A LINEARIZED GERBER FATIGUE MODEL

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Abstract

A new bending fatigue design model based on the linearization of the Gerber failure criterion is presented here. The model divides the design diagram into two failure regimes: dynamic fatigue and static fatigue. Fatigue failure is generally of a brittle fracture nature. Dynamic fatigue failure is associated with fatigue strength of a material, while static fatigue failure is associated with the ultimate tensile strength of a material. Some examples are presented in order to demonstrate the use of the model in design verification and sizing. This model provides solutions that are slightly more conservative than the Gerber criterion but not as conservative as the Goodman criterion. This means that when bending fatigue is a significant serviceability criterion, it will yield design solutions with smaller sizes, giving more cost-effective designs. Generally, smaller components weigh less and are often cheaper to manufacture, enhancing profitability. Also, smaller products will reduce the rate of depletion of scarce materials. This new model, then, has the potential to help achieve economical designs for machine and structural members.

Introduction

Many machine and structural members are loaded by repeated or cyclic forces that can lead to fatigue failure. About 80% to 90% of the failures of machine and structural members result from fatigue [1], [2]. Hence, fatigue failure represents a significant proportion of failure problems in mechanical and structural systems. The importance of fatigue failure was first recognized by Pencelot in 1839. Rankine made similar observations about fatigue phenomena in 1843. Between 1843 and 1870, Wholer designed fatigue testing machines and used them to conduct many fatigue tests, including the investigation of the influence of stress concentration due to changes in cross sections [4]. Fatigue failure since then has been studied by numerous scientists and engineers such as Gerber, Goodman, Soderberg, Miner, Petersen, Marin, and a host of others [3-5]. Norton, [3] provides a time line and summary of many contributors in fatigue science and technology.

Several approaches are available for fatigue design and analysis [3]. The focus in this study is the stress-life (*S-N*) approach. An *S-N* diagram [6] displays three distinct portions judging by its slope. Hence, in the *S-N* approach, the fatigue load cycles may be divided into low-cycle fatigue, high-cycle fatigue and infinite-life fatigue regimes. However, there is no universal agreement on the dividing line between these regimes as overlap exists from classification by different authors [3], [7], [8]. Low-cycle fatigue is generally in the range of 1 to 10^3 load cycles and high-cycle fatigue is between 10^3 and 10^7 load cycles. Infinite-life fatigue is generally 10^6 load cycles and above. In low- and high-cycle fatigue, the life of a component is measured as the number of load cycles before failure. In infinite-life fatigue, the material is able to sustain an unlimited number of load cycles at some low stress levels. For most steel materials, infinite-life is observable between $2x10^6$ and 10^7 load cycles. For materials without apparent infinite life, it is often taken to be 10^8 or $5x10^8$ cycles.

The stress state in bending fatigue is appraised from the maximum and minimum stress values imposed on the structural or machine member during one load cycle. The exact variation of the stress during the cycle does not seem to be particularly relevant [1], [7]. The damage from a fluctuating bending stress state is assessed on the basis of the mean and alternating stress components. The alternating and mean stress components (please refer to Appendix for nomenclature) per cycle are, respectively:

