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Improved Learning of Engineering Mathematics through Hands-on, Real-world Laboratory Experiments

P. J. Shull
The Pennsylvania State University, 3000 Ivyside Park, Altoona, PA
pjs18@psu.edu

Abstract

Many engineering and engineering technology majors believe that their mathematics courses are not connected to concrete concepts that they can readily grasp. This (perceived) lack of real-world connection is a well-documented barrier to student learning and is pervasive in mathematics education. Often the mathematical material itself is not overly complex; it is more that the concepts are new and alien to the students' experience. This, unfortunately, creates an increased level of anxiety that interferes with their learning. While this newness is true for many engineering concepts as well, a solid connection to the real world reduces the newness anxiety. In this work, the authors have developed and implemented a prototype of a mathematical, laboratory-based assignment to connect Fourier transforms to real-world applications using experiential learning methodology. In this way, students will experience hands-on manipulation of the data in the transform plane and observe the effect on the object in the inverse transform plane. Our long-term intention is to develop a series of real-world, hands-on laboratory experiments for the more difficult mathematical concepts. This work will describe the details of the transform laboratory exercise and the learning results in the context of our overall goal of developing experiential methods to realistically and effectively make mathematical concepts more engaging and intuitively available to the students.

Introduction

"This is just theory—they don't connect any of this to the real world!" "I do fine in all my engineering courses, but I just don't get the math [courses]." How often have engineering and mathematics faculty heard these statements from students frustrated with their performance in math courses? Regardless of whether a math faculty member relates the concepts to real engineering applications, students' comments and frustrations remained markedly similar—"I just don't get it," or "I don't see why we really need to know this." Beyond the excuses, real or not, engineering and engineering technology students, in general, find the mathematics courses the most difficult and least intellectually accessible of all the technical courses.

"Math anxiety" is the common jargon that expresses students' frustration that significantly interferes with learning [1, 2, 3, 4]. There is a host of literature that presents current thinking and mitigation techniques to reduce this debilitating math anxiety and improve student learning [5, 6, 7, 8]. However, very little of this work relates subject familiarity and/or students' ability to connect the concepts to something in the real world. As evidenced by engineering students learning in technical fields other than mathematics (e.g., engineering courses, physics, and chemistry, where their performance is significantly better), this connection to the real world and a resultant level of familiarity can have significant impact on student learning. Further, there is an obvious lack of work relating the pedagogical advantages of experiential learning to a mathematics education [9, 10, 11, 12, 13, 14].

Two common distinctions between the pedagogy of engineering and mathematics are the teaching-learning format and the connection of the material to everyday life. Engineering has a long tradition of student-centered experiential learning through its extensive use of laboratories and capstone courses. Recently, this format has been expanded in many engineering programs that emphasize the use of industrial design projects as a vehicle for learning specific engineering concepts. In contrast, mathematics courses are typically taught in a more teacher-centered environment of material/concept presentation, practice problems, and testing. Most mathematics curriculums attempt to connect the theoretical material to the field of engineering by presenting examples and assigning practice problems related to the field. However, the problems are typically somewhat randomly chosen, and while engineering related, they are not coherently related to a specific topic. Thus, students do not make any real and meaningful connections.

This work presents an initial effort to increase experiential learning of mathematics through the creation of a laboratory-based mathematics modules and analysis of the concomitant results.

The Class/Laboratory

This initial effort was to design, develop, and implement a mathematics module that monopolizes on the positive elements of experiential student learning that are commonly and successfully employed in engineering classes and that makes clear connections to the real world, providing a clear need for learning the material.

While experiential learning has shown to significantly improve learning, the method has some downsides to be considered. Listed below are the positive elements that were incorporated into the module design and negative elements that were minimized.

Positive Elements:

- Active student engagement.
- Increased excitement about learning.
- Material becomes more relevant to student's interests.
- The mathematical concepts are perceived as less threatening than when presented in a traditional format.
- Hands-on learning.

Negative Elements:

- Perceived as being more time consuming than traditional methods.
- Effect of students having different academic backgrounds, which places constraints on specifics of the laboratory.
- Requires specialized laboratory equipment.

The specific topic chosen for the initial module was Fourier transforms, which is taught in an upper division engineering mathematics course. Fourier transform/analysis is a common mathematical tool used in many practical areas of engineering, such as control systems. It is also one that students find particularly obtuse.

