

# A NUMERICAL METHOD FOR PERMEABILITY ESTIMATION

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## Abstract

Correlation between porosity and permeability for a certain rock type is a basic procedure used in core-data interpretation. However, the correlation may not always be satisfactory due to pore heterogeneity and pore geometry. In a reservoir, it is very common for rocks to have similar porosities but different permeabilities. Apparent formation factor was defined as true resistivity divided by water resistivity. In previous work, curves of the apparent formation factor versus water saturation were used to interpolate permeabilities. Unfortunately, the accuracy was very poor at high water saturation because the curves were horizontal in that region. Even in the regions which are not horizontal, chart reading can be very subjective and accuracy is limited. In this study, a numerical method was proposed in order to solve this problem. Based on combining a correlation between the formation resistivity factor and permeability proposed by Ogbe and Bassiouni [1], Archie's Equation, and the apparent formation factor definition, a new equation was derived in order to estimate permeabilities for clean formations from resistivity logs.

An algorithm was developed to carry out the procedures of the estimation, an example of which is included here in order to illustrate how to use the algorithm. This example shows that the permeability estimate from this study was more accurate than that from the previous work. In addition, this study yielded an objective permeability estimate while, the previous work gave only a subjective estimate. A computer program developed from the algorithm can be incorporated into a reservoir simulator as an improved way to input permeability values.

## Introduction

Permeability is one of the most important properties to estimate for a reservoir [2]. A good correlation between the porosity and permeability of a certain rock type is desirable for reservoir characterization or simulation. Unfortunately, the correlation may not always be satisfactory due to pore heterogeneity and pore geometry [3]. It is very common for rocks in a reservoir to have similar porosities but different permeabilities.

By analyzing laboratory measurements on 155 sandstone samples from three different oil fields in North America, Timur [4] found the empirical correlation

$$k = f^{4.4}/(S_{wr})^2 \quad (1)$$

where  $k$  is permeability,  $f$  is porosity, and  $S_{wr}$  is residual water saturation. Morris and Biggs [5] and Coates and Dumanoir [6] also developed correlations between  $k$ ,  $f$  and  $S_{wr}$ . However, these methods are only applicable at irreducible water saturation. Saner et al. [7] defined the apparent formation resistivity factor as true formation resistivity divided by water resistivity. Saner et al. [3] used curves of the apparent formation resistivity factor versus water saturation to interpolate permeabilities. However, it was difficult to get an accurate permeability reading from the plots, especially at high water saturations, because the curves became asymptotic or horizontal. This current study extends the work by Saner et al. [3] by developing a numerical method to solve the accuracy problem of the plot reading. A new equation was derived and an algorithm developed in order to estimate permeabilities for clean formations from resistivity logs.

## Approach

In the following sections, the derivation of the equation and the development of the algorithm are presented. An example is also provided to illustrate the application of the algorithm.

## Equation and Algorithm Development

By Combining  $\tau = F^b$  (2)

where  $F$  is the formation resistivity factor,  $\tau$  is tortuosity, and  $b$  is an exponent depending on the rock texture, and

$$k \propto \frac{1}{\tau} \quad (3)$$

where  $k$  is permeability, a correlation between the formation resistivity factor,  $F$ , and permeability,  $k$ , was found such that

$$F = Ak^{-B} \quad (4)$$

where  $A$  and  $B$  are constants for a specific formation.

According to Archie's Law,

$$F = S_w^n \frac{R_t}{R_w} \quad (5)$$

where  $S_w$  is water saturation,  $R_t$  is true resistivity,  $n$  is the water saturation exponent, and  $R_w$  is water resistivity. Let  $R_o$  refer to formation resistivity when water saturation is 100%. At water zones  $S_w = 1$ ,  $R_o$  is equal to  $R_t$ . From Equation (5), it can be seen that only when  $S_w = 1$  will  $F = R_t/R_w$ . Apparent formation factor,  $F_a$ , was defined by Saner et al. [7] as