$$\sigma_a = \frac{1}{2}(\sigma_{max} - \sigma_{min}) \tag{1}$$

$$\sigma_m = \frac{1}{2}(\sigma_{max} + \sigma_{min}) \tag{2}$$

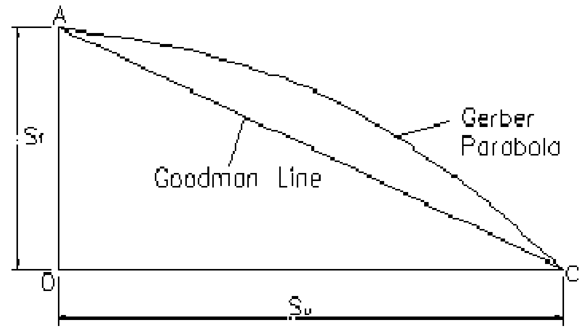
When σ_m is positive or tensile during a fatigue load cycle, the material can fail at stress levels lower than the yield strength. Several models [3], [4], [9-12] are available in that address the influence of tensile mean stress on fatigue life. Among these are the Gerber (Germany, 1874), Goodman (England, 1899), and Soderberg (USA, 1930) models. According to Norton [3], the Gerber criterion is a measure of the average behavior of ductile materials in fatigue resistance, while the modified Goodman criterion is that of minimum behavior. Shingley and Mischke [10], state that the Gerber criterion falls centrally on experimental data while the modified Goodman criterion does not. The modified Goodman criterion is often used as a design criterion because it is more conservative than the Gerber criterion. Also, the modified Goodman criterion is simpler in application, especially in determining the size of members due to its linear nature. The use of the Gerber criterion in the determination of member size is generally more computationally intensive and so rather unattractive for many designers. If the Gerber criterion is linearized, it can be used to determine the size of machine and structural members like the modified Goodman criterion.

The objective of this study was to develop a linearized model of the Gerber criterion so that it could be used in design sizing, like the modified Goldman criterion, without iterations. Because the Gerber criterion represents average behavior of ductile materials, one can expect 50% reliability. Hence, using this rule in a probabilistic model for design sizing means definite probability goals can be achieved. Over-design can be avoided by using probabilistic methods, while still ensuring the safety of a component [13]. Because it is less conservative than the modified Goodman criterion, a linearized Gerber rule will lead to reduced component sizes so that designs can be more cost effective as smaller components are lighter and often easier to make. In a global and technologically advancing world economy, cost-effective designs are a competitive edge. Lastly, material usage per product will be reduced, which will help to conserve scarce resources.

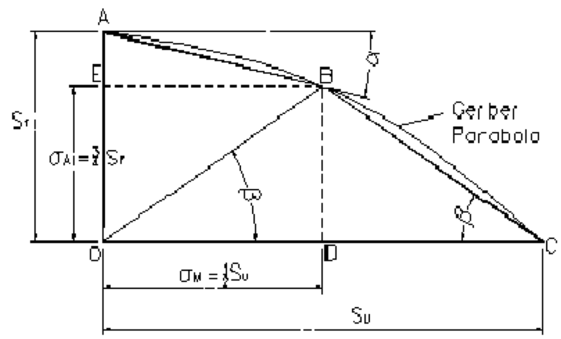
Linearizing the Gerber Criterion

Figure 1a shows a bending fatigue diagram with tensile mean stress indicating the Gerber parabola and the modified Goodman line. In this figure, the fatigue strength of the material is on the vertical axis and the ultimate tensile strength is on the horizontal axis. The Soderberg model is not shown because it is said to be more conservative than the modified Goodman rule [10] and is seldom used.

The basic idea of a linearized Gerber criterion is the approximation of the Gerber parabola with two line segments. Figure 1b shows two line segments AB and BC as approximations of the Gerber curve. This effectively divides the allowable design space into two triangles OAB and OBC with line OB common between these two triangles. In region OAB, material failure will most likely result from the predominant influence of the alternating stress and is called the dynamic fatigue regime. Brittle fracture is expected to be the dominant mode of failure in this regime. The failure line in triangle OAB is line AB and it makes angle α with the horizontal line. In region OBC, material failure will most likely result from the predominant influence of the mean stress and is called the static fatigue regime. Brittle fatigue failure is still expected in this regime, but some type of yielding is conceivable, especially at the micro-level, before gross fracture. The failure line in triangle OBC is line BC and it makes angle β with the horizontal line. It can be seen that lines AB and BC are on the conservative side of the Gerber parabola. The linearized Gerber model is defined by angles α (or ψ) and β (or η_t).



