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# Spatial Visualization by Isometric Drawing

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## Abstract

Spatial visualization is a fundamental skill in technical graphics and engineering designs. From conventional multiview drawing to modern solid modeling using computer-aided design, visualization skills have always been essential for representing three-dimensional objects and assemblies. Researchers have developed various types of tests to measure students' spatial visualization abilities, in which most pictorial views of three-dimensional objects are represented by isometric drawings. Isometric drawings have many advantages such as being simple to draw, and equally representing the front, top, and side surfaces. However, they are also susceptible to drawing mistakes in spite of their relatively simple construction. This paper discusses the typical styles of visualization tests designed for engineering and technology students. Three formats for spatial visualization tests (developments, rotations, and isometric views) are compared through the students' performance at a community college. A popular spatial visualization test is used as example to show some mistakes in its isometric drawings. Recommendations are made to improve isometric drawing in spatial visualization tests.

## 1. Introduction

Spatial visualization ability has played an important role in civilization, from ancient architectural wonders to modern space exploration. Spatial visualization skills are especially important in engineering and technology fields. From the traditional board drawings of multiviews, sections, and assemblies, to modern solid modeling using computer aided design (CAD) tools, almost all product designs require the visualization of three dimensional (3D) objects. Spatial visualization ability is equally vital in some hot technologies such as geographic information systems (GIS), biomedical information technology, and robotics.

Spatial visualization has long been considered fundamental in engineering graphics courses. In the 1980s, along with the development of microcomputers, CAD was introduced into classrooms. Since then, both computer hardware and software have significantly improved, such that 3D solid modeling CAD has become ubiquitous in industrial applications. Therefore, spatial visualization has become a required skill for engineering and technology students. These developments have revitalized educators' interest in spatial visualization [1, 2].

In recent decades, many formats have been developed for testing spatial visualization skills. Psychologists have intensively studied spatial visualization from the perspective of cognition and perception. In the early 1970s, Shepard and Metzler [3] designed a test to investigate the reaction time of visualizing rotated 3D objects. Vandenberg and Kuse [4] later developed a test, based on Shepard and Metzler's model, known as the mental rotation test (MRT). Ekstrom [5] also included spatial visualization in a set of cognitive tests, which were included in the Educational Testing Service's (ETS) catalog of standardized tests. In the late 1970s, engineering and technology educators also investigated the relationship between spatial visualization ability and technical graphics skills. Among these educators, Guay [6] developed a test called the Purdue Spatial Visualization Test (PSVT).

The PSVT is a multiple-choice test. The test originally consisted of three sections (developments, rotations, and views), each containing twelve problems. The first section is developments (folding 2D flat patterns along fold lines) into 3D objects (surface models). The second section is the orthogonal rotations of 3D objects about the axes of the Cartesian coordinate system. The third section is the isometric views of 3D objects. By adding 18 new problems to the original 12 problems in the second section (rotations) of the PSVT test, Guay extended it into a 30-problem test on spatial visualization of rotations (PSVT-R) [7]. The PSVT-R was included in the ETS test collection and has since been widely used by researchers in engineering and technology fields. An excerpt of the PSVT test was also included in a textbook [8], containing the first five problems from each of the three sections of the PSVT test for a total of 15 problems.

Sorby [9] developed a workbook on 3D spatial visualization, which contains a total of 513 problems involving a variety of testing formats in spatial visualization (not counting additional exercises in the bundled software package). Most of the problems are variations of the basic formats in the PSVT test [6] and textbooks [8]. For example, there are 135 problems on orthogonal rotations in the workbook. In addition to the traditional multiple-choice questions on rotated views, there are also multiple-choice and free-form questions relating to degree, direction, and sequence of rotations. Some problems require students to sketch the isometric views after specified orthogonal rotations.

Spatial visualization tests have been used by educators as both teaching and research tools at many colleges. Textbooks include various formats of spatial visualization exercises, quizzes and tests on spatial visualization [8, 9]. Spatial visualization tests have been commonly used by educators as pre-test and post-test assessments to diagnose and improve students' visualization skills in graphics and CAD courses [10-15]. Spatial visualization tests have also been frequently used to assess and improve the retention rate of freshmen, women, and minority engineering and technology students [10, 14-19]. Educators have used spatial visualization tests to study human visualization behaviors in the context of age, gender, education, training, etc. [10, 12, 20-25] Researches have also been done to improve the testing tools for spatial visualization. For example, to help students visualize 3D objects from the approximate and simplified drawings in visualization tests, some researchers have added reference axes and grids to the views. Branoff [11, 26, 27] added the Cartesian coordinate axes to the PSVT-R test to highlight the visual relationship between the object and the axes it rotates about. Sorby [9, 28] exploded variations of the conventional test formats of pictorial views and used isometric grids to provide dimensions to the features of an object. Recently, educators have used CAD software to create solid models in spatial visualization tests for more realistic 3D views [9, 17, 29].

Usually, spatial visualization tests represent 3D objects in 2D pictorial views on paper or screen. Due to their simplicity, axonometric views have been dominantly used in spatial visualization tests. Isometric views, a sub set of axonometric views, have been particularly favored by engineering and technology instructors in developing spatial visualization tests. The MRT test [4] problems are all axonometric views. The PSVT test [6] consists of 36 problems evenly distributed among developments, rotations, and isometric views. However, all of the problems show pictorial views in isometric. All of the 30 questions in the PSVT-R test [7] are also exclusively in isometric views. Sorby's [9] workbook has a total of 513 questions involving pictorial views (not counting additional pictorial views in the bundled software package.) All of the views are axonometric and the majority of them (457 or 89%) are in isometric views. This paper discusses the role of the isometric view, especially its pros and cons, in spatial visualization tests.

The PSVT-R test is popular in engineering schools. This is probably partly due to the fact that the test uses isometric drawings for spatial visualization, a simple orthogonal projection taught in engineering graphics courses. Thousands of engineering and technology students in many colleges and universities have taken the PSVT-R test [10-17, 20-27, 29-35]. Due to its popularity and the use of isometric drawings, this paper will primarily use examples from the PSVT-R test for the discussion of spatial visualization by isometric drawing.

## 2. Isometric drawings in spatial visualization tests

Due to its simplicity in displaying 3D objects, isometric drawing has been predominant among engineering and technology educators in designing tests for spatial visualizations [6, 9]. In the three formats of the PSVT test [6], isometric drawings were the only pictorial views used to show 3D objects. They were used not only in the format of isometric views, but also to illustrate the rotations and developments. In the variety of spatial visualization tests in Sorby's workbook [9], isometric drawing was also the only format for pictorial views in almost all formats except for the intersected solids, in which more general axonometric views were used to avoid overlapping of edges.

The PSVT and PSVT-R tests [6, 7] are popular and representative. An abridged PSVT test is readily accessible in a textbook [8], and many instructors and students are familiar with the tests. A variety of test formats may be derived from the PSVT test [9]. Because all pictorial views in the tests are also exclusively in isometric drawings, this paper introduces the three formats of the PSVT test to discuss the use of isometric views in testing spatial visualization ability.

2.1 Spatial visualization by developments

True development is an un-stretched forming process that bends sheet metals into various shapes. Spatial visualization by developments takes the format of folding a 2D flat pattern into a 3D surface model. As shown in an example with the same format as the PSVT test (Figure 2.1), a 2D flat pattern and five isometric views are given. The five views represent different 3D objects, and only one of them can be developed from the flat pattern. In Figure 2.1, the correct choice is D.