$$F_a = \frac{R_t}{R_w} \quad (6)$$

From Equations (4) - (6), one gets

$$S_w^n F_a = Ak^{-B} \quad (7)$$

Taking a logarithm for both sides of Equation (7), and then rearranging, yields

$$\log(F_a) = -n \log(S_w) + (\log(A) - B \log(k)) \quad (8)$$

It can be inferred from Equation (8) that  $\log(F_a)$  is linear to  $\log(S_w)$  for a certain group of core samples, which have constant  $n$ ,  $A$ ,  $B$  and  $k$  values. If many core samples are obtained, they can be grouped according to the procedures similar to those provided by Saner et al. [3], and a series of straight lines of  $\log(F_a)$  versus  $\log(S_w)$  can be plotted. The difference of intercepts between different lines is due to the difference of the permeabilities of the groups represented by the lines. The higher the permeability, the lower the  $\log(F_a)$ -axis intercept of the line. Based on the above discussion, it can be inferred that the curves of  $\log(F_a)$  versus  $\log(S_w)$ , plotted from different groups of core samples, can be used to estimate permeability. Also the more data obtained, the more accurate the final estimate.

For a certain part of the formation, true resistivity can be obtained from a deep induction log, and water resistivity can be obtained from a water catalog, water analysis, SSP (static spontaneous potential), or other kinds of cross-plots. Then, the apparent formation resistivity factor can be calculated from Equation (6). Porosity,  $f$ , can be obtained from the density log, sonic log or neutron log. If parameters such as cementation factor  $a$ , cementation exponent  $m$  and saturation exponent  $n$ , are known, then for a clean formation (without shale), water saturation can be calculated from Archie's Equation as follows:

$$S_w = \left[ \frac{aR_w}{\phi^m R_t} \right]^{\frac{1}{n}} \quad (9)$$

McCoy and Grieves [8] present procedures to calculate water saturation at Prudhoe Bay. If parameters such as  $a$ ,  $m$  and  $n$  are unknown for a formation, procedures similar to those illustrated by McCoy and Grieves [8] can be applied to solve for these parameters. Alfosail and Alkaabi [9] developed an equation to calculate water saturation in shaly formations. If the formation is a shaly formation, it is required that a suitable equation be developed to calculate the water saturation following similar steps [9].

Suppose point  $\log(S_w)$ ,  $\log(F_a)$  is located between the two straight lines defined by the following two equations:

$$\log(F_{a1}) = -n_1 \log(S_{w1}) + (\log(A) - B \log(k_1)) \quad (10)$$

and

$$\log(F_{a2}) = -n_2 \log(S_{w2}) + (\log(A) - B \log(k_2)) \quad (11)$$

where  $k_1$  and  $k_2$  refer to the permeabilities of the individual straight lines, and  $n_1$  and  $n_2$  are the slopes. In Appendix A, a new equation is derived to estimate permeabilities for clean formations from resistivity logs, that is,

$$k = k_1 \left[ \frac{k_2}{k_1} \right]^{\frac{b_1 - b_2}{b_1 - b_2}} \quad (12)$$

where  $b_1 = \log(A) - B \log(k_1)$ ,  $b_2 = \log(A) - B \log(k_2)$ . If  $n_1 \neq n_2$ , then  $b$  is calculated by