a) Common



b) Linearized Gerber
Figure 1. Bending Fatigue Design Diagrams

The Gerber parabola in bending fatigue is described by the equation:

$$\sigma_A = S_f\left[1-\left(\frac{\sigma_M}{S_u}\right)^2\right] \tag{3}$$

Referring to Figure 1b, when $\sigma_M = \frac{S_u}{2}$ in Equation (3), then:

$$\sigma_A = \frac{3}{4}S_f \tag{4}$$

Therefore,

$$\tan\alpha = \psi = \frac{OA-OE}{OD} = \frac{S_f-\sigma_A}{\frac{S_u}{2}} = \frac{S_f}{2S_u} = \frac{\psi_s}{2} \tag{5}$$

$$\tan\beta = \eta_t = \frac{OE}{OD} = \frac{3S_f}{2S_u} = \frac{3\psi_s}{2} \tag{6}$$

Equations (5) and (6) depend on ψ_s , which is obtained from the basic fatigue ratio ψ_o. ψ_o is based on standard polished laboratory specimens. ψ_s is obtained by multiplying ψ_o by adjustment factors such as temperature, size or reliability [3], [9], [10], depending on service conditions. Some representative values of ψ_o are wrought steel at 0.5; cast steel, nodular cast iron, aluminum and copper alloys at 0.4; gray cast iron at 0.35; and normalized nodular cast iron at 0.33 [3], [10]. Some scatter can be expected with fatigue ratio data. Also, at relatively high ultimate tensile strengths, the fatigue ratio drops, so care is needed in using these values. Note that ψ_s will normally be smaller than ψ_o in service conditions.

Effective Bending Stress

With the angles α and β now known, the effective bending stress, resulting from a combination of alternating stress and mean stress, needs to be determined. Any combination of these stresses will have a load line that passes through the origin with a slope given by:

$$\eta = \frac{k_\sigma \sigma_a}{\sigma_m} \tag{7}$$

Equation (7) has the stress concentration factor k_σ applied to the alternating normal stress. This is necessary for realistic estimates of stresses at cross-sections with notches. According to Collins et al. [9], experimental studies indicate that stress concentration factors should be applied only to alternating components of stress for ductile materials in fatigue loading. However, stress concentration factors should be applied to both alternating and mean stresses in brittle materials when loaded in fatigue. Now, the load line factor η determines the fatigue failure regime that is appropriate for a particular situation. If η is equal to or greater than the load line transition factor, η_t , then the design point will be inside triangle OAB in Figure 1b, and the dynamic fatigue failure regime would apply. If η is less than η_t , the design point will be inside triangle OBC in Figure 1b, and the static fatigue failure regime would apply.

Dynamic Fatigue Failure Regime: $\eta \geq \eta_t$

Figure 2a shows a fatigue bending stress state with a load line in the dynamic fatigue failure regime. The effective bending stress is represented by OF and alternating stress by OE. OD represents the mean, or steady, stress. Note that lines FG and AB, the failure lines, are parallel. This ensures that effective stress is being mapped with the appropriate failure rule. If these two lines are not parallel, a different failure criterion would apply to line FG, introducing distortion to the failure rule.

Referring to Figure 2a:

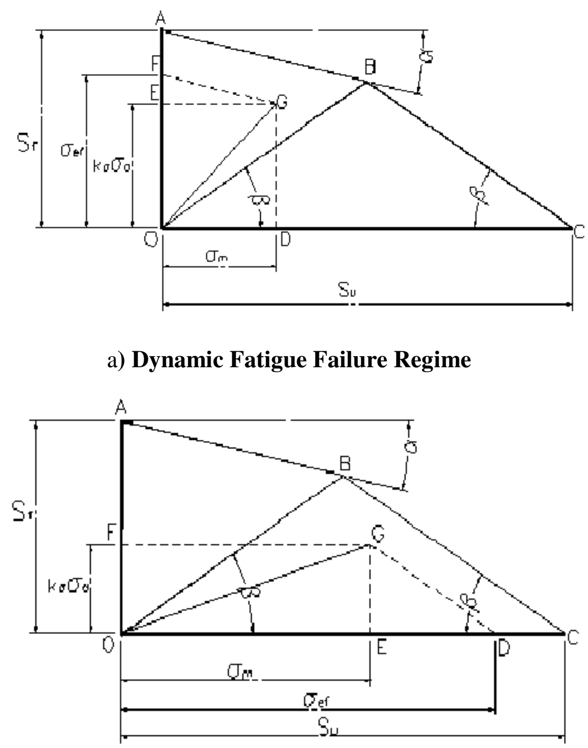
$$OF = \sigma_{ef} \qquad OE = k_a \sigma_a \qquad OD = EG = \sigma_m$$

$$EF = EG \tan \alpha = \psi \sigma_m \tag{8}$$

$$OF = OE + EF \tag{9}$$

$$\sigma_{ef} = k_a \sigma_a + \psi \sigma_m = k_a \sigma_a + \frac{1}{2} \psi_s \sigma_m \tag{10}$$