The specific goals of this effort were to maximize the positive learning elements, minimize the negative ones, and assess the overall value and feasibility of a laboratory-based mathematics courses.

Brief Introduction to Fourier Transforms, Analysis, and Optics

Fourier transforms are used to convert (transform) events in time or space, such as images and sound, into their frequency representations. While our interaction with events and objects in time and space occurs constantly, visualizing them as a series of frequencies is far from intuitive. The practical/engineering advantage of this transform is that a signal representing the event or object can be manipulated more easily in its frequency form than in time. The common example is adjusting the bass, midrange, or treble on a sound system. While we listen to the changes in amplitude of pressure in time (sound), the adjustments to improve the sound are done on specific ranges of frequency (i.e., the changes are made in the frequency domain and not in the time domain).

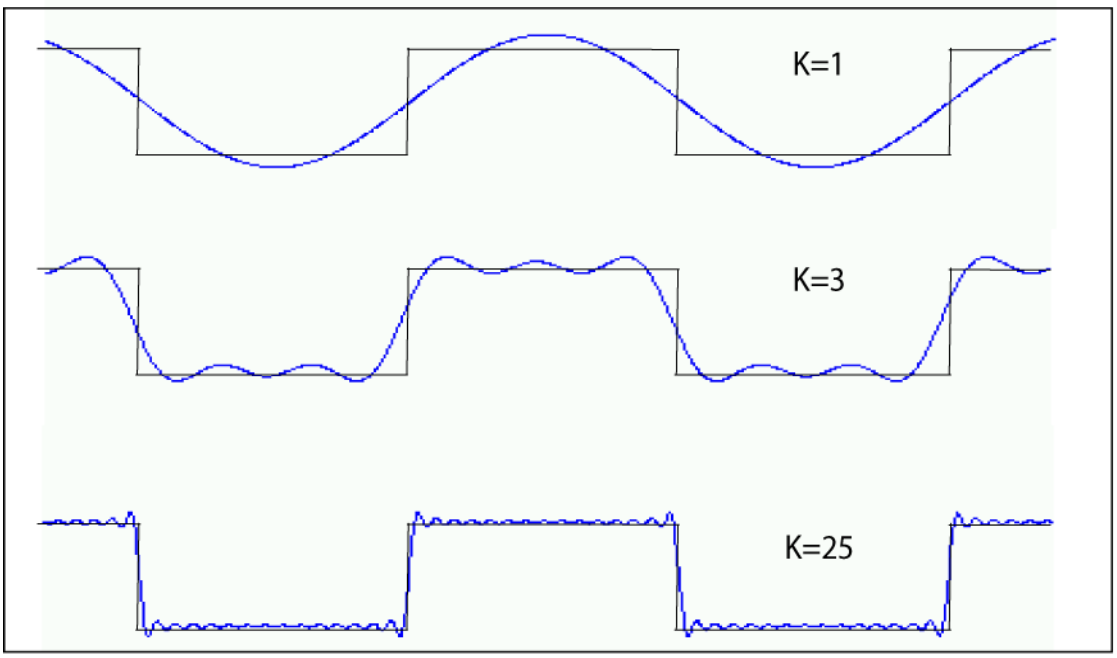

Figure 1: A sum of sinusoidal waves of differing frequencies and amplitudes are used to recreate an event in time or space—a square wave. K is the number of sine waves of different frequencies and amplitudes added to approximate the image. While reconstructing a simple square wave is used for this illustration, this could represent the image of white dashed lines on a highway—dash, no dash, or white line, black line.

Fourier analysis is predicated on the theory that all of our interactions that occur in time or space can be expressed as a sum of sinusoidal waves of differing frequencies and amplitudes. Figure 1 shows a square wave being constructed by a sum of sinusoidal waves—the more sine waves added, the more accurately the square wave is reproduced. The frequency representation of an event in time or an object in space can be obtained mathematically by integrating the function that represents the event or object over all time or space. This strictly mathematical format of presenting the Fourier transform concept is neither intuitive, nor engaging, nor connected to students' knowledge of the world, even when an engineering problem is used as an example.