$$b = \frac{x_2 y_1 - x_1 y_2}{x_2 - x_1} \quad (13)$$

where  $x_2 = \log(S_w)$ ,  $y_2 = \log(F_a)$ , and

$$x_1 = \frac{b_2 - b_1}{n_2 - n_1} \quad (14)$$

$$y_1 = \frac{n_2 b_1 - n_1 b_2}{n_2 - n_1} \quad (15)$$

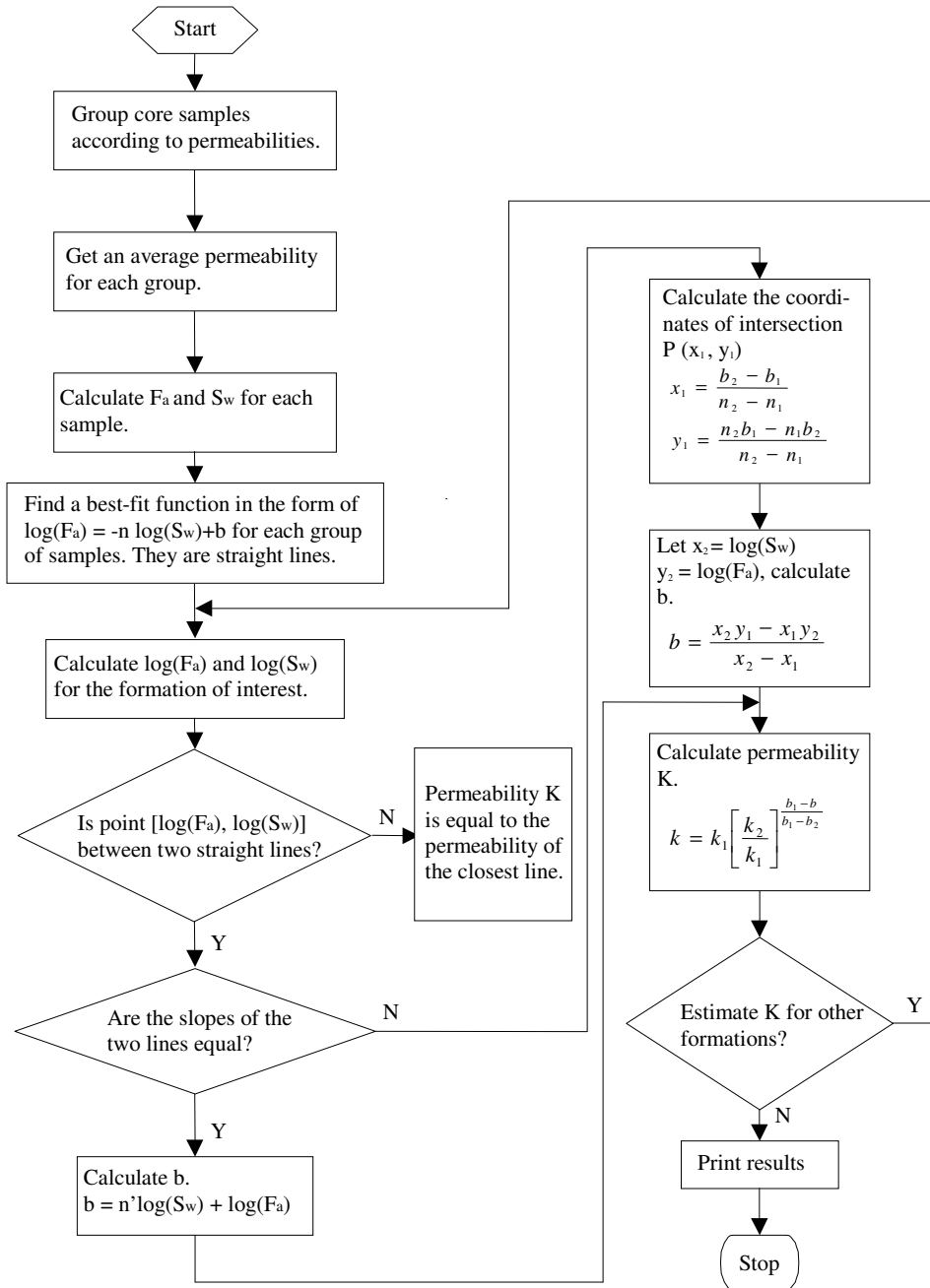
If  $n_1 = n_2$ , Equations (14) and (15) are not valid. If  $n_1 = n_2 = n'$ , then  $b$  is calculated by

$$b = n' \log(S_w) + \log(F_a) \quad (16)$$

Based on the above analysis, an algorithm was developed to estimate permeability, as shown in Figure 1. In the following two sections, (Synthetic Example and Analysis), an example is given to illustrate the permeability estimation process for a carbonate formation.

## Synthetic Example

Suppose that measurements for six groups of core samples are available from a carbonate formation and each



**Figure 1. Algorithm to Estimate Permeabilities for Clean Formations from the Resistivity**

group has a different average permeability,  $K_{ave}$ . Correlations of the apparent formation resistivity factor  $F_a$  versus water saturation,  $S_w$ , are listed in Table 1, where data are created by following the example given by Saner et al. [3]. There are log measurements for a part of the formation, and then water saturation and apparent formation resistivity factor values that can be calculated from Equations (9) and (6), respectively. Suppose further that the water saturation is

found to be 0.50 and the apparent formation resistivity factor is 100.00, then estimate the permeability for this part of the formation.

