913). Harris's Economic Order Quantity (EOQ) model has been widely studied. His model makes the assumptions that:

1. Production is instantaneous
2. Delivery is immediate
3. Demand is deterministic
4. Demand is constant over time
5. Each production run incurs a constant setup cost

With these assumptions, he derived the following formula for calculating the total inventory cost per product:

$$Y(Q) = \frac{hQ}{2D} + \frac{A}{Q} + c$$

Where D = Demand rate in units per year
c = Unit production cost
A = Constant setup cost to produce a lot
h = Holding cost in dollars per unit per year
Q = Lot size in units

The lot size that minimizes Y(Q) in the previous equation is:

$$Q^* = \sqrt{\frac{2AD}{h}}$$

The Economic Production Lot Model (EPL) propose by Taft [11], modifies the EOQ model to include finite and predictable production rate P.

$$Y(Q) = \frac{AD}{Q} + \frac{h(1 - D/P)Q}{2} + Dc$$

Minimizing equation 14 yields:

$$Q^* = \sqrt{\frac{2AD}{h(1 - D/P)}}$$

Wagner-Whitin model [12] considers the problem of determining production lot size when demand is deterministic but varies with time.

2. Stochastic Models

Statistical modeling of production and inventory control dates back to Wilson's work [13]. Wilson breaks inventory control problems into two parts:

1. Determining the **order quantity**, which is the amount of inventory that will be purchased or produced with each replenishment.
2. Determining the **reorder point**, or the inventory level at which a replenishment will be triggered.

Following three models have attempted to address this issue with three different approaches:

1. Newsboy Model – Considers only a single replenishment so only issue is to determine the **order quantity** in face of an uncertain demand.
2. Base Stock Model – Considers the replenishment of inventory one unit at a time as random demand occurs. Thus, the only issue here is to determine **reorder point**. The target inventory set for the system is known as the base stock level.
3. (Q,r) Model – In this case the inventory is monitored continuously and demand occurs randomly and possibly in batches. When the inventory level reaches r, an order of size Q is placed. After a lead time l, during which a stockout may occur, the order is received.

Newsboy model while being useful in certain cases, is not realistic in case of a supply chain where multiple replenishments may be required. Thus we will look at the last two models in detail and compare them in the context of a two tier supply chain. Base stock model is closer to the Lean concept of make one move one since the replenishment quantity is one here.

This paper deals with supply chain issues related to stochastic demand. There are various inventory models such as (Q, r) model, News Vendor model and Base stock model, are available to address issues related to stochastic demand. We decided to apply Base Stock Model to supply chain and find out the reorder point at each stage. The mathematical model is developed and applied to supply chain (Figure 1).

Physical simulation is then designed to verify the validity of the results obtained by mathematical model. Physical simulations can quickly and effectively demonstrate the effect of organizational and process change to participants.

The Base Stock Model

The Base stock Model uses a continuous time frame and makes the following assumptions:

1. Demands occur one at a time.

2. Any demand not filled from stock is backordered.
3. Replenishment lead times are fixed and known.
4. Replenishments are ordered one at a time.
5. Products can be analyzed individually.

We make use of the following notations:
l = Replenishment lead time (in years)
x = Demand during replenishment lead time (in units), a random variable
G (x) = P (X<=x), cumulative distribution function of demand during replenishment lead-time; we will allow G to be continuous or discrete.
θ = E [X] = mean demand (in units) during lead time l
h = cost to carry one unit of inventory for one year
b = cost to carry one unit of backorder for one year
r = reorder point which represents the inventory level that triggers a replenishment order
R = r + 1 base stock level
S = r - θ, safety stock level
The fraction of demands filled from stock (as opposed to backordered), which we call the service level or fill rate.

As the order is placed every time a demand occurs, the relationship

Inventory + orders = R

The probability that the order arrives before its demand (i.e. does not result in a backorder) is given by P (X<R).
The fraction of demands that are filled from stock is equal to the probability that an order arrives before the demand it has occurred.

P (X<R) = G (R) if demand is continuous
G (r) if demand is discrete

Hence G(R), G (r) represents the fraction of demands that will be filled from stock (i.e. fill rate). Base stock model is equivalent to the Japanese Kanban System (with kanban size of one) since, order quantity is one

The primary insights from the model:

1. Reorder points control the probability of stockouts by establishing safety stock.
2. To achieve a given fill rate, the required base stock level (and hence safety stock) will be an increasing function of both mean and standard deviation of the demand during replenishment lead time.
3. Base stock levels in multistage production systems are very similar to kanban.