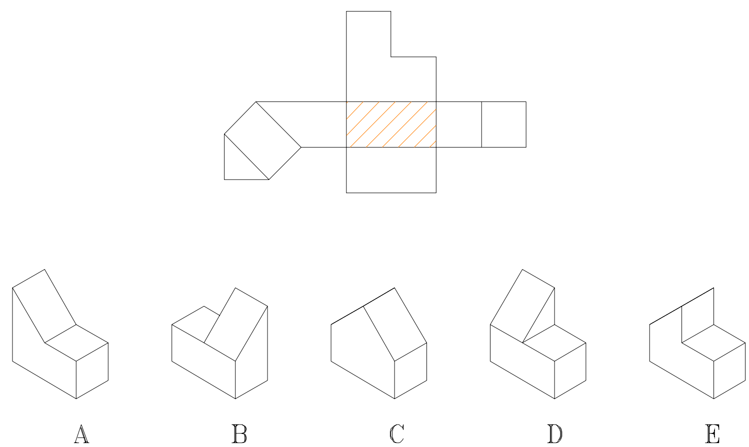


Figure 2.1 Spatial visualization by developments of folding
a 2D flat pattern into a 3D surface model

To help students visualize the developments, fold lines are usually provided. A suggested base surface, around which adjacent surfaces are folded, is also marked. The marked base surface may be viewed as the bottom of the developed 3D model in contact with the tabletop or ground. In a 2D flat pattern, all interior lines must be fold lines, and all boundary lines must be seam lines. The flat pattern must be folded along the fold lines and then sealed along the seam lines to form the correct choice of the surface model. It is also assumed that the flat pattern is folded upward. The correct answer in Figure 2.1 would be B if it is folded downward.

The test of developments may also take the reverse format of unfolding a 3D surface model into its flat patterns. In this case, there is no need to mark the base surface because it is the bottom of the object as it is placed. The seam lines now become cut lines on the 3D surface model along which the edges are cut open and the surfaces are unfolded into the flat pattern. Normally, the edges corresponding to the cut lines are left for the students to figure out, and are not indicated on the 3D surface model.

Usually, a surface model may be unfolded into multiple flat patterns. On the other hand, the cut lines must be carefully chosen on the 3D models to form a continuous boundary of a 2D flat pattern. The flat pattern can be made clearer for visualization by using the dashed line type instead of continuous lines for the fold lines.

The developments of simple objects are usually fairly easy. For example, a rectangular box requires no more than a maximum of three consecutive folds and all fold lines can be aligned with horizontal or vertical lines on a flat pattern. However, the fold lines for oblique surfaces generally do not follow horizontal or vertical lines on flat patterns. The developments of some complex and embedded surfaces may also require a considerable number of consecutive folds. In these situations, spatial visualization of mentally folding flat patterns into a surface model could be a challenging task.

## 2.2 Spatial visualization by rotations

Another very popular format in spatial visualization tests is rotating a 3D object in its axonometric views. Examples of spatial visualization by rotations include the MRT test [4] and the PSVT-R test [7]. In the MRT test, students are to recognize an object of ten face-to-face connected cubes after rotating it at an arbitrary angle in axonometric views. In the PSVT-R test, students are to visualize orthogonal (single or multiple 90º) rotations of a 3D object about the Cartesian coordinate axes in isometric views.

Figure 2.2 shows an example with the same format as the rotation problems in the PSVT-R test. An object and its rotated view are given as a sample, and then another object is presented with its five different views after various rotations. Students are to choose one view that is resulted from the same manner of rotation as the given example. In Figure 2.2, the correct choice is B. All of the views in the PSVT-R test are bird's-eye regular isometric views, and the five views (A through E) are rotated views of the same object. The rotations are orthogonal, i.e. multiples of 90º about a single Cartesian axis or multiple axes in sequence.

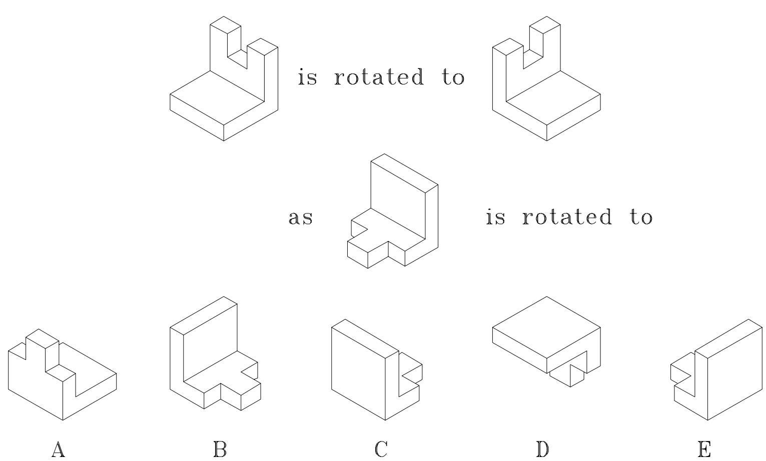


Figure 2.2 Spatial visualization by orthogonal rotations

Due to the controllability of limited number of isometric views, as well as due to their simplicity, the format of orthogonal rotations of isometric views has been chosen by most engineering and technology educators [10-17, 20-27, 29-35].

2.3 Spatial visualization by isometric views

Another format of applying isometric views in spatial visualization test is to identify the isometric view of a 3D object at a given vertex of its confining rectangular box. Figure 2.3 shows an example with the same format as the isometric view problems in the PSVT test [6]. An object is placed inside a confining cube and the point of view is denoted by a dot at one of the vertices of the cube. Then five isometric views of the object are given, and students are to choose one that represents the view from the given point of view. The right choice is C in Figure 2.3.

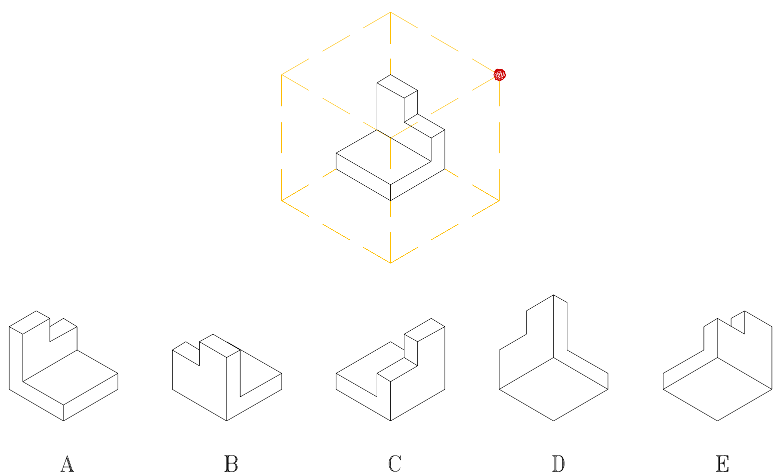


Figure 2.3 Spatial visualization by isometric views

This format of spatial visualization requires a prior knowledge in engineering graphics, such as the concepts of point of view and isometric projection. Therefore, it may not be appropriate for testing a general audience. There are also some problems with the application of the format in spatial visualization. As discussed later, multiple isometric views may exist at each vertex and the available isometric views are limited by imposing constraints to avoid possible ambiguities.

2.4 Comparison of spatial visualization test formats in a community college classroom

Bertoline's textbook [8] includes an excerpted version of the PSVT test [6]. The abridged test excerpts the first five problems from each part of the PSVT test, with a total of 15 problems in three formats of spatial visualization: developments, rotations, and isometric views. These excerpts are relatively simple problems drawn from the complete sets of the PSVT test. For example, the rotations are 90° or 180° about a single axis, and the isometric views are all regular bird's-eye views at the four top vertices.  Since the textbook is readily available, it is a good testing tool to introduce students to spatial visualization.