## Analysis

Following the above procedures, the permeability can now be estimated. Curves of apparent formation resistivity

**Table 1.  $F_a$  vs  $S_w$  for Six Groups of Core Samples**

Sw	Fa					
	Group 1 $K_{ave}=0.01$ md	Group 2 $k_{ave}=0.15$ md	Group 3 $k_{ave}=1.5$ md	Group 4 $K_{ave}=10$ md	Group 5 $K_{ave}=85$ md	Group 6 $K_{ave}=750$ md
0.07					1681.77	1317.99
0.08					1326.37	1042.14
0.09					1075.99	847.17
0.10					892.54	703.88
0.11	82037.79	31771.31	4962.92	1770.31	897.84	795.26
0.12	63578.29	20724.38	3748.99	766.72	410.26	360.80
0.13	50094.78	18290.74	5174.57	908.39	335.01	573.73
0.14	41443.91	18960.95	1025.63	1082.82	615.24	260.51
0.15	33484.65	9619.26	3391.17	301.58	627.89	465.01
0.16	28659.62	12725.17	2114.59	887.76	269.11	207.99
0.17	23324.28	7137.20	2188.68	280.15	343.82	366.84
0.18	21575.76	7568.38	1927.41	252.68	310.47	155.37
0.19	16588.56	7736.30	1709.04	663.06	281.90	314.66
0.20	17359.51	3342.57	1234.79	184.57	198.24	130.02
0.22	10657.45	5192.25	1693.54	315.95	292.99	255.92
0.23	9498.35	2993.49	817.42	347.98	160.44	113.69
0.24	10962.10	3469.12	1016.51	236.32	185.77	190.96
0.21	12328.30	3982.76	967.99	346.88	235.78	150.92
0.22	9769.02	5892.25	1433.54	354.95	266.99	210.92
0.23	11103.78	2893.49	838.42	288.98	133.44	129.69
0.25	7136.82	3905.60	628.29	244.44	221.72	109.50
0.27	7495.67	2520.66	782.25	269.45	219.55	122.72
0.28	4896.33	2283.95	721.47	116.70	111.08	115.11
0.29	6195.73	1676.64	667.31	230.46	104.52	108.22
0.30	5566.46	2194.25	418.85	119.52	180.74	101.96
0.32	3157.54	1590.14	734.10	187.92	81.16	72.02
0.33	3006.30	1762.84	300.64	140.00	127.22	101.23
0.35	3314.17	891.11	539.23	167.40	64.73	64.75
0.36	2938.82	879.39	313.56	117.57	126.08	97.99
0.38	2541.43	1277.84	444.81	138.47	52.80	38.28
0.39	2415.72	620.97	266.28	61.11	78.09	87.28
0.41	2788.32	522.04	388.94	90.55	71.42	36.87
0.42	711.35	949.67	223.82	135.28	68.41	86.42
0.44	1937.33	670.52	341.03	49.58	82.96	32.99
0.45	1383.01	860.88	251.16	75.12	39.49	68.98
0.47	1201.82	460.71	228.01	98.84	88.97	26.30
0.48	1464.94	529.60	217.58	46.99	53.90	44.61
0.50	919.47	774.10	148.70	63.80	25.12	41.52
0.51	1133.57	249.32	220.14	58.43	48.37	30.10
0.52	1151.93	526.27	182.10	36.19	65.73	68.76
0.53	995.48	315.81	174.55	73.09	31.16	14.48
0.54	966.84	484.80	217.44	52.09	55.68	46.27
0.56	963.76	302.67	154.43	48.43	30.94	16.02
0.58	891.45	367.02	94.84	27.13	38.45	31.98
0.60	721.10	248.18	132.46	42.16	36.19	40.13
0.62	578.20	322.58	123.15	55.48	47.14	13.44
0.64	546.53	175.75	154.75	37.04	17.25	36.90
0.66	651.07	223.32	74.16	53.82	48.53	14.48
0.68	438.97	328.95	129.28	32.80	28.95	37.18
0.70	385.29	78.38	94.02	20.94	36.49	15.98
0.72	491.11	210.38	58.31	29.24	26.14	29.87
0.74	396.12	163.75	83.09	27.68	17.89	8.84
0.76	473.46	202.33	100.30	33.23	23.73	26.88
0.78	227.41	141.97	93.91	12.90	19.66	19.00
0.80	326.44	94.55	40.86	23.67	31.66	12.17
0.82	329.27	95.97	84.13	12.52	20.72	24.40
0.84	278.77	146.12	46.68	21.46	9.85	13.67
0.86	226.74	37.95	79.48	25.47	24.03	16.00