To actively engage students, an experiential Fourier transform module was developed utilizing Fourier optics. This optical setup allows one to see and/or manipulate the Fourier transform of an object or event in space or time. The frequency representation of the object is visualized by the intensity and spacing of dots of light. Figure 2 shows a simple example of a sinusoidally varying image in space and the associated optical Fourier transform. Note: the transform contains three dots of light. The two outer dots are redundant, with each representing the sinusoidal image. The frequency is contained in the dot's distance from the center, and the amplitude information is given by the intensity of light of the dot. The central dot represents a constant level of background light and is referred to as the DC frequency. Finally, the frequency representation is transformed back into a reconstructed image of the original.

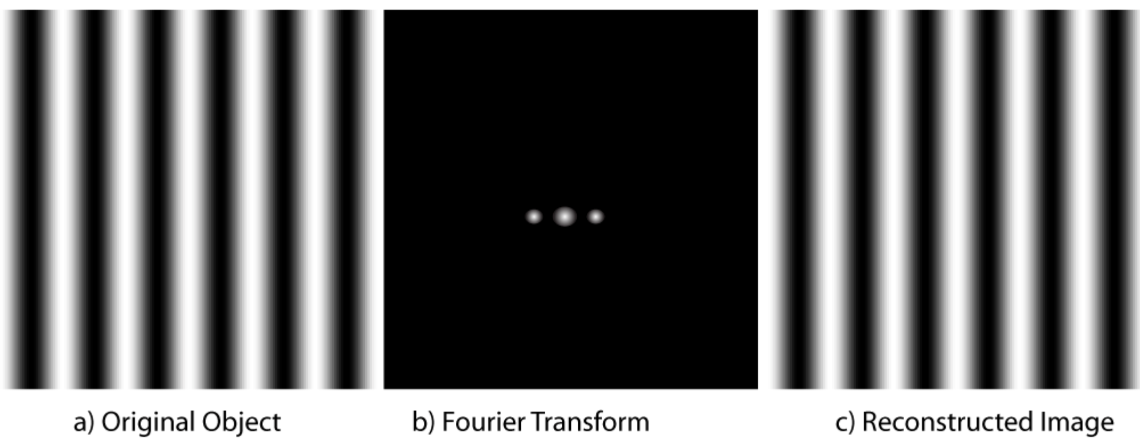

Figure 2: Example of a Fourier transform—a) the original object is a sinusoidally varying image, b) the Fourier transform of the image, and c) a reconstructed image created from the transform data.

An outline of the optical laboratory setup is presented below, followed by the laboratory procedure. Student response to participation in this module will be presented.

Experimental Setup

The basic Fourier transform setup is shown in Figure 3. The system has four parts: light source and beam expander, working area, Fourier transform plane, and image reconstruction. A 22 Watt HeNe laser is used as the light source. The laser beam is aligned with a microscope objective that expands the beam to a working diameter of approximately 8.5 cm. To obtain a columnated beam, where the light is neither diverging nor converging, lens l_1 is placed a distance f_1 from the microscope objective. f_1 is the focal distance of the lens. The working area, where the beam is columnated, is where objects or images to be transformed are placed. The Fourier transform of the real object is viewed in the focal plane of lens l_2—the Fourier transform plane. Finally, as the light beam continues, the inverse Fourier transform reconstructs the object that is imaged on a screen. Figure 2 shows what would appear at three positions within the system for a transparent plastic sheet imprinted with a sinusoidally varying image in the working area—the object in the working area, the Fourier transform, and the reconstructed image of the original object.

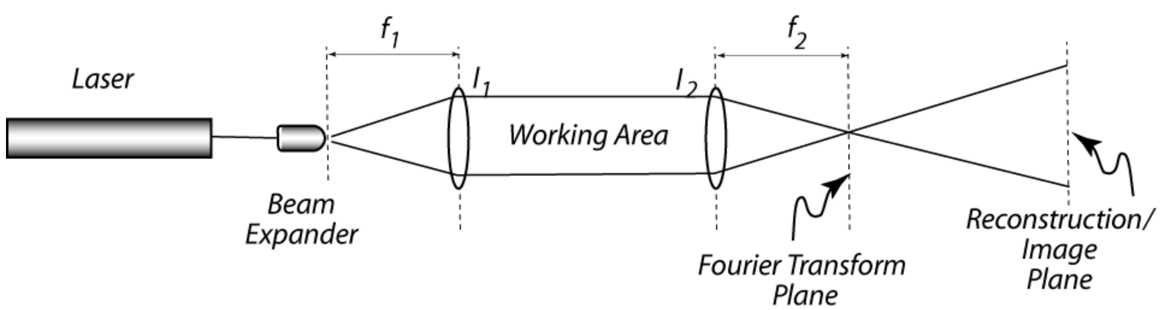

Figure 3: Optical Fourier transform laboratory setup.