Two classes at Essex County College have taken the abridged PSVT test.  Eighteen students in *ENR 103 Engineering Graphic*, and eleven students in *ENR 105 Applied Computer Aided Design* took the test at the beginning of the spring semester and summer term I in 2006 respectively. *ENR 103* is an introductory hand drafting course and *ENR 105* is a CAD skills course using the AutoCAD software package.  Both courses are required for engineering and technology majors, and ENR 103 is a prerequisite for *ENR 105*.  None of the students in both classes had previously taken the PSVT test; in particular, none of the students in the *ENR 105* class came from the *ENR 103* class that took the test.  The classes' test scores in percentile are listed in Table 2.4.

Table 2.4 Comparison of the average scores in different parts of the PSVT

| Part | 18 students in *ENR103* | 11 students in *ENR105* |
|---|---|---|
| I. Developments | 77% | 67% |
| II. Rotations | 60% | 71% |
| III. Isometric views | 60% | 60% |
| Overall | 66% | 66% |

As suggested by Table 2.4, previous training in engineering drawing, including isometric drawing, may not have much of an impact on the improvement of spatial visualization skills. The overall average scores of the two groups were the same.  The results from a previous investigation on 84 students in *ENR 103* and 47 students in *ENR 105* taking the PSVT-R test showed overall average scores of 65% and 69% respectively, which was also not a statistically significant difference [24].

Among the three formats of spatial visualization, the developments should logically be the easiest.  Paper folding of flat patterns into various shapes is a popular hands-on activity for students as early as kindergarten.  Since paper folding closely resembles the developments, everyone should have some personal experiences with it.  Non-technical students should also be familiar with this format of spatial visualization without prior graphics training.  The students in the *ENR 103* class indeed had the highest score (77%) in developments, and lower scores in rotations and isometric views (60% each).  The rotations may also be mentally visualized without graphics training, though it may be more beneficial to be able to associate the rotations to Cartesian coordinate axes.  The students in the *ENR 105* class scored highest in rotations (71%), better than developments (67%).  The isometric views do require some basic knowledge in engineering graphics such as points of view and isometric projections.  Students in both classes scored poorly on isometric views (both are 60%).  It is especially surprising that, after intensive practice on isometric drawings in *ENR 103*, the students in *ENR 105* still scored the lowest in this format (60%).  Due to the small sample sizes, the comparisons are preliminary and further investigations are necessary.

## 3. Available isometric views for spatial visualization

Theoretically, there are an infinite number of isometric views available. However, due to certain practical constraints, only a very limited number of isometric views have been used in spatial visualization tests.

An isometric drawing is a special axonometric view with equal angles of projection (120º) between any pair of the axes of the Cartesian coordinate system, where the axes are projected on the view (Figure 3.1.) If we place an object in a cubic glass box and look at the object diagonally downwards through the two vertices of the box, we will see the object as an isometric view. Real objects have various shapes, but any object can be confined in a rectangular box with its size matching the overall width, depth, and height of the object. Therefore, we may use a generic rectangular object in our discussion of isometric views.

Theoretically, there are an infinite number of isometric views. By rotating either the object or the eyes (or camera lens) about the line of view, different isometric views are generated at various orientations. However, if we place an object in a fixed Cartesian coordinate system and align its width, depth, and height with the coordinate axes, keeping our heads upright and our eyes in the horizontal position, only a very limited number of isometric views exist (Figure 3.1.) This is equivalent to the usual views of human eyes on the Earth, designated regular isometric views in this paper. After all, people normally look at still objects with their heads upright, rather than tilting their heads sidewise, or rotating any object larger than handheld size. Therefore, regular isometric views are usually the most common views in applications.

You may visualize a clearer picture by placing an object in a rectangular glass box on tabletop or ground. The width, depth, and height of the object are aligned with those of the box, and they are aligned also with the Cartesian axes. This is usually a typical layout of an isometric view, which shows the top and two side surfaces of the object (Figure 3.1a). This kind of isometric view represents a bird's-eye view looking diagonally downwards at an object on the ground. Other types of isometric views are also possible. For example, if one craws under a table and looks diagonally upwards at an object on the tabletop (the tabletop must be transparent), a worm's-eye isometric view would be obtained, which shows the bottom and two side surfaces of the object (Figure 3.1b). However, even if a tabletop is made of glass so that it would not block the sight of the worm's-eye view, people normally would still prefer the bird's-eye views than the worm's-eye views. Consequently, if people's viewing preferences or habits are taken into consideration, there are only four practical isometric views at the top vertices of a fixed object (Figure 3.1a). For example, AutoCAD provides only the four isometric views at the top vertices as direct view options or toolbar buttons, and calls them SW, SE, NE, and NW isometric views [36].

(a)

(b)

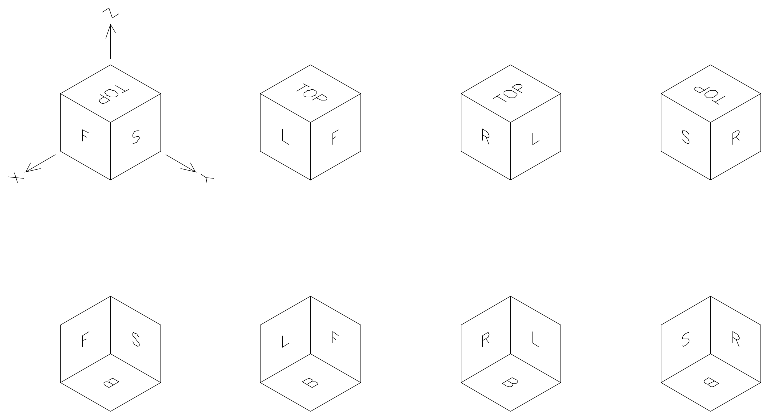


Figure 3.1 Eight regular isometric views for the top and bottom surfaces

Fortunately, more isometric views are available by placing an object on its six different surfaces (top, bottom, front, back, right side, and left side) one at a time. There are three isometric views with different orientations at each vertex (Figure 3.2), and a total of 24 regular bird's-eye isometric views for the eight vertices (Figure 3.3) [37].