We have assumed Poisson distribution for demand and found out reorder point, order quantity and the safety stock in supply chain.

Application runs of Base Stock Model to Three-Tier Supply chain

Replenishment lead time = 12 months

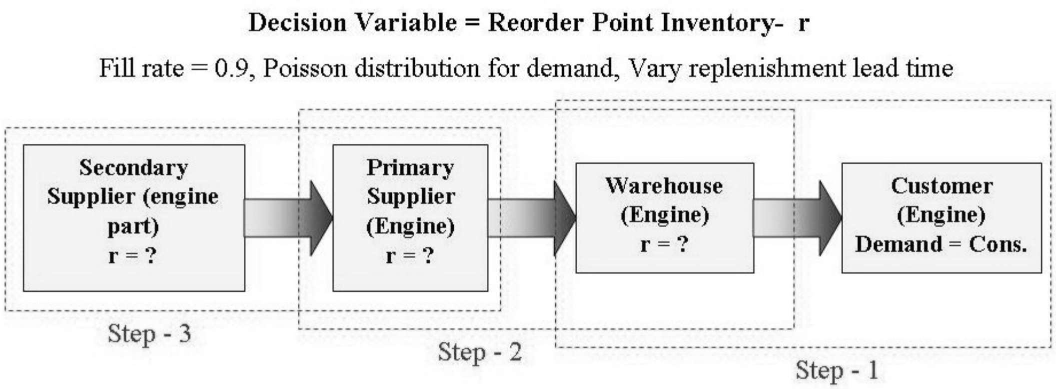

Figure 1. Supply chain considered for Base Stock model

At Warehouse

Demand during 12 months is 10 units /year

Average Demand = 10 units per year

P (k) = Probability (Demand during lead time, k)

$$= \frac{\theta^k e^{-\theta}}{k!} = \frac{10^k e^{-10}}{k!} \quad ; \quad G(r) = \sum_{k=0}^{r} p(k)$$

r	P(r)	G(r)
0	0.00	0.00
1	0.00	0.00
2	0.00	0.00
3	0.01	0.01
4	0.02	0.03
5	0.04	0.07
6	0.06	0.13
7	0.09	0.22
8	0.11	0.33
9	0.13	0.46
10	0.13	0.58
11	0.11	0.70
12	0.09	0.79
13	0.07	0.86
14	0.05	0.92
15	0.03	0.95
16	0.02	0.97

Table 1: Fill rate for various values of r

If the customer has an average demand of 10 units (say engines) per year then, for a fill rate of 90%, we see from Table 1, that the value of reorder point, r = 14 units per year at warehouse. Similarly we identify reorder point, r at Primary Supplier and Secondary Supplier for various replenishment lead time of 8, 6, 4 and 2 months. (Table 2)

Results from Base Stock Model

Table 2 summarizes all the results for base stock model and frequency of order. Order cost is assumed to be $ 25 per order. The total cost is calculated by using

$$TC = c\left(\frac{Q}{2} + r - \theta\right) + \text{Order cost.}$$

Total cost VS. Replenishment Lead-time

The total inventory cost is plotted against replenishment lead time in Figure 2.

Replenishment Lead Time (months)	Warehouse ($)	Primary Supplier ($)	Secondary Supplier ($)
12	925	1175	1450
8	741.25	925	1175
6	775	925	1225
4	725.5	975	1350
2	316.25	450	650

Table 2: Summary of results of costs (Base Stock Model)

Replenishment Lead Time	Demand	Reorder Point(r)	Q	Location	Frequency of order (F=D/Q)	Average Demand	Order Cost	Total Cost
12	10	14.00	1.00	Warehouse	10.00	10	250	925
	14	19.00		PS	14.00	14	350	1175
	19	25.00		SS	19.00	19	475	1450
8	6.67	10.00	1.00	Warehouse	6.67	6.67	166.75	741.25
	10	14.00		PS	10.00	10	250	925
	14	19.00		SS	14.00	14	350	1175
6	10	8.00	1.00	Warehouse	10.00	5	250	775
	16	11.00		PS	16.00	8	400	925
	22	15.00		SS	22.00	11	550	1225
4	10	6.00	1.00	Warehouse	10.00	3.33	250	725.5
	18	9.00		PS	18.00	6	450	975
	27	13.00		SS	27.00	9	675	1350
2	1.67	3.00	1.00	Warehouse	1.67	1.67	41.75	316.25
	3	5.00		PS	3.00	3	75	450
	5	8.00		SS	5.00	3	125	950