Equation (10) gives the effective normal stress for the new model in the dynamic fatigue failure regime.



a) **Dynamic Fatigue Failure Regime**

b) **Static Fatigue Failure Regime**

Figure 2. Effective Bending Stresses in Failure Regimes

The effective bending stress in this situation is projected on the fatigue strength (vertical) axis. Fatigue strength is a dynamic material property. The effective stress based on the modified Goodman model can be expressed as:

$$\sigma_{ef} = k_a \sigma_a + \frac{S_f}{S_u} \psi \sigma_m = k_a \sigma_a + \psi_s \sigma_m \tag{11}$$

The effective normal stress based on the Gerber model [9] is:

$$\sigma_{ef} = \frac{k_\sigma \sigma_a}{1 - \left(\frac{\sigma_m}{S_u}\right)^2} \quad (12)$$

The factor of safety in the dynamic fatigue failure regime is obtained as:

$$n_s = \frac{S_f}{\sigma_{ef}} \quad (13)$$

Static Fatigue Failure Regime: $\eta < \eta_t$

Figure 2b shows a fatigue bending stress state with a load line in the static fatigue failure regime. In this regime, the effective normal stress is projected on the horizontal axis. The effective bending stress is represented by OD and alternating stress by OF. Note that lines DG and BC are parallel, which makes the slope of line DG equal to that of line BC, the failure line. As in the static fatigue failure regime, this ensures that effective stress is being mapped with the appropriate failure rule.

Referring to Figure 2b:

$$OD = \sigma_{ef}$$

$$EG = OF = k_a \sigma_a$$

$$FG = OE = \sigma_m$$

$$\tan\beta = \eta_t = \frac{OF}{ED} \qquad ED = \frac{OF}{\tan\beta} = \frac{k_\sigma \sigma_a}{\eta_t} = \frac{2k_\sigma \sigma_a}{3\psi_s} \quad (14a)$$

$$OD = OE + ED \quad (14b)$$

Thus:

$$\sigma_{ef} = \sigma_m + \frac{k_\sigma \sigma_a}{\eta_t} = \sigma_m + \frac{2k_\sigma \sigma_a}{3\psi_s} \quad (15)$$

Equation (15) gives the effective mean normal stress by the new model for the static fatigue failure regime. The effective bending stress in this situation is projected on the tensile strength (horizontal) axis. Tensile strength is a "static" material property. The effective stress based on the modified Goodman model can be expressed as:

$$\sigma_{ef} = \sigma_m + \frac{k_\sigma \sigma_a}{\psi_s} \quad (16)$$

An expression for the Gerber effective mean normal stress can be found by replacing S_u with σ_{ef} in the Gerber criterion and simplifying it. That is,

$$\sigma_{ef} = \frac{\sigma_m}{\sqrt{1 - \frac{k_\sigma \sigma_a}{S_f}}} \quad (17)$$

The factor of safety in the static fatigue failure regime is obtained as:

$$n_s = \frac{S_u}{\sigma_{ef}} \quad (18)$$

Avoiding Yield at the First Load Cycle

There is a possibility that when a static fatigue failure condition exists, a member may yield at the first load circle [3], [10]. The Langer or the yield line shown in Figure 3 is inclined at 45^o to the horizontal. If a stress state lies to the right of line DF, then yielding would have occurred. If this happens, local yielding can occur, which can lead to changes in straightness and strength (local strain hardening is also possible), resulting in unpredictable loading [10]. The line DE in Figure 3 is parallel to the failure line BC. Now, if the angle β is smaller than 45^o, then yielding can be prevented in the static fatigue failure regime by translating line BC to the position of line DE, which does not change the failure criterion. For most materials used in fatigue design, β will be smaller than 45^o since the high value of ψ_o is about 0.5 as was indicated previously. This limits the angle β from Equation (7) to about 37^o in a worst-case scenario. However, ψ_s is normally smaller than ψ_o , so the value of β in service will even be lower than 37^o. To translate line BC to position DE means the safety factor, n_s , on the ultimate strength should be sufficiently high to preclude yielding. This condition is satisfied when