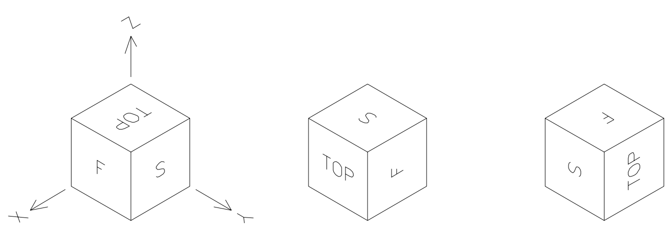


Figure 3.2 Three possible isometric views at each vertex

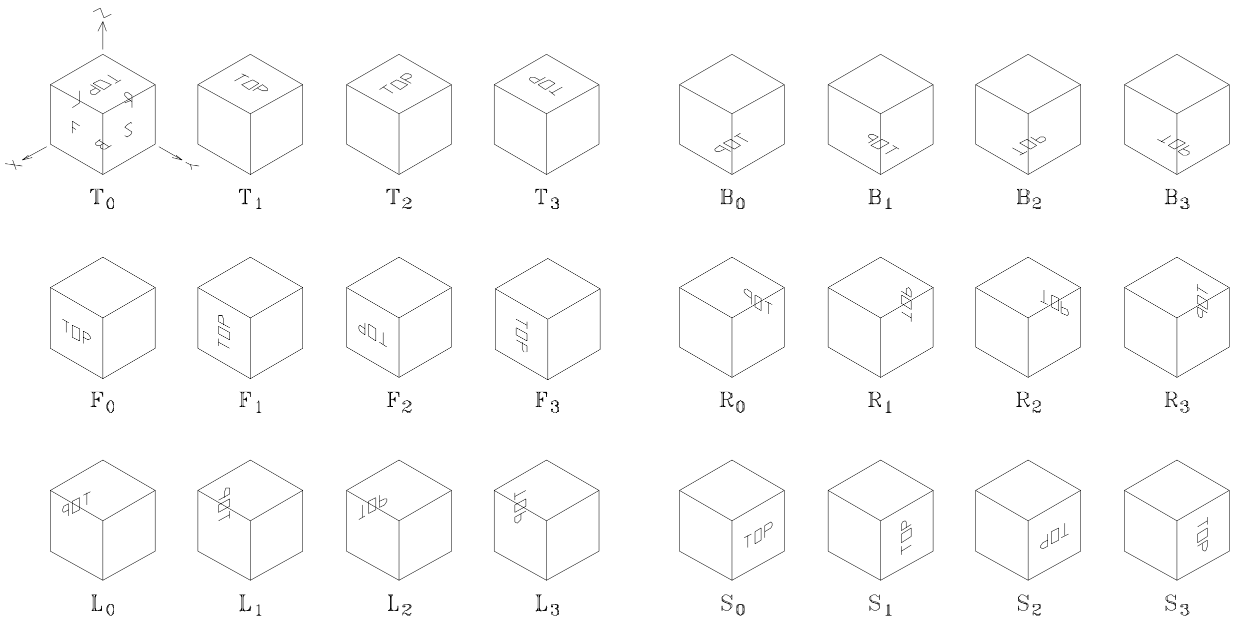

Figure 3.3 All 24 bird's-eye isometric views

In Figure 3.3, each view is denoted by a subscript composed of a letter and a number. The letters indicate the surfaces as an original top surface (T) rotated to bottom (B), front (F), back or rear (R), left side (L), and right side (S). The number in the subscript indicates the number of positive orthogonal rotations about the axis along the surface vector of the specified surface (0 is $0 \times 90^o = 0^o$, 1 is $1 \times 90^o = 90^o$, 2 is $2 \times 90^o = 180^o$, and 3 is $3 \times 90^o = 270^o$). For example, the $T_3$ view is obtained by rotating the $T_0$ view 270º about the Z-axis. The views in the figure represent all 24 available bird's-eye isometric views of an object, though the notations and sequences of these views may be rearranged. Similarly, there are also a total of 24 worm's-eye isometric views of any object.

## 4. Representation of isometric views in spatial visualization by rotations

A total of 24 available bird's-eye regular isometric views may be formed by rotating the object about the Cartesian axes orthogonally, i.e. with 90º angles. Their difficulties of rotation vary depending on the number of axes involved and the degrees of rotations. A well designed visualization test makes an effort to represent all available views and across the spectrum of the difficulty of rotations. Let us look at the PSVT-R test as an example.

4.1 Representation of the isometric views in example rotations

In the PSVT-R test [7], two very similar objects are used in the exemplary rotations of all 30 problems to show the rotations you must visualize when selecting the right choice which follows the same rotations.  The two objects are both cubes with a central slot on one surface and a cut on one side wall of the slot.  One of the objects has a left cut and the other a right cut as shown as the view $T_0$ in Figure 4.1.1 and Figure 4.1.2.   A total of 48 isometric views exist for the two objects (Figure 4.1.1 and Figure 4.1.2.)  If each of the initial and rotated views in the examples of a 30-problem test uses a unique view, it needs a total of 60 different views.  There is a shortage of 12 views.  Therefore, either some views will be repeated or an additional object is required.  Alternately, we may use the same initial view for as many problems as possible and different views or rotations from problem to problem.  For example, one object can provide the same initial view and different rotated views for 23 problems, which cover all 24 available isometric views.  Two objects would provide 46 problems with different rotations for each object, more than enough for a 30-problem test.  Therefore, a 30-problem test with two sample objects is capable of representing all possible isometric views.

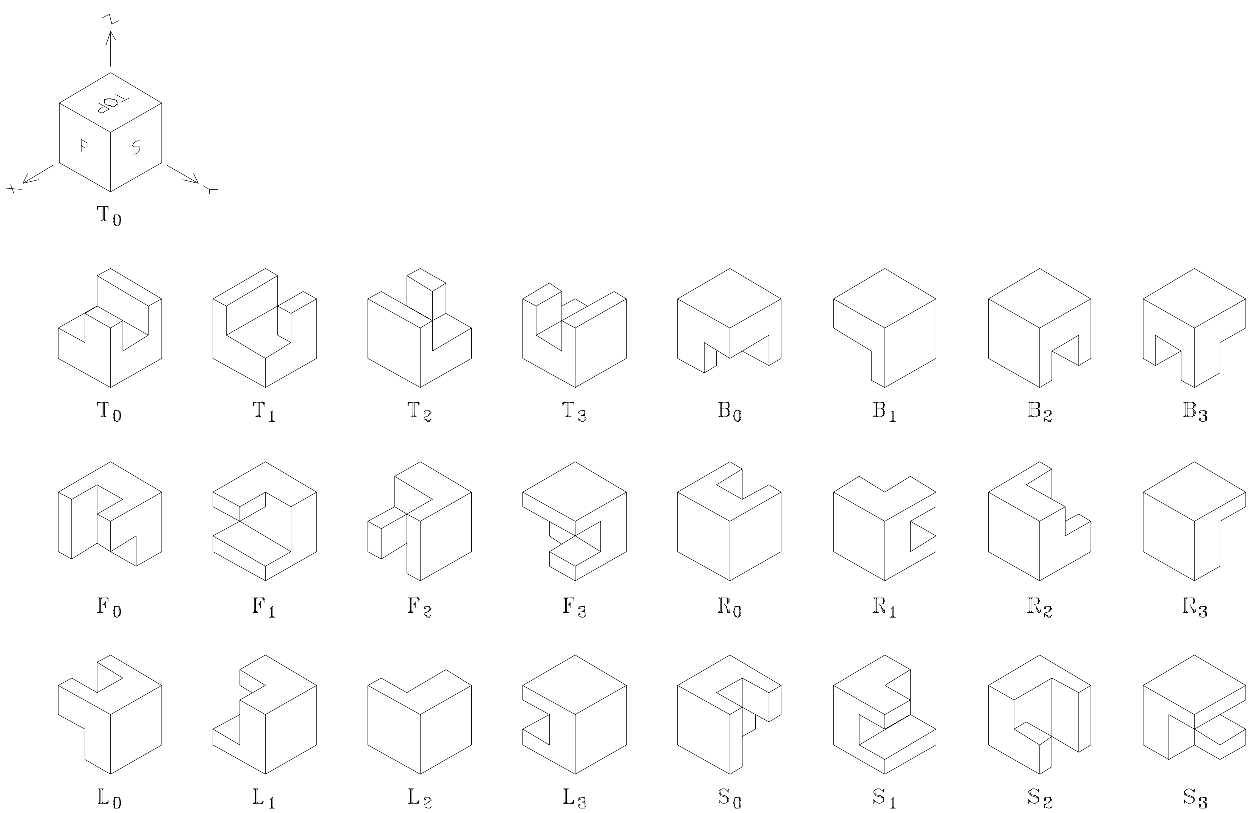


Figure 4.1.1 Twenty-four isometric views by orthogonal rotations for the object A