**Table 1.  $F_a$  vs  $S_w$  for Six Groups of Core Samples (continued)**

Sw	Fa					
	Group 1 $K_{ave}=0.01$ md	Group 2 $k_{ave}=0.15$ md	Group 3 $k_{ave}=1.5$ md	Group 4 $K_{ave}=10$ md	Group 5 $K_{ave}=85$ md	Group 6 $K_{ave}=750$ md
0.88	258.56	90.36	37.52	19.55	8.27	15.37
0.90	233.27	124.31	53.76	18.68	17.55	19.77
0.92	169.43	64.74	32.20	11.88	25.87	11.21
0.94	206.68	85.60	48.80	20.12	16.24	18.68
0.96	224.12	115.85	69.57	12.41	20.64	13.18
0.98	147.16	57.45	44.48	20.75	8.08	8.72
1.00	195.98	89.38	15.53	12.12	20.54	12.27

factor versus water saturation are plotted in Figure 2, along with the regression fit equations. Figure 3 shows the curves of  $\log(F_a)$  versus  $\log(S_w)$  plotted from the functions obtained for the six groups of core samples. From Figure 2, it can be determined that point (0.50, 100) is between the curve for the average permeability of 1.5md and that the average permeability of 10md. That is, in Figure 3, point  $\log(0.5)$ ,  $\log(100)$  is between the straight lines defined by the following two equations:

$$\log(F_{a1}) = -2.1866 \log(S_{w1}) + 1.6096 \quad (17)$$

and

$$\log(F_{a2}) = -1.998 \log(S_{w2}) + 1.162 \quad (18)$$

From Equations (14) and (15), the coordinates of the intersection for the two straight lines was calculated to be (2.3732, -3.5798). The measurement is at point  $\log(0.50)$ ,  $\log(100.00)$ , that is, point (-0.3010, 2.0000). The intercept for the straight line which passes points (2.3732, -3.5798) and (-0.3010, 2.0000) can be calculated by Equation (13). The intercept (b) is 1.3720. With  $k_1 = 1.5\text{md}$ ,  $k_2 = 10\text{md}$ ,  $b = 1.3720$ ,  $b_1 = 1.6096$  and  $b_2 = 1.162$ , the permeability for this part of the formation, k, was calculated from Equation (11) and found to be 4.106md.

## Results and Discussions

From the example just presented, the authors feel that the numerical method does improve the accuracy of the permeability estimation. From Figure 2, to estimate the permeability by reading the plot, 3md can be used as an estimate, albeit a very subjective one. Using the numerical method, the permeability was estimated to be 4.106md. Figure 4 is a plot of the apparent formation factor versus water saturation from the previous work [3]. With a water saturation of 0.5 and apparent formation factor of 100, from Figure 4, one estimate of permeability may give 5md while another may give 3md. The results are very subjective and inaccurate. The accuracy of the estimate from the work presented here is much improved over the previous work and improves

permeability estimates by several digits beyond the decimal point. The importance of this work is to improve the accuracy of permeability estimation and reduce human errors in figure reading. In addition, the new equation and algorithm developed in the permeability estimation process can be programmed into computers, thus speeding up the estimation process.

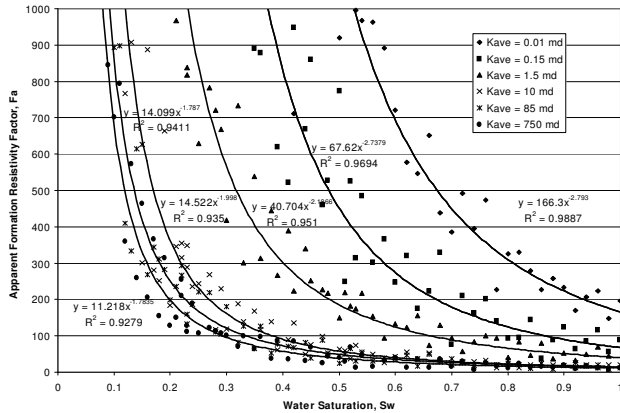


Figure 2. Plot of Apparent Formation Resistivity  $F_a$  versus Water Saturation  $S_w$