Experimental Procedure

As an introductory comment to this procedure, it should be made clear that the students do not need to have any special knowledge of optics beyond elementary knowledge of the function of a lens. The optical system is simply used as a tool that allows students to be directly involved in using and manipulating images, their Fourier transforms, and the resultant reconstructed images.

Introduction

1. Present the idea that any object or image in space or time can be represented by a sum of sinusoidal waves of different frequencies and amplitudes. Use the standard square wave example shown in Figure 1.

Basic Fourier Optics (Focal Plane Equals Fourier Transform Plane)

2. Now moving to the experimental system and monopolizing on student's basic understanding of a lens, an object is placed in the working area, and students are asked to describe what they would expect to see at the image plane (i.e., the screen).
3. Student predictions are tested, and the image of the object is shown on a screen.
4. A transparency imprinted with a sinusoid (Figure 2a) is placed into the working area and is viewed on the image screen. Students are instructed to find the focal point and predict what one would see.
5. Using card stock, the students find the focal point, which shows a bright dot in the center of the beam.
6. Turn out the lights and let the students' eyes become dark adjusted.
7. Ask the students to repeat step five.
8. Discuss what is observed and the fact that the focal plan of a lens is a Fourier transform plane, where the frequencies and amplitudes are represented by the spacing of the dots and their intensity.
9. Place a more complicated object into the working area—a standard sieve (wire mesh)—which acts as a 2-D hat function in space where light passing and no light passing are periodic in both the *x*- and *y*-directions. Students observe the response in the Fourier plane, Figure 4.

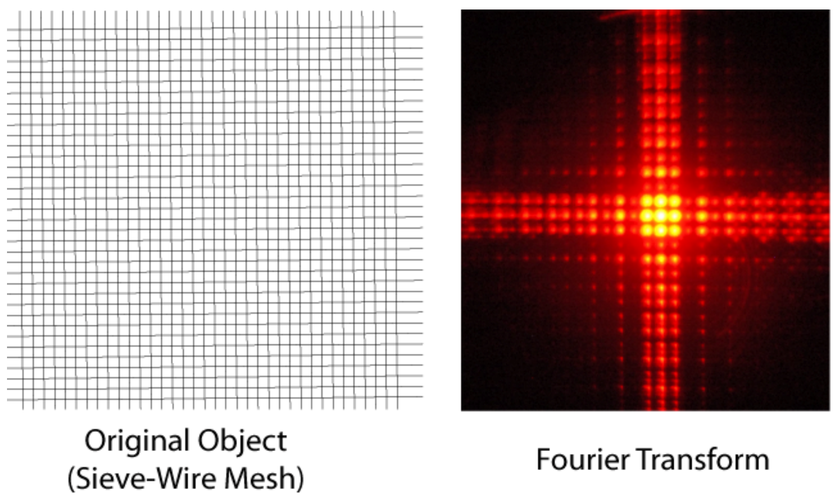

Figure 4: Optical Fourier transform of a standard sieve (2-D square wave function)

Optical Fourier Analysis/Applications (Filtering in the Fourier Plane)

10. With the sieve still in the working area, perform the filtering shown in Figures 5b and 5c. Observe and discuss the results.
11. As a separate filtering, remove all the dots except the central dot, as shown in Figure 5d. Observe and discuss the results. This discussion should make use of the example in Figure 1 and the initial discussion in step one.
12. Using the filtering setup, as in Figure 5e, remove the central dot. Observe and discuss results. Again, relate the results to the discussion in step one. This specific filtering format is called Schlieren imaging.