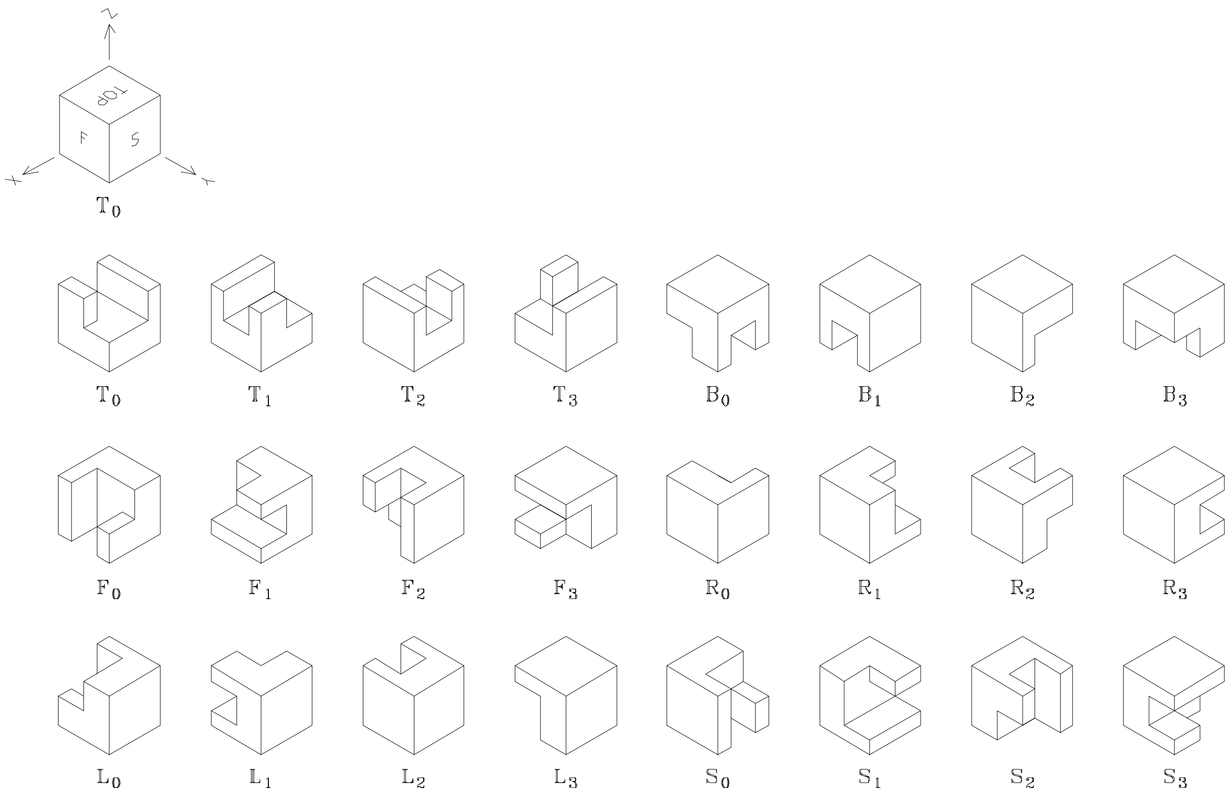


Figure 4.1.2 Twenty-four isometric views by orthogonal rotations for the object B

Table 4.1 lists the views used in the 30 problems of the PSVT-R test. From Table 4.1, some of the available isometric views are not represented in the PSVT-R test. Only 15 of the 24 (63%) available isometric views have been used for object A, and 12 of the 24 (50%) views for object B. On the other hand, some other views have been used multiple times. The $S_2$ view and $F_1$ view have been used 9 and 6 times respectively for object A, and the $T_0$ view and $S_1$ view 6 and 5 times respectively for object B. In the PSVT-R test, there are also repeated rotations (the same original and rotated views) in two problems (#8 and #10) for object A, and another two problems (#7 and #9) for object B. An improvement is possible in the future design of visualization tests to include more views and to have a better representation of the available isometric views.

Table 4.1 Isometric views in the example rotations of the PSVT-R test

| | $T_0$ | $T_1$ | $T_2$ | $T_3$ | $B_0$ | $B_1$ | $B_2$ | $B_3$ | $F_0$ | $F_1$ | $F_2$ | $F_3$ | $R_0$ | $R_1$ | $R_2$ | $R_3$ | $L_0$ | $L_1$ | $L_2$ | $L_3$ | $S_0$ | $S_1$ | $S_2$ | $S_3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $A_b$ | | 3 | | | | | | | | 6 | | | | | | | | | | | | | 8 | |
| $A_a$ | | 1 | | | 1 | 1 | | | 1 | | 1 | | 2 | 1 | 1 | 1 | 2 | | 1 | | 2 | | 1 | 1 |
| $B_b$ | 6 | | | | | | | | 2 | | | | | | | | | | | | | 5 | | |
| $B_a$ | | 1 | 1 | | | 1 | | | | | | | | | 1 | 1 | | 3 | 3 | | | | 1 | 1 |

Note: $A_b$ and $A_a$ indicate the object A before and after rotations (similar meanings for $B_b$ and $B_a$.) $T_0$, $T_1$, etc. are the possible isometric views in Figure 4.1.1 and Figure 4.1.2.

4.2 Representation of the difficulty of rotations

The total of 24 bird's-eye isometric views of an object (Figure 3.3) is obtained by orthogonal rotations about the Cartesian axes. They may be sorted into four types of rotations based on the degree of difficulty [37]. Each orthogonal or 90º rotation about an axis is a basic unit of rotation. The degree of difficulty is dependent on two factors: the number of axes involved and the number of orthogonal rotations. The four types are Type I: 90º rotation about one axis, Type II: 180º rotation about one axis, Type III: 90º rotation about one axis and 90º rotation about another axis, and Type IV: 90º rotation about one axis and 180º rotation about another axis (Table 4.2.) Clockwise and counterclockwise rotations are treated as the same degree of difficulty.

For example, the view $T_1$ is rotated 90º counterclockwise about the Z axis from the initial view $T_0$, and $T_3$ clockwise. Both of them are Type I rotations. Each of the views may be obtained by a number of different combinations of rotations. For example, the view $S_0$ may be obtained from $T_0$ by rotating 90º clockwise about the X axis and then 180º clockwise about the Z axis; or 90º counterclockwise about the X axis and then 180º clockwise about the Y axis; or 90º clockwise about the Y axis and then 90º clockwise about the X axis and finally 90º clockwise about the Z axis; etc. These are the Type IV rotations. If more than one type of rotation produces the same view, the simplest type of rotation always has precedence.

Table 4.2 Types of Orthogonal Rotations

| Type of Rotation | Isometric Views | Number of Views | PSVT-R Views |
|---|---|---|---|
| Type 0: Initial view | $T_0$ | 1 | |
| Type I: 90º one axis | $T_1$, $T_3$, $F_3$, $R_1$, $L_0$, $S_2$ | 6 | 6 |
| Type I: 180º one axis | $T_2$, $B_0$, $B_2$ | 3 | 8 |
| Type III: 90º one axis & 90º another axis | $F_0$, $F_2$, $R_0$, $R_2$, $L_1$, $L_3$, $S_1$, $S_3$ | 8 | 8 |
| Type IV: 90º one axis & 180º another axis | $B_1$, $B_3$, $F_1$, $R_3$, $L_2$, $S_0$ | 6 | 8 |

The rotations in the PSVT-R test do not represent all 24 available isometric views (Table 4.1.) Therefore, they do not include some rotations. However, they do represent all four types of rotations (Table 4.2.)

**5. Problems of using isometric views in spatial visualization tests**

Due to its simplicity, isometric drawing has not only been favored to illustrate pictorial views in engineering and technology textbooks [8], but also been chosen in designing spatial visualization tests [6, 9]. However, the oversimplifications of the isometric view could also cause problems in its application in spatial visualization tests.