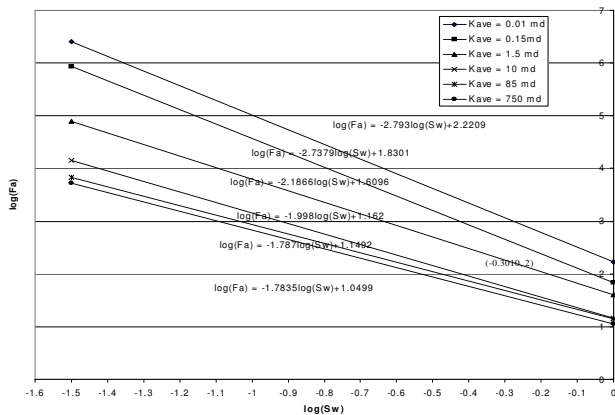


Figure 3.  $\log(F_a)$  vs  $\log(S_w)$

As mentioned previously in reference to Equation (4), A and B are constants for a certain formation. Now let us investigate what parameters A and B may be related to. The Carman-Kozeny equation is

$$k = \frac{\phi}{(k_z \tau) S_{pv}^2} \quad (19)$$

where  $k_z$  is Kozeny's constant and  $S_{pv}$  is the internal surface area of the pores per unit of pore volume. The generalized  $\tau$ -F relationship is in the form of

$$\tau = (F\phi)^y \quad (20)$$

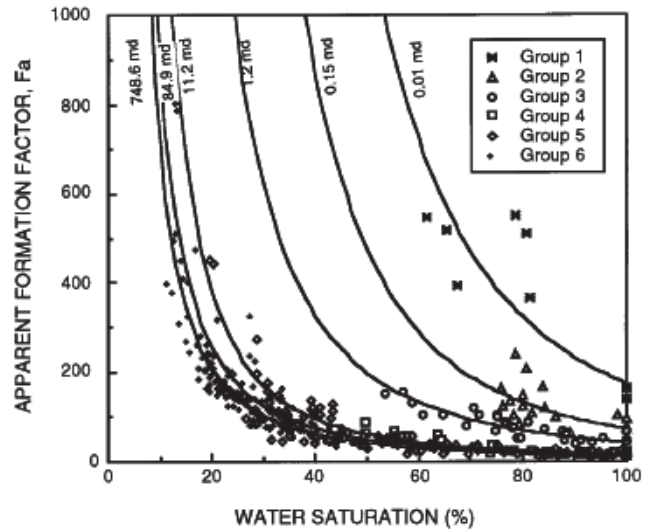


Figure 4. Apparent Formation Factor vs Water Saturation for Various Permeability Groups [3]

where  $y$  is an exponent. Combining Equations (18) and (19) gives

$$k = \frac{\phi^{1-y}}{k_z F^y S_{pv}^2} \quad (21)$$

Substituting the Salem & Chilingarian [10] relationship

$$k_z = 2.24(F\phi) \quad (22)$$

into Equation (21) leads to

$$k = \frac{\phi^{-y}}{2.24 F^{y+1} S_{pv}^2} \quad (23)$$

Rearranging Equation (23) yields

$$F = \left[ \frac{\phi^{-y}}{2.24 S_{pv}^2} \right]^{\frac{1}{1+y}} k \left[ \frac{1}{1+y} \right] \quad (24)$$

Comparing Equation (24) with Equation (4), we have

$$B = -\frac{1}{1+y} \quad (25)$$

and

$$A = \left[ \frac{\phi^{-y}}{2.24 S_{pv}^2} \right]^B \quad (26)$$

Equations (25) and (26) show that the constant, B, is related only to  $y$ , the exponent in the generalized  $\tau$ -F relationship, while the other constant, A, is a function of  $y$ , the pore

-volume-based specific surface,  $S_{pv}$ , and porosity  $f$ . For a certain formation,  $y$ ,  $S_{pv}$  and  $f$  are referred to average properties and can be considered constants; thus, from Equations (25) and (26), it can be said that A and B are constants.

## Conclusions

From this study, three conclusions can be reached:

1. A new equation was derived and an algorithm developed to calculate permeabilities for clean formations from resistivity log measurements. By applying the algorithm, accuracy was improved in the estimation of permeability.
2. The algorithm can be incorporated into a reservoir simulator so that it will provide an improved permeability input.
3. Equations to predict the constants in the correlation of the formation resistivity factor versus permeability were derived. Parameters related to the constants were also found.