Table 3: Summary of Application Runs of Base Stock Model

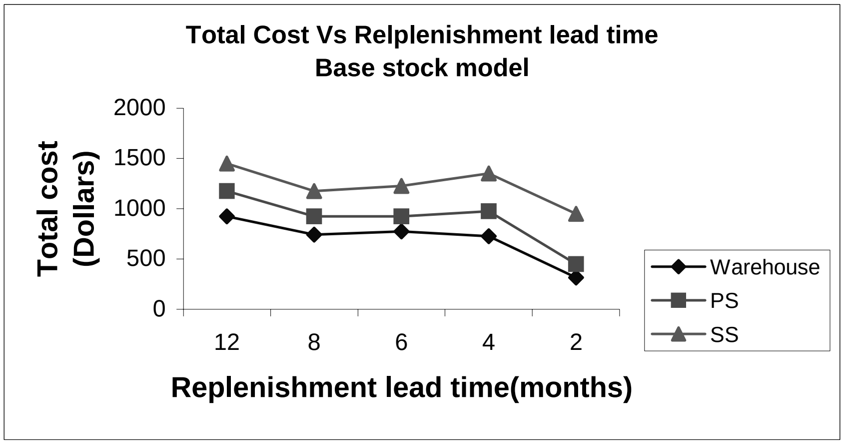

Figure 2: Total cost vs. replenishment lead- time (Base Stock Model)

Reorder point vs. Replenishment Lead time

The reorder point decreases with replenishment lead- time. Reorder point is plotted against replenishment lead time in Figure 3.

Replenishment lead time (months)	Reorder point at warehouse	Reorder point at primary supplier	Reorder point at secondary supplier
12	14	19	25
8	10	14	19
6	8	11	15
4	6	9	13
2	3	5	8

Table 4: Reorder point for Base Stock Model

Summary of Base Stock Model

The graph in Figure 3 shows the decreasing trend in reorder point from warehouse to secondary supplier for the same lead time. The total inventory cost decreases with replenishment lead-time for Base Stock Model. We can conclude from Figure 2 that there is decreasing trend in costs of warehouse, primary supplier and secondary supplier for the same replenishment lead-time.

Base stock model emphasizes on order quantity of 1. Base stock model can be used where demand is stochastic. Base stock model proves to be better for small lead-time.

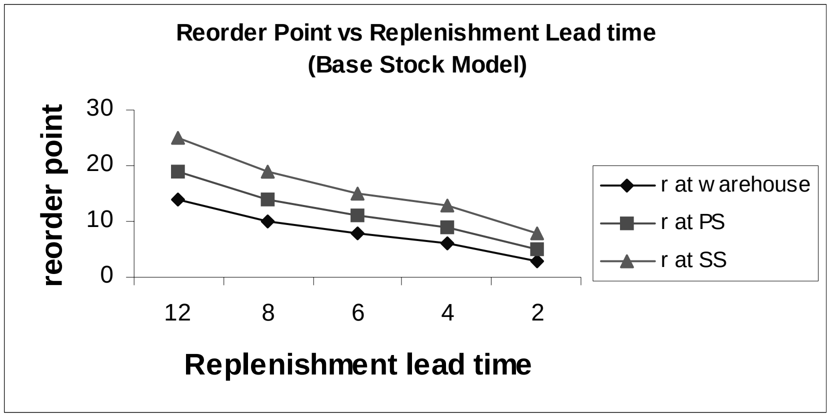

Figure 3: Reorder point vs. replenishment lead-time (Base Stock Model)

Physical Simulation of Base Stock Model

Primary goal of conducting the physical simulation is to validate the results obtained from the mathematical models. Simulation was run to confirm that optimum inventory levels i.e. reorder point at warehouse, primary supplier and secondary supplier are realistic values. Physical simulations are being used very effectively as a teaching tool for Lean training.

This physical simulation models a three-tier single-product supply chain. ABC Company produces engines. Final assembly department of the company withdraws these engines from the warehouse as needed. The *Warehouse* receives engines from *Primary Supplier***.** *Primary Supplier* receives the engine parts like cylinders from *Secondary Supplier*. We will make the assumption that only one cylinder is needed per engine. We are interested in inventory levels at *Warehouse, Primary Supplier* and *Secondary Supplier*. Excessive inventory results in increased holding costs while inadequate inventory results in backorders. Thus it is necessary to keep the optimum level of inventory at *Warehouse, Primary Supplier and Secondary Supplier*.