### 5.1 Lack of 3D features in isometric views

Isometric drawings are far from true or realistic isometric views. A realistic view of a 3D object is composed of many necessary features. These features include dimensions, colors, external lighting and shades, light transmission, surface textures, and perspectives. The dimensions of a 3D object include overall sizes in width, height, and depth, as well as feature sizes and locations. The lighting and shading are affected by the luminance (brightness) of the light sources, whether point or ambient, and their colors (warm or cold). The light transmission may be transparent, translucent, or opaque. The surface textures reflect material properties and finishes such as wood grain, polished metal, and cloth fiber. Obviously, isometric drawing lacks many of these 3D features. Most importantly, it lacks the perspective effect. The visual rays or projection lines of an isometric view are parallel to each other. As a result, isometric views are distorted and distant features appear significantly enlarged. Isometric drawing also shows only the unblocked edges or boundary lines of a 3D view without surface features and light effects.

An isometric drawing is based on isometric projection with an adjusted proportional ratio. In an isometric projection of a rectangular box, its edges are foreshortened into approximately 82% of their original length. In an isometric drawing, though, they are usually still drawn as their original length for convenience. Sometimes, the depth dimensions are drawn as half of the original length, known as the cabinet view as opposed to the full length cavalier view. In this paper, the terms of isometric view, isometric projection, and isometric drawing are sometimes used interchangeably to mean isometric drawing.

As a matter of fact, isometric drawing has so many approximations that it may be the simplest among all types of pictorial drawings. In a spatial visualization test, though, it is necessary to display, as much as possible, realistic pictorial views to help visualize 3D objects. Therefore, the weaknesses of isometric drawing may impose certain limits on its application in spatial visualization tests.

### 5.2 Misinterpretations of isometric views

Since the hidden lines are not shown in isometric views, blocked rear features are not fully specified and left to the interpretation of the viewer. For example, Figure 5.2.1a shows two intersected cubes from four different points of view. In the first three views, it is very hard to tell whether the blocked portion extends half way or all the way unless we know its size. The fourth view shows a worst-case scenario in which the rear portion is entirely blocked and the object appears as though it were a single cube. In order to clarify rear features, hidden lines are sometimes drawn in isometric views (Figure 5.2.2a.) However, because the parallel edges of an object may overlap, this may cause even more confusion and misinterpretation. In both cases, general axonometric views, such as the trimetric views as shown in Figure 5.2.1b and Figure 5.2.2b, usually can avoid the overlapping and give a much clearer picture. For example, in Sorby's workbook [9], a small number of problems (approximately 10% of the total problems) are constructions of solid models, such as revolution, union, subtraction, and intersection. In order to make the intersections clearer, hidden lines are drawn in some of the objects, and general axonometric views are also used whenever necessary. In Sorby's workbook [9], all edges of each individual object shown as continuous lines as if they are visible and the objects are overlapping each other, and the intersections of solids are not shown no matter visible or invisible. Figure 5.2.2 illustrates the continuous hidden lines and missing intersection lines similar to the workbook problems. This treatment may highlight features on individual objects, but does not represent realistic joined solids. When it is necessary to show hidden lines in an axonometric drawing to make the drawing clear, it is still better to draw the hidden lines in dashes as usual.

(a)

(b)

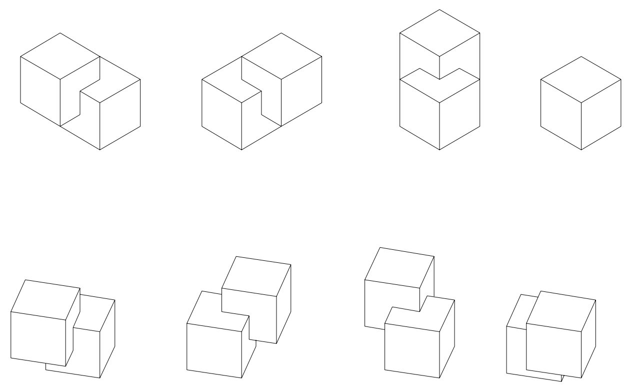

Figure 5.2.1 Two intersected cubes with hidden lines invisible

(a)

(b)

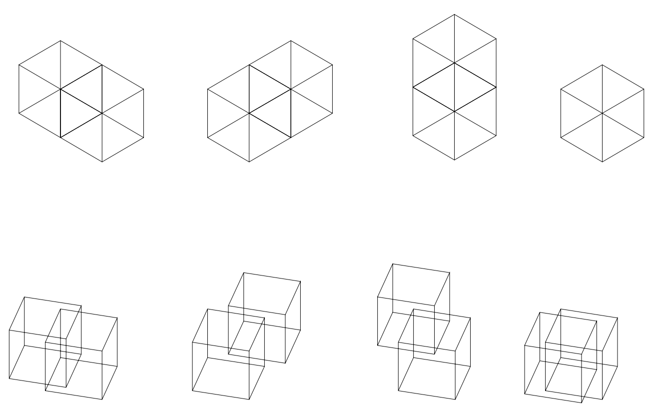

Figure 5.2.2 Two intersected cubes with hidden lines shown

5.3 Ambiguities in spatial visualization by isometric views

One format for applying isometric views in spatial visualization test is to identify the isometric view of a 3D object, seen from a given vertex of its confining rectangular box [6]. There are two major problems with this format of spatial visualization. First, multiple isometric views exist at each vertex.  Second, when constraints are imposed to avoid the ambiguity, only limited isometric views are available.

As discussed above, theoretically there are 3 regular isometric views possible at each vertex (Figure 3.2), and a total of 24 bird's-eye views (Figure 3.3.)  If we include all views in the spatial visualization by isometric views, it would be ambiguous to choose one of the three views at a vertex.  For example, an object and its three views at the indicated vertex are shown in Figure 5.3a.  If these three views were included in the multiple choices in a test problem, such as the C, D, and E as shown in Figure 5.3b, which one should we choose from?   This problem may be solved by adding some constraints to clarify the ambiguity.  For example, we could allow only the regular isometric views (Figure 3.1) in this format of spatial visualization test.  With this constrain added, there are only a total of 8 regular isometric views, 4 bird's-eye views and 4 worm's-eye views (Figure 5.3c, $T_0$ - $T_3$ and $B_0$ - $B_3$ respectively), available for spatial visualization by isometric views.  Back to Figure 5.3b again, the correct choice now is C.  Because choices D and E are not regular isometric views, they are no longer valid and can not be used in the test as options.  This solution seems to have solved the problem, but it naturally brings up another problem: there are now only very limited views left for use as answer choices.