## Nomenclature

### English Symbols

A	= constant
a	= cementation factor
b	= exponent of formation resistivity factor; intercept
B	= constant
F	= formation resistivity factor
k	= permeability, $L^2$ , md; Kozeny's constant
$K_{ave}$	= average permeability, $L^2$ , md
m	= cementation exponent
n	= water saturation exponent
R	= resistivity, W.m
S	= saturation; specific surface, 1/L, 1/m
x	= $\log(S_w)$ axis
y	= $\log(F_a)$ axis; exponent

### Greek Symbols

$\Delta$	= intercept difference
$\phi$	= porosity
$\tau$	= tortuosity

## Subscripts

1	= property of line 1
2	= property of line 2
a	= apparent
o	= 100% water saturation
pv	= pore volume
r	= residual
t	= True
w	= water
z	= Kozeny

## References

- [1] Ogbe, D. & Bassiouni, Z. (1978). Estimation of Aquifer Permeabilities from Electrical Well Logs. *The Log Analyst*, 19(5), 21-27.
- [2] Lopez, C. & Davis, T. L. (2011). *Permeability Prediction and its Impact in Reservoir Modeling. Postle Field, Oklahoma*. AAPG Search and Discovery Article #90129, presented at the AAPG Southwest Section Meeting, Ruidoso, New Mexico.
- [3] Saner, S., Kissami, M. & Nufaili, S. A. (1997). Estimation of Permeability from Well Logs Using Resistivity and Saturation Data. *SPE Formation Evaluation*, 12(1). 27-31.
- [4] Timur, A. (1968). An Investigation of Permeability, Porosity, and Residual Water Saturation Relationship for Sandstone Reservoirs. *The Log Analyst*, 9(4).
- [5] Morris, R. L. & Biggs, W. P. (1967). Using Log-Derived Values of Water Saturation and Porosity. *Proceedings of the SPWLA Eight Annual Logging Symposium*.
- [6] Coates, G. R. & Dumanoir, J. L. (1973). A New Approach to Improved Log-Derived Permeability. *Proceedings of the SPWLA Fourteen Annual Logging Symposium*.
- [7] Saner, S., Orcan, A. & Wajid, M. (1994). Apparent Cementation Factor Concept for Water Saturation Determination from Well Logs. *The Arabian Journal for Science and Engineering*, 19(3).
- [8] McCoy, D. D. & Grieves, W. A. (1997). Use of Resistivity Logs to calculate Water Saturation at Prudhoe Bay. *SPE Reservoir Engineering*, 12(1), 45-51.
- [9] Alfossail, K. A. & Alkaabi, A. U. (1997, March). *Water Saturation in Shaly Formation*. Paper presented at the Middle East Oil Show and Conference, Bahrain.
- [10] Chilingarian, G., Torabzadeh, J., Rieke, H., Metghal-

chi, M., & Mazzullo, S. (1992). Interrelationships Among Surface Area, Permeability, Porosity, Pore Size, and Residual Water Saturation. In Chilingarian, G., Mazzullo, S., & Rieke, H. (Eds.), *Carbonate Reservoir Characterization: A Geologic-Engineering Analysis* (pp. 379-397). Amsterdam: Elsevier.

## Appendices

### Derivation of the New Equation

Suppose point  $\log(S_w)$ ,  $\log(F_a)$  is located between the two straight lines defined by the following two equations:

$$\log(F_{a1}) = -n_1 \log(S_{w1}) + (\log(A) - B \log(k_1)) \quad (A-1)$$

and

$$\log(F_{a2}) = -n_2 \log(S_{w2}) + (\log(A) - B \log(k_2)) \quad (A-2)$$

The difference of the intercepts for these two straight lines is

$$\Delta = B(\log(k_2) - \log(k_1)) \quad (A-3)$$

Now, if we let

$$b_1 = \log(A) - B \log(k_1) \quad (A-4)$$

and

$$b_2 = \log(A) - B \log(k_2) \quad (A-5)$$

then from Equations (A-3), (A-4) and (A-5) we get

$$B = \frac{b_1 - b_2}{\log(k_2) - \log(k_1)} \quad (A-6)$$

If in Equations (A-1) and (A-2)  $n_1 \neq n_2$ , then the lines are not parallel and the two straight lines defined by Equations (A-1) and (A-2) cross at point P. From Equations (A-1), (A-2), (A-4) and (A-5), the coordinates of point P ( $x_1$ ,  $y_1$ ) can be calculated as