$$n_s \ \geq \ n_o = \frac{S_u}{S_Y} \quad (19)$$

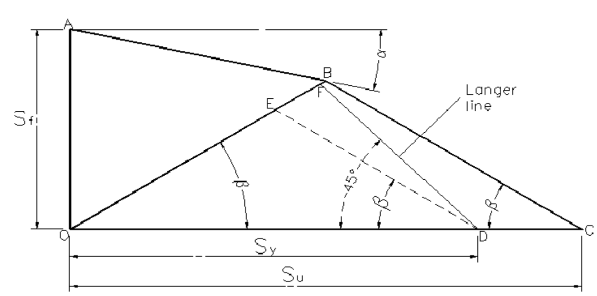


Figure 3. Avoiding Yield at First Load Cycle

Design Sizing Application

Bending stress can be expressed as a function of the bending moment and section modulus:

$$\sigma_a = \frac{M_a}{Z} \tag{20}$$

$$\sigma_m = \frac{M_m}{Z} \tag{21}$$

$$\eta = \frac{k_\sigma \sigma_a}{\sigma_m} = \frac{k_\sigma M_a}{M_m} \tag{22}$$

Dynamic Fatigue Failure Design: $\eta \geq \eta_t$

For design sizing applications in a dynamic fatigue failure regime, the task is to find a Z value that will satisfy the strength serviceability criterion. This condition is expressed as:

$$\sigma_{ef} \leq \frac{S_f}{n_s} \tag{23}$$

From Equations 10, 20, 21 and 23:

$$\sigma_{ef} = k_a \sigma_a + \psi \sigma_m = \frac{k_\sigma M_a}{Z} + \frac{\psi_s M_m}{2Z} \leq \frac{S_f}{n_s} \tag{24}$$

So that:

$$Z \geq \frac{n_s}{S_f}\left\{k_\sigma M_a + \frac{1}{2}\psi_s M_m\right\} \tag{25}$$

Static Fatigue Failure Design: $\eta < \eta_t$

For design sizing applications in a static fatigue failure regime, the task is to find a Z value that will satisfy the strength serviceability criterion:

$$\sigma_{ef} \leq \frac{S_u}{n_s} \tag{26}$$

Combining Equations 13, 20, 21 and 26:

$$Z \geq \frac{n_s}{S_u}\left\{\frac{k_\sigma M_a}{\eta_t} + M_m\right\} = \frac{n_s}{S_u}\left\{\frac{2k_\sigma M_a}{3\psi_s} + M_m\right\} \tag{27}$$

The section modulus, Z, depends on the shape and dimensions of the shape of the cross-section of a member. For simple shapes such as circles and rectangles, the formula for Z is available in structural and machine design textbooks. In structural design, values of Z can be obtained from tables for structural steel shapes; for example, AISC Steel Construction Manual.

Some Applications of the Linearized Gerber Model

The linearized Gerber model (LGM) was applied in three examples. The first is a case of possible dynamic fatigue failure taken from Norton [3], while the second example is a case of possible static fatigue failure and a modification of the first example. This example was used because it is described as a typical design problem [3]. The model application in these examples is that of design verification in which the adequacy of a design is assessed on the basis of a factor of safety for a member with a known form or 3D figure. A design is accepted as adequate if the factor of safety is at least equal to a desired value. A factor of safety greater than unity is necessary for failure avoidance. Design verification is a task often performed in the detail/prototype phase of a design project. The third example is a redesign of the components of the first example, demonstrating the application of the new model in design sizing. The task in design sizing is choosing the form and determining the size of a member for a desired factor of safety.