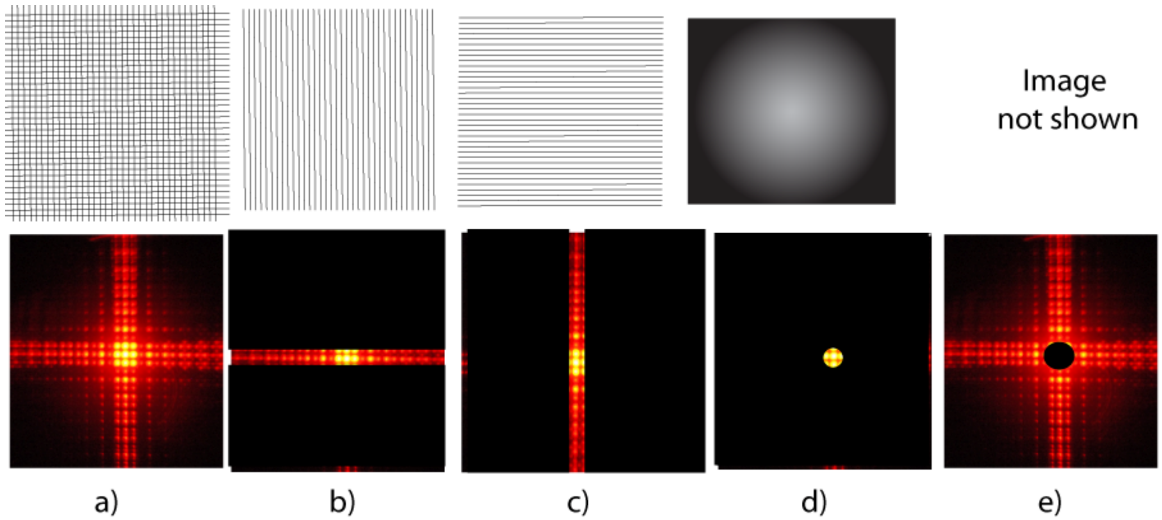

Figure 5: Removal of specific frequency components in the Fourier plane by filtering (blocking) and the resultant reconstructed image, where a) original object-no filtering, b) filtering all but one horizontal row, c) filtering all but one vertical row, d) filter all but central dot, and e) filtering of the main or central dot.

Fourier Transforms: Advanced Applications Using Schlieren Imaging

Scenario 1: You work for an advanced jet engine manufacturing company and were assigned to verify the actual direction of the exhaust of the company's new vectoring engine that varies the direction of thrust to change the direction of the aircraft. Recalling your knowledge of Fourier optics, you set up a Schlieren system to analyze the exhaust. Present a demonstration for your boss and explain how the system works.

13. Using the filtering setup, as in Figure 5e, set up a Schlieren imaging system. Verify the system as one that detects the edges or rapidly changing parts of an image, and place a finger into the working space. The edges/outline of the finger should be shown by a bright red line on a dark background.
14. Now that the system is working, place the exhaust of the jet engine—a hair dryer—into the working area. Discuss what you observe. Is the exhaust uniform?

Scenario 2: You work for an optical glass company. Currently, quality control is done by hand, with people visually inspecting the glass for minute scratches. You believe you could dramatically improve the inspection process using Fourier filtering (Schlieren).

15. Using the Schlieren/Fourier filtering system, show how you can detect fine scratches in a piece of glass plate. Discuss the results. Make a mental note of the quantity and size of the scratches detected in the Fourier filtered image.
16. Turn on the lights and observe the scratches on the glass plate. Discuss the enhancement of scratch detection using the Fourier filtering system.

Results/Discussion

The intent of the laboratory format was to actively engage the students, increase their excitement in learning about Fourier transforms, make the material more relevant and connected to the real world, and present the material in a hands-on format and in a less threatening environment.

Beginning the laboratory with a topic, such as the physics of a simple lens, which is common knowledge to any engineering student, immediately set the tone of a more relaxed environment. This quickly got them over any anxiety regarding a mathematics laboratory and the complex-looking optical setup. Further, this format of beginning with material that students understand intuitively gave them grounding when the material became more complex and unfamiliar. At moments of confusion, students may return to the familiar basic concepts and return to the unfamiliar material in a more relaxed state.

It is important to move quickly to new material because some students begin to disengage thinking (e.g., "I already know this!"). For this reason, within a few minutes of beginning the assignment, the students are presented with the complete picture of the focal plane and the presence of an optical Fourier transform. Apart from the revelation of this new information and a more complete picture, this directly challenges their early training in optics and lens, which only presented a single dot of light at the focal point—a dramatic simplification of the reality that is almost universally taught in physics classes. This introduction of the bigger picture and the notion of an optical Fourier transform took students by surprise and held their attention.