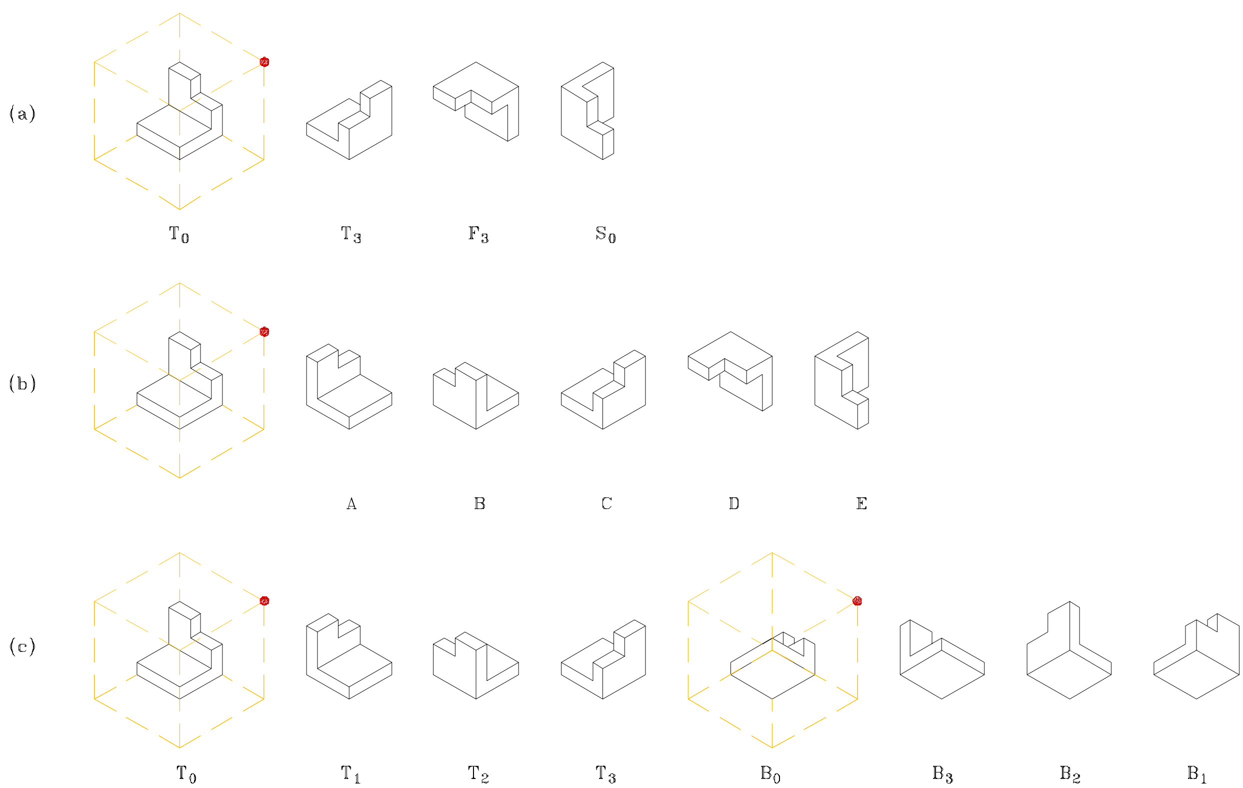


Figure 5.3 An example of spatial visualization by isometric views

5.4 Limited choices of isometric views

In order to avoid the ambiguity of multiple isometric views at each vertex, we have limited our views to the 8 regular isometric views (Figure 3.1) for spatial visualization by isometric views. Among the 8 views, the 4 bird's-eye views are more popular than the 4 worm's-eye views due to their similarity to natural human vision. If we limit the isometric views to bird's-eye views, only 4 views are left. After one of the views is used for the initial view, there are only 3 views left for choices, not even enough to make up five multiple choice answers. It seems that we have no choice but to include the worm's-eye views. Even so, we still only have 7 views available for choices. The limited choices available for spatial visualization by isometric views hinder the possible variations of this test format.

For example, in the 12 isometric view problems of the PSVT test [6], worm's-eye views are included to boost the number of choices. Except for two view point (TFS and BFS), all other six view points (TRS, TFL, TRL, BRS, BFL, and BRL) are presented in the test both as specified view points and in the multiple choices. Here the three letters denote the surfaces that form the vertex or point of view. The initial views in the PSVT test are all bird's-eye views of TFS, thus can not be used for choices, The BFS is a worm's-eye view and not used in the PSVT test, neither as the initial view nor as multiple choices. In the 5 isometric view problems of the abridged PSVT test [8], all specified views are bird's-eye views (TRS, TFL, and TRL).

5.5 Necessary training in visualizing isometric views

An isometric view itself is merely the simplest approximated graphic representation of a 3D object. People usually are able to easily visualize isometric views. Most spatial visualization tests choose isometric or axonometric views [3, 4, 6, 7, 9]. However, because of oversimplifications, some isometric views could be very confusing even to professionals (as discussed in the section below).

The test of spatial visualization by isometric view (Figure 2.3) [6] asks students to choose the isometric view that is created by viewing at the given point of view. This format involves more than just looking at isometric views. It addresses several concepts in engineering graphics. The students need to know the concepts of the confining "glass box", the projection plane, the lines of projection, and the point of view. They also need to have some familiarity with the creation of an isometric drawing. Therefore, the test of spatial visualization by isometric views may not be appropriate for a general audience.

## 6. Case study of isometric drawing errors in spatial visualization test

An isometric drawing is able to show the approximation of a pictorial view of a 3D object. Isometric views are usually easy to draw, especially with the aid of isometric grids. Therefore, isometric views have been widely used in spatial visualization tests [6, 7, 9]. However, due to its misrepresentation and distortion of a true 3D view, isometric drawing is prone to errors. For example, in the widely used PSVT-R test [7], the isometric drawings in seven out of the thirty test questions contain errors (23% error rate). Each of them had at least one or more errors, which accounted for 10 rotated views (with two of them the same object and rotation in the exemplary rotations.) These errors include missing features, misrepresented features, and the inclusion of extra features (Table 6).

Table 6 Summary of Errors in the PSVT-R Test

| Item | Question Number | Drawing Number | Error |
|---|---|---|---|
| 1 | 8 * | Example rotation | Missing features |
| 2 | 10 * | Example rotation | Missing features |
| 3 | 13 | Example rotation | Missing features |
| 4 | 13 | A | Extra features |
| 5 | 13 | D | Missing features |
| 6 | 14 | A | Missing features |
| 7 | 14 | E | Missing features |
| 8 | 15 | C | Extra features |
| 9 | 17 | Example rotation | Missing features |
| 10 | 25 | B | Misrepresented feature |

* Questions #8 and #10 share the same exemplary object and its rotations.

Question #14 of the PSVT-R [7] test is also the question #17 in the original PSVT test [6], and included in a textbook [8] (Question E in the part of visualization by rotations, Figure 5.169, p.301). Since question #14 of the PSVT-R test is readily available, it has been chosen as an example to show some details of the errors (Figure 6.1).

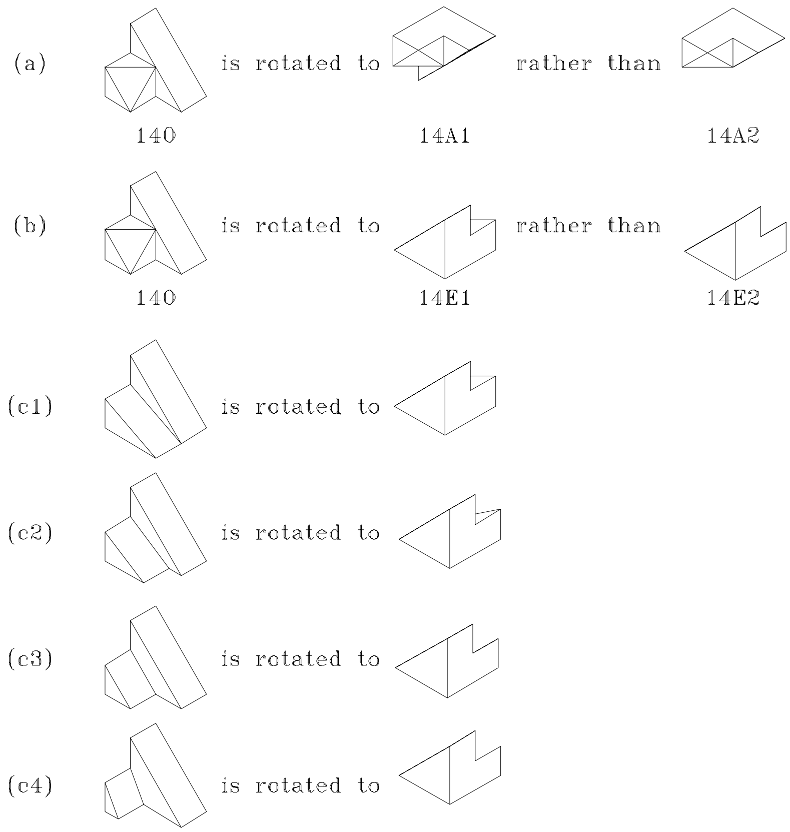


Figure 6.1 Errors in the question #14 of the PSVT-R test

In question #14 of the PSVT-R test, both the original isometric drawings of the choices A and E missed some features in the rear part of the views. As shown in Figure 6.1, 14O is rotated to 14A2 and 14E2 in the test, as compared to the correct isometric views 14A1 and 14E1 respectively.