$$x_1 = \frac{b_2 - b_1}{n_2 - n_1} \quad (A-7)$$

$$y_1 = \frac{n_2 b_1 - n_1 b_2}{n_2 - n_1} \quad (A-8)$$

Point  $\log(S_w)$ ,  $\log(F_a)$  is located between the two straight lines defined by the Equations (A-1) and (A-2). Next, let

$$x_2 = \log(S_w) \quad (A-9)$$

and

$$y_2 = \log(F_a) \quad (A-10)$$

then the straight line which passes points ( $x_1$ ,  $y_1$ ) and ( $x_2$ ,  $y_2$ ) has a function of

$$y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_1 \quad (A-11)$$

Its y intercept is

$$b = \frac{x_2 y_1 - x_1 y_2}{x_2 - x_1} \quad (A-12)$$

The difference between the intercept of the straight line defined by Equation (A-11) and that of the line defined by Equation (A-1) is

$$b_1 - b = -B[\log(k_1) - \log(k)] \quad (A-13)$$

From Equations (A-6) and (A-13), k can be solved as

$$k = k_1 \left[ \frac{k_2}{k_1} \right]^{\frac{b_1 - b}{b_1 - b_2}} \quad (A-14)$$

From Equation (A-14), when  $b = b_1$ , it can be seen that  $k = k_1$ ; and when  $b = b_2$ ,  $k = k_2$ . So, the calculation of permeability is effective for the whole interval  $b \in [b_2, b_1]$ . If  $n_1 = n_2 = n$ , then the two straight lines defined by Equations (A-1) and (A-2) are parallel to each other. From point ( $x_2$ ,  $y_2$ ), a straight line can be drawn parallel to these two lines. Letting  $b$  be the intercept of this new line, then

$$\log(F_{a1}) - \log(F_a) = b_1 - b = -B[\log(k_1) - \log(k)] \quad (A-15)$$

and

$$\log(F_{a1}) - \log(F_{a2}) = b_1 - b_2 = -B[\log(k_1) - \log(k_2)] \quad (A-16)$$

From Equations (A-15) and (A-16), k can be solved as

$$k = k_1 \left[ \frac{k_2}{k_1} \right]^{\frac{b_1 - b}{b_1 - b_2}} = k_1 \left[ \frac{k_2}{k_1} \right]^{\frac{\log(F_{a1}) - \log(F_a)}{\log(F_{a1}) - \log(F_{a2})}} \quad (A-17)$$

where  $b = n \log(S_w) + \log(F_a)$ ;  $b_1$  and  $b_2$  were defined in Equations (A-4) and (A-5);  $\log(F_{a1})$  was calculated from Equation (A-1), assuming that  $S_{w1} = S_w$ ,  $\log(F_{a2})$  was calculated from Equation (A-2) and that  $S_{w2} = S_w$ .

Furthermore, from Equation (A-17), when  $b = b_1$ , it can be seen that  $k = k_1$ ; and when  $b = b_2$ ,  $k = k_2$ . So, the calculation of permeability is also effective for the whole interval  $b \in [b_2, b_1]$ . The first part of Equation (A-17) is the same as Equation (A-14). If the permeability is calculated from in-

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tercept differences, for both cases,  $n_1 = n_2$  and  $n_1 \neq n_2$ , the same equation can be used. If  $n_1$  and  $n_2$  are equal to each other, another method for calculating the permeability would be to use the second part of Equation (A-17).

## SI Metric Conversion Factors

$$\text{md} \cdot 9.869\,233 \text{ E-04} = \text{mm}^2$$

## Biography

**DACUN LI** is featured in *Who's Who in America 2011 (65<sup>th</sup> Edition)*. He is Coordinator and Assistant Professor of Petroleum Engineering Program at the University of Texas of the Permian Basin, and Editorial Review Committee member of the Society of Petroleum Engineers (SPE). Holding three degrees respectively in three different areas (a Bachelor's degree in Aerospace Engineering, a Master's degree in Health Physics, and a Ph.D. in Petroleum Engineering), he has international, academic, and industrial work experiences. He was one of the main characters in the TV documentary titled *Red Capitalism* (1994 Canada's Golden Sheaf Award winner), produced by Canadian Broadcasting Corporation (CBC) in July 1993. With a personable character, Dr. Li likes singing, dancing, photographing, practicing calligraphy, sledding, jogging, playing table tennis, and traveling. He can be reached at [li\\_d@utpb.edu](mailto:li_d@utpb.edu)