The form of a member is defined by its length, cross-sectional shape and dimensions over its length. In general, the cross-section may vary along the length of a member, but this makes analysis more complicated and costly. Constant cross-sectional members are usually the first choice, especially during preliminary design, but modifications often occur later in the design process. The length of a member is often based on space limitation and may be estimated in a preliminary layout diagram but can be refined later, taking into consideration rigidity and strength. The cross-section can be sized for an assumed shape based on fatigue strength or other serviceability criteria. Design sizing is a task often performed in the preliminary phase of a design project.

Example 1

Figure 4 shows one of two brackets attached to a machine frame. The brackets carry a combined fluctuating load varying from a minimum of 890N to a maximum of 9,786N, (data converted to SI Units) [3]. The load is shared equally by the brackets; the maximum allowable lateral deflection was 0.51mm for each bracket, each of which should be designed for 10^9 load cycles. The load-time function was sinusoidal, maximum cantilever length was 152mm, and the operating temperature was 50°C. Trial dimensions were b = 51mm, h = 25.4mm, H = 28.6mm, r = 12.7mm and l = 127mm. The brackets were machined to size from stocks. Norton [3] recommends values of k_σ = 1.16 and Z = 5463.45mm^3. The brackets were made from SAE 1040 steel with S_u = 550MPa, S_y = 414MPa and S_f = 150MPa at 99.9% reliability.

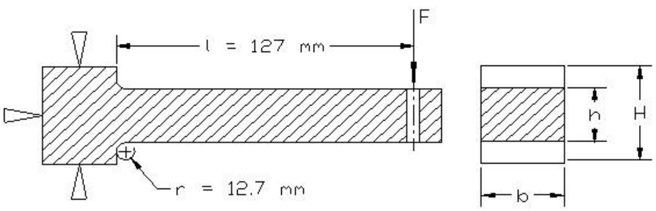


Figure 4. Sample Bracket

Solution 1

The expected 10^9 load cycles for the brackets was in the range of infinite-life regime. This simplified the estimation of the service fatigue strength [3]. Now S_f = 150MPa because it was evaluated at 99.9% reliability. At 50% reliability, the reliability adjustment factor was 1.0; at 99.9% reliability, it was 0.753 [3]. Because the Gerber model is said to represent average behavior of materials, it was consistent to use the model at a 50% reliability level. Hence, the fatigue strength at 50% reliability was S_f = 200MPa.

Equations 7, 10-13, 20 and 21 were coded in Excel, Microsoft spreadsheet software for analysis in dynamic fatigue bending fatigue failure. Table 1 shows the layout of the Excel page. It consists of two main sections of Input and Output. The maximum bending moments and other data were provided as input data and the codes generated the output data. The same data were used to run the codes developed for the static fatigue failure mode. The critical section of the bracket is at the fillet position where the bending moment is maximum. The maximum bending moments were evaluated to be 339,280.5Nmm for M_m and 282,130.5Nmm for M_a.

Table 2 summarizes the safety factor results for the design verification for Example 1. For the same design conditions, a more conservative model will give a smaller safety factor. From Table 2, the LGM was observed to be slightly more conservative than the Gerber model but less conservative than the modified Goodman model. This shows that the new model is an improvement on the modified Goodman criterion and, thus, will help to conserve material resources if used. The Gerber model gives a safety factor of 7.18 in static fatigue failure analysis, indicating that dynamic fatigue failure is actually more likely (lower safety factor value). The modified Goodman model shows no such discrimination. Certainly, the new model classification into dynamic fatigue and static fatigue failure regimes seems realistic.