Customer, Warehouse, Primary Supplier and *Secondary Supplier* are the four departments in the simulation. The movement of the parts is as shown in the Figure 4. The Secondary Supplier provides cylinders to Primary Supplier. The Primary Supplier assembles the cylinders in the Engine Block and sends the Engine to the Warehouse. Engines are pulled from warehouse based upon a demand that follows Poisson distribution.

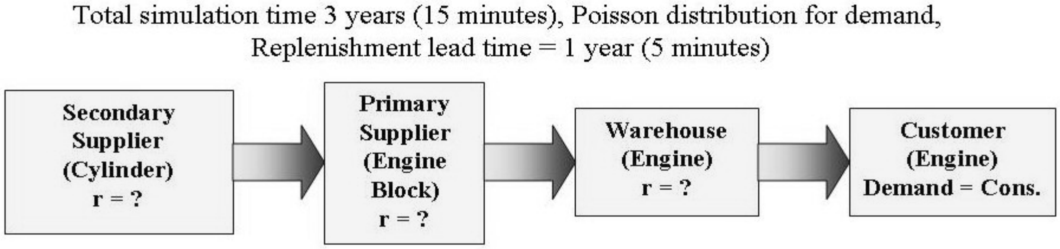

Figure 4. Layout of Supply Chain for Physical Simulation

Simulation Activity Time Frame

The total duration of simulation for each phase is 15 minutes (3 years). Customer sends the Order Requirement Form to the *Warehouse* at the start of simulation. Inventory at *Warehouse* goes below reorder point when the customer demands parts from *Warehouse* (at 1st min). *Warehouse* then sends Order Requirement Form to *Primary Supplier*. This triggers production activity at *Primary Supplier*, which has a replenishment lead time of one year. Replenishment lead time at *Secondary Supplier* is also one year. *Warehouse* has initial inventory (equal to reorder point). Demand at *Customer* is satisfied with this initial inventory.

In second year *Primary Supplier* sends the parts to *Warehouse* as per the schedule provided by *Warehouse*. Demand at *Warehouse* also follows Poisson distribution. When inventory level at *Primary Supplier* goes below reorder point (at 6th min), it sends Order Requirement Form to *Secondary Supplier*. This initiates production at *Secondary Supplier*. *Secondary Supplier* takes one year to replenish the items at *Primary Supplier*. *Customer* sends second order at 6th minute to the warehouse and subsequently *Warehouse* sends Order Requirement Form to *Primary Supplier*. Thus the production for third year starts at *Primary Supplier.*

In third year, *Secondary Supplier* starts sending parts to *Primary Supplier* (11th min). *Primary supplier* sends engine to *Warehouse* as per the schedule received in second year. *Warehouse* fulfills the *Customer* demand as per the Order Requirement Form provided by *Customer* in third year.

Distribution of Demand

We ensure that the demand at *Warehouse*, *Primary Supplier* and *Secondary Supplier* follows Poisson distribution as in the case of mathematical models. This is done by using Stat-Fit software to calculate demand quantities for *Customer*, *Primary Supplier and Secondary Supplier*. Figure 5 shows the Stat-Fit screen for demand calculation for a typical year.

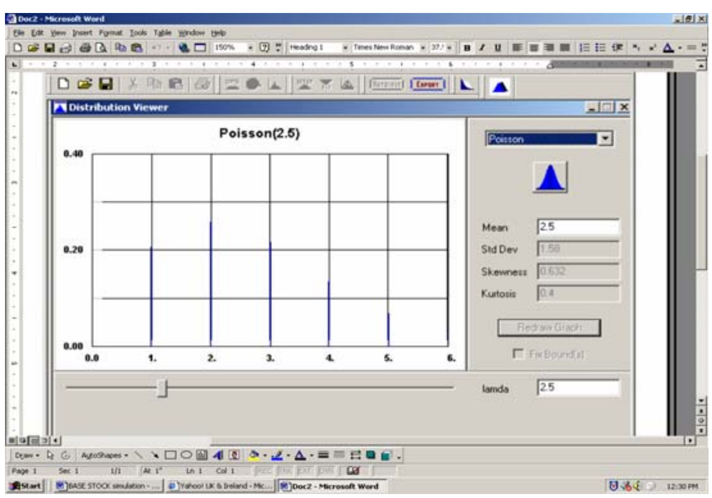

Figure 5. Stat-Fit Screen Showing Poisson distribution

The values obtained are shown in Table 5.