The laboratory was designed around the idea of letting concepts build without instilling excessive anxiety. While this began by starting at a common knowledge point and, thus, low anxiety, a second anxiety reducing concept was integral to the design—active engagement of the students in their learning. The students were broken into the following working groups:

- Image (reconstruction)—accurately describe the image and any changes as a result of filtering in the Fourier plane.
- Fourier transform (focal) plane—find the focal plane and accurately describe the details of the Fourier transform.
- Working area—correctly introduce objects into the working area and accurately describe the object.
- Analysis (entire class)—develop logical arguments that connect the object to the transform and the transform to the reconstructed image.

Although students were instructed to rotate groups, many of them could not wait. Students commonly left their group after hearing a description by another group to see the response for themselves. The response of, "Let me see," was common. All this occurred in a completely darkened room.

The level of direct, hands-on student engagement and excitement continued and, for most, increased as the laboratory progressed. However, at the beginning, students needed some training on what constituted a quality description of the images. Initially, they were a little reticent to fully describe the image and would make comments such as, “I just see a bunch of lines.” However, with a little coaching and with the revelation of the Fourier plane, they quickly became more active and engaged and would provide accurate descriptions such as, “I see a set of perpendicular lines with equal spacing,” (these represented the sieve wires.) Further, they were able to pay close attention to detail that they could detect small changes, such as the lines doubling when Schlieren filtering enhanced the edges of the sieve wires and removed the actual image of the wires. Similarly, their descriptions of what they saw in the Fourier plane and the filtering used became more accurate as they became more engaged.

Apart from some explanations, procedural directions, and clues, students were required to draw their own conclusion from the data they acquired. Although they were given clues such as, “relate your observations to the introductory example of a Fourier transform” (see Figure 1), their deductions were very astute.

Students reported that the laboratory module gave them a solid grounding, connecting Fourier concepts to real-world application. This connection allows students to increase or maintain interest in the concepts when presented in the more theoretical and abstract environment of the lecture room by allowing the material to be grounded to clear and intellectually accessible use in the real world.

In general, it was amazing how quickly and easily students adapted to the experiential format of learning a complex mathematical concept. In review of student comments on the laboratory experience, no one mentioned a concern regarding their lack of background in optics. Although this was a concern in the design of the module, the students readily adapted. Beyond their initial surprise at finding more than a simple dot in the focal plane, the students did not report any difficulty in accepting the optical Fourier concept and working with the equipment. Nearly all students reported that this module was helpful. They found it particularly useful to have had the experiential component prior to the traditional, formal lecture.

Regarding the length of the module of 50 minutes, a little more time would be useful. The initial laboratory ran long by approximately 10 minutes. Additionally, many students wanted to stay and continue working with the equipment or spend time discussing the concepts and applications. One hour and 15 minutes would probably be a more appropriate amount of time.

As stated, many students wanted to stay and either experiment with the system or further discuss Fourier transforms and their uses. This level of excitement and interest is in contrast to a typical mathematical lecture on FT, where engineering students might, at best, stay to discuss parts of the math that they did not understand. It was impressive to see the students actively engaged in the laboratory to the point where the discussion continued after the formal laboratory was over. In many ways, it appeared that they forgot that they were learning mathematics and were simply excited about the material.

Conclusion

An experiential mathematic module on Fourier transforms was designed and implemented in an advanced engineering mathematics course. The module was designed to improve student learning through direct hands-on experiential learning and to make the material relevant to the student by connecting the material to real-world applications.

This initial effort showed positive results for all of the project goals: actively engage students, increase student excitement about learning, increase relevance of the material, decrease the threatening nature of the learning environment, and increase hands-on learning.

The effort also attempted to minimize the effects of time constraints, variances in student knowledge of optics, and need for specialized equipment. The module was taught during one regular class period of 50 minutes. However, a longer period of one hour and 15 minutes would be a more appropriate amount of time. No student reported any concern regarding their lack of knowledge of optics. However, specialized equipment is needed. For our setup, equipment was borrowed from other laboratories.

Overall, this initial effort demonstrated that students can learn complex mathematical concepts using a combination of experiential and traditional methods. This effort will continue to include larger groups of students so that statistical data can be obtained.

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Biography

Peter J. Shull is an Associate Professor of Engineering at Penn State University at Altoona. The two primary areas of his research are nondestructive evaluation and pedagogy. He is the author of numerous articles in both fields and the author of a popular textbook in nondestructive evaluation. Dr. Shull is a Fulbright Scholar (Argentina). He received his Ph.D. from Johns Hopkins University.