Table 1. Calculations for Example 1

Example 2

DESIGN VERIFICATION: DYNAMIC FATIGUE FAILURE		
INPUT DATA		
Material Properties		
Tensile Strength (MPa), S_u	550	
Yield Strength (MPa), S_y	414	
Fatigue Strength (MPa), S_f	200.0	
Design Factors		
Overload factor	1.00	
Stress concentration factor, k_σ	1.160	
Bending Moments		
Alternating moment (Nmm), M_a	282448.0	
Mean moment (Nmm), M_m	338938.0	
Section modulus (mm^3), Z	5463.45	
OUTPUT DATA		
Service fatigue ratio: ψ_s		0.3636
Alternating stress (MPa), σ_a		59.9694
Mean stress (MPa), σ_m		62.0374
Load line transition factor, η_t		0.5455
Load line factor, η		0.9667
Linearized Gerber Model		
Model effective stress (MPa), σ_{ef}		71.249
Design safety factor, n_s		***2.81***
Modified Goodman Model		
Model effective stress (MPa), σ_{ef}		82.528
Design safety factor, n_s		***2.42***
Gerber Model		
Model effective stress (MPa), σ_{ef}		60.742
Design safety factor, n_s		***3.30***

The problem of Example 1 was analyzed with a fluctuating load on a bracket varying from a minimum of 3,114N to a maximum of 4,893N. Other factors in the problem remained unchanged.

Table 2. Models of Comparison for the Dynamic Fatigue Failure Design

Model	Dynamic Fatigue Failure		Static Fatigue Failure	
	Safety Factor	% Difference	Safety Factor	% Difference
Gerber	3.30	0	7.18	0
LGM	2.81	14.85	Not applicable	
Modified Goodman	2.42	26.67	2.43	66.16

Solution 2

As in the previous problem, Equations 7 and 15-21 were coded in Excel for analysis in static fatigue bending fatigue failure. This Excel page was similar to that of Table 1 and, thus, not shown here. The maximum bending moments and other input data were supplied to the codes for both failure modes. The critical section is at the fillet location as in Example 1. The maximum bending moments were determined to be 508,508Nmm for M_m and = 112,966.5Nmm for M_a.

Table 3. Models of Comparison for the Static Fatigue Failure Design

Model	Dynamic Fatigue Failure		Static Fatigue Failure	
	Safety Factor	% Difference	Safety Factor	% Difference
Gerber	8.1	0	5.50	0
LGM	Not applicable		4.02	27.6
Modified Goodman	3.46	57.28	3.47	37.8

Table 3 summarizes the results for design verification for Example 2. The minimum safety factor, n_o , to avoid yield at the first load cycle was 1.33. From Table 3, the LGM was again observed to be slightly more conservative than the Gerber model but less conservative than the modified Goodman model. The safety factor for the Gerber model in the static fatigue failure regime was smaller than that of the dynamic fatigue failure regime, indicating that static fatigue failure is more likely. The 27.6% deviation of LGM from the Gerber model can be explained by taking a closer look at Figure 1b. The maximum deviation of line BC from the Gerber parabola is expected about its midpoint. Since the transition load line angle is $\beta = 28.54^o$ and the load line angle is 14.56^o (about half of β) in this example, 27.6% deviation represents about the maximum error to be expected from the new model for this design condition. Again, the modified Goodman model showed no such difference in safety factor due to its single linear relationship. Therefore, the new model classification into dynamic fatigue and static fatigue failure regimes appear to be very realistic.

Example 3

Example 3 was a redesign of the brackets of Figure 4 such that b was half h , and where h and H maintain the same ratio for a minimum safety factor of 2.5. The material and other conditions remained the same as stated in Example 1.

Solution 3

Equations 20-22, 25 and 27 were coded in Excel for design sizing for the new model. The task in this problem was to determine the section modulus Z for the critical section which was at the fillet location in Figure 4. From the section modulus value, the dimensions of the cross-sectional shape can be determined once the shape type was chosen. The shape of the cross-section was rectangular, as shown on the right side of Figure 4. The shape factor was taken to be the ratio of section width to section height, which was 0.5 in this example. From Example 1, M_m = 339,280.5Nmm and M_a = 282,130.5Nmm. Now, n_s = 2.5 and k_σ was taken as 1.3 as a trial value.