Demand at Customer	Demand at Primary Supplier	Demand at Secondary Supplier
2	3	4
3	4	5
2	3	4
2	2	3
1	2	3
10	**14**	**19**

Table 5. Order Quantity vs. Replenishment Lead Time

Performance Metrics

The assumptions about backorder cost and inventory holding costs match with the mathematical models. It is assumed that each backorder costs $100 and unit inventory holding cost is $20. The order cost is assumed to be $25 per order. In Base Stock model, the order quantity is one therefore; total numbers of orders are same as order quantity. Following spreadsheet is used to collect the data:

Performance Criteria	**Phase I**	**Phase II**	**Phase III**
Total number of orders	24	33	44
Order cost	$600.00	$825.00	$1,100.00
Excess Inventory	6	24	41
Total number of backorders	10	0	0
Cost of each backorder ($)	$100.00	$100.00	$100.00
Total cost of backorder	$1,000.00	$0.00	$0.00
Cost of inventory cost	$10.00	$10.00	$10.00
Excess Inventory cost	$60.00	$240.00	$410.00
TOTAL COST	$1,660.00	$1,065.00	$1,510.00

Table 6. Performance metrics

Summary of Physical Simulation

Excess inventory and number of backorders is documented at the end of each phase. The inventory holding cost and backorder cost are calculated in each phase. Ten backorders were observed during phase-I because of inadequate inventory at Warehouse. Therefore, total backorder cost is $1000 in phase-I. During phase-III, excess inventory exists and cost associated with this inventory is $410.

Phase-II includes the optimum level of inventory as predicted by mathematical models. In this case, backorder cost is zero and excess inventory cost is higher than phase-I but lower compared with phase-III. Total cost of inventory is the lowest in Phase-II as predicted by the mathematical models. Figure 4 shows the blocks used during simulation for engine blocks, cylinders and assembled engines.

Agile Inventory Management

A supply chain process includes customer orders, order processing, inventory, scheduling, transportation, storage, and customer service. Agile supply chain is flexible and is able to adapt to changing customer requirements. This paper looks at the agility of supply chain from inventory point of view. The customer demand is always changing and hence the inventory should also be changed in order to optimize inventory-holding cost. The supply chain should be able to act according to the change in demand. The mathematical model and simulation discussed in the paper represents real life supply chain where the demand from customer is changing.

Conclusions

Physical simulation is designed to include all the assumptions made by mathematical model. Hence, mathematical Base Stock Model and Physical simulation are comparable. Demand follows Poisson distribution in physical simulation as indicated in Figure 5. The backorder cost and inventory holding cost are calculated in each phase of simulation and summarized in Table 6. We can conclude that the total inventory cost is optimum in phase II, during which the reorder point is same as that calculated by the mathematical model. In phase I, the total inventory cost is more than that of phase II because of backorders. In phase III, excess inventory increased the total cost. Thus, the values obtained from mathematical model provide optimal inventory cost.

Base Stock Model is effective when the demand is not deterministic and service factor assumed in mathematical model is 0.9, which is quite acceptable. Base stock model assumes replenishment order quantity as 1 and the total inventory cost decreases with replenishment lead time. Base stock model is beneficial for supply chains having short replenishment lead time.

Physical simulation results indicate that physical simulations can be used to model stochastic systems like organizational supply chains and even to validate the results from mathematical models.

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Biography

Alok K. Verma is Ray Ferrari Professor and, Director of the Automated Manufacturing Laboratory at Old Dominion University. He also serves as the Chief Technologist of the Lean Institute at ODU. Alok received his B.S. in Aeronautical Engineering, MS in Engineering Mechanics and PhD in Mechanical Engineering. Alok is a licensed professional engineer in the state of Virginia, a certified manufacturing engineer and has certification in Lean Manufacturing and Six Sigma. Alok Verma has co-edited the proceedings of the International Conference on CAD/CAM & Robotics for which he was the general chairman. He was also general chair for ICAM-2006 international conference. He is serving as the associate editors for the International Journal of Agile Manufacturing (IJAM) and International Journal of Advanced Manufacturing Systems (IJAMS). Dr. Verma has developed and delivered training program in Lean Enterprise & Design for Manufacturing for Northrop Grumman Newport News, STIHL and several other companies in U.S. He has developed simulation based training programs for shipbuilding and repair industry under a grant from the National Shipbuilding Research Program (NSRP). He is well known internationally and has been invited to deliver keynote addresses at several national and international conferences on Lean/Agile manufacturing. He is active in ASME, ASEE and SME.