Along a particular line of view and at a specific orientation, there is one and only one isometric view of an object, while an isometric view could be the result of different objects even along the same line of view and orientation. As shown in Figure 6.1a and Figure 6.1b, the given object can only be rotated to the correct isometric views 14A1 and 14E1. The erroneous view 14E2 could be the projections of various objects such as the examples shown in Figure 6.1c3 and Figure 6.1c4. The views c1 through c4 in Figure 6.1 show the effects of the change of angle or size of the small wedge to their rotated views. The widths of the wedge change from the full width W to 0.75W, 0.50W, and 0.25W (the views c1 through c4). Through the progressively larger change, we see that different objects can produce the same isometric views as shown in the views 14E1 and c1, and c3 and c4 in Figure 6.1.

In the 30 problems of the PSVT-R test [7], each problem provides a rotation example. Students follow these exemplary rotations to visualize and select from multiple choices the view that has the same manner of rotations. Their errors are critical in that they are likely to lead to student selection of erroneous answer choices and therefore worthy of discussion. There are two similar objects in the exemplary rotations. Both objects are a cube with a slot, and one side wall of the slot is truncated by half. The only difference between the two objects is that the truncation is on the left for one object and on the right for another (Figure 6.2a, objects A and B). Among the 30 exemplary rotations, 17 of them are object A and 13 are object B. Also among the total of 24 available regular bird's-eye isometric views for each object [37], 15 views are represented for object A and 12 views for object B. Some of the initial views and/or rotated views are used more than once (Figure 6.2c and 6.2d.) Among the thirty exemplary views, four of them have errors (two for each object), and the error rate is more than 13%. The errors for object A are in the problems #8 and #10 (they share the same exemplary object and rotations), and for object B are in the problems #13 and #17 (Figures 6.2b through 6.2d). In Figure 6.2, the views 8EG1, 13EG1, and 17EG1 are the correct views that should replace the incorrect views of 8EG2, 13EG2, and 17EG2 respectively as given in the PSVT-R test. All errors in exemplary rotations are missing features in the rear part.

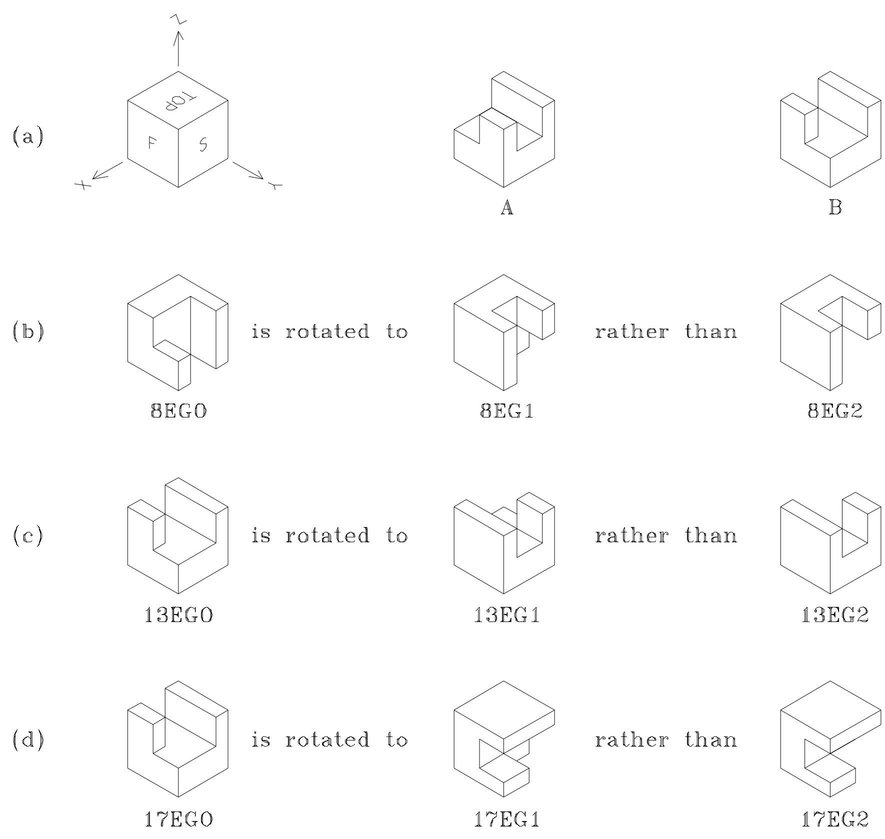


Figure 6.2 Errors in the exemplary rotations of the PSVT-R test

## 7. Conclusion

Isometric views have been widely used in spatial visualization tests, especially those tests designed for engineering and technology students. Isometric drawings may be the simplest of all graphic representations or realistic views of 3D objects. They are usually easy to draw and visualize. However, due to the over simplification and lack of many 3D features in isometric drawings, they can sometimes also be confusing and prone to errors. As examples, this paper discussed errors occurring in the isometric drawings of a widely used spatial visualization test, the Purdue Spatial Visualization Test – Visualization of Rotations (PSVT-R) [7]. In the total of 30 problems, 7 of them contain errors, an astonishing 23% error rate. These errors are mostly missing features and some are extra features or misrepresented features. Each of the test problems first provides an example of rotations of an object, then gives another object and asks the students to select a view from 5 multiple-choice answers, resulting from the same rotations as the example. Even 4 exemplary rotations have errors. The most erroneous problem contains 3 drawing errors, one in the exemplary view and two in the multiple-choice views. The PSVT-R test was designed in late 1970s and probably drawn by hand, without the precession of today's CAD hardware and software. The discussions of the errors in the PSVT-R test are only intended to show the cons of isometric drawing that it is prone to errors. With modern computing technology and insights of spatial visualization, we should redesign the testing tools for spatial visualization in order to avoid confusion and better help students to improve their skills in spatial visualization. For example, many CAD software packages are capable of 3D solid modeling. We may avoid potential isometric drawing errors by first crating a 3D solid model and then obtaining its isometric view, rather than using the conventional method of constructing isometric drawings on a 2D plane with lines. 3D solid models can also provide more realistic views. Therefore, they should be applied in spatial visualization tests to improve the visual effects. Many features of realistic 3D views, such as color, lighting, shading, perspective effect, and surface textures, are also available through rendering solid models. Other supplemental effects, such as animation and audio-visual, may also be included using CAD and multimedia software.

The various aspects of isometric drawings in spatial visualization are systematically discussed to provide more insights into the topic. From the results of the abridged PSVT test [8] given to the students in an engineering graphics class and a CAD class at a community college, comparisons are made to the three formats in the test: spatial visualization by developments, rotations, and isometric views. The results show that the average scores of the two classes are the same (66% each), despite the fact that the students in the CAD class already took the graphics course and went through some basic graphics skills and hand sketch training. The students in the graphics class scored highest in the developments (77%), and the CAD class had the highest score (71%) in the rotations. Due to the small sizes of the classes (eighteen students in the graphics class and 11 students in the CAD class), though the results are interesting, they are preliminary and more research is necessary to compare the data from larger sample sizes and different populations.

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