Table 4 shows the layout of a portion of the Excel pages that made up the codes in Excel. The full layout consisted of three sections of Input, Processing and Output. The processing page is not shown in Table 4 and only a portion of the output section is shown. The parameters used in the developed equations are indicated. The bending moments evaluated previously along with other data were provided as input and the codes generated the output data. The section depth, h, was calculated to be 40.06mm but was chosen to be 40mm; the width was chosen to be 20mm. With these dimensions:

$$H = 1.125x40 = 45\ mm; \qquad \frac{H}{h} = \frac{45}{40} = 1.125$$

$$\frac{r}{h} = \frac{12.7}{40} = 0.3175$$

Based on the ratios of 1.125 and 0.3175, and from Figure 4.36 in the book by Norton [3], k'_σ was read to be 1.3 and k_σ was evaluated to be 1.2627. k'_σ and k_σ are related by the notch sensitivity factor. Using the same procedure as in Example 1, the design safety factor, n_s was evaluated to be 2.55 for the new model and 2.95 for the Gerber model. These values are relatively close to the desired value of 2.5. The deflection at the point of load application was calculated to be 0.151mm and 0.196mm at the end of the bracket. These values were much lower than the maximum allowable value of 0.51mm. The cross-sectional area of the old bar was 1290mm^2. The new bar had a cross-sectional area of 800mm^2. This is a 38% reduction in area and consequently a 38% reduction in weight or material cost at 36.4% of maximum allowable deflection. A 38% reduction in material cost could translate into thousands if not millions of dollars in savings in a large volume production!

Table 4. Calculations for Example 3

DESIGN SIZING: DYNAMIC FATIGUE FAILURE	
INPUT DATA	
Material Properties	
Tensile Strength (MPa), S_u	550
Yield Strength (MPa), S_y	414
Fatigue Strength (MPa), S_f	200.00
Factors	
Safety factor, n_s	2.50
Overload factor	1.00
Stress concentration Factor, k_σ	1.3000
Bending Moments	
Alternating moment (Nmm), M_a	282448
Mean moment (Nmm), M_m	338938
Section Shape	
Rectangular: shape factor	0.50
OUTPUT DATA	
Failure Type Assessment	
Load line transition factor, η_t	1.0500
Load line factor, η	0.5691
Failure Type (DFF* or SFF*)	DFF
Dynamic Fatigue Design	
Rectangle	
Chosen Dimensions	
Height (mm), h	40.00
Width (mm), b	20.00
Section modulus (mm^3), $Z = \frac{bh^2}{6}$	5333.33

*DFF = Dynamic Fatigue Failure
*SFF = Static Fatigue Failure.

Conclusion

A linearized Gerber model (LGM) was developed in order to simplify the use of the Gerber model in design sizing. The model divides the fatigue design diagram into two portions of dynamic fatigue failure and static fatigue failure regimes. As shown in Examples 1 and 2, this division correctly identifies the more likely mode of failure in design situations. The linearized model is less computationally intensive than the Gerber model and less conservative than the modified Goodman model. It defines a minimum safety factor for static fatigue failure design if yielding must be precluded.

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Appendix: Nomenclature

α = dynamic fatigue failure angle in fatigue
ψ = dynamic fatigue failure slope factor
$\psi_s = \frac{S_f}{S_u}$ = service fatigue ratio
$\psi_o = \frac{S_f^{'}}{S_u}$ = basic fatigue ratio
β = load line transition angle in fatigue
η = load line slope factor in fatigue
η_t = load line slope transition factor in fatigue
σ_A = Gerber alternating failure stress
σ_M = Gerber mean failure stress
σ_a = nominal normal alternating stress
σ_m = nominal normal mean stress
σ_{ef} = effective bending stress
$k_\sigma^{'}$ = theoretical bending stress concentration factor
k_a = service bending stress concentration factor
k_a = $q(k_\sigma^{'} - 1) + 1$
q = notch sensitivity factor
h = depth of rectangular cross-section
b = width of rectangular cross-section
M_a = alternating bending moment
M_m = mean bending moment
n_s = factor of safety
n_o = minimum factor of safety to avoid yield at first load cycle
$S_f^{'}$ = fatigue strength of polished laboratory specimen
S_f = service fatigue strength
S_Y = yield strength
S_u = ultimate tensile strength
Z = section modulus of member

Biography

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