

# ELECTRIC VEHICLE ENERGY USAGE MODELING AND MEASUREMENT

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## Abstract

Although electric vehicles show significant promise as a viable means of transportation [1], technical and societal challenges remain. One of the largest consumer concerns related to electric vehicles is their perceived lack of range. Although range estimates for particular vehicles are available, these estimates may be unreliable, especially if the vehicles will experience duty cycles that are dissimilar to standard test duty cycles. Speed, load and topography all play a significant role in the real range of an electric vehicle. Although vehicles produced by large manufacturers are well characterized by their manufacturer, highly detailed information is seldom shared. Vehicles from smaller manufacturers (such as utility vehicle manufacturers) may not be as well characterized. Fleet managers and other potential owners do not have a reliable way to estimate vehicle range along their routes prior to purchase and operation. This study explored an energy usage modeling approach that could be applied by fleet managers and other owners to help them make appropriate decisions regarding electric vehicle deployment. This paper describes an experiment in which road load calculations, vehicle efficiency and route data (route elevation and travel speed profiles along a specified route) were used to model the energy requirements of an electric utility vehicle. Road testing of an electric utility vehicle was performed along the specified route to validate model predictions. Results show that the total energy required along the route by the test vehicle was 0.50kWh, while the model prediction was 0.58kWh.

## Introduction

The primary goal of this effort was to accurately model energy usage along potential routes of travel in order to predict energy usage for a vehicle that is scheduled to travel a known route. Of particular interest were the energy requirements of electric vehicles since these vehicles are the subject of "range anxiety" and because of the current sparseness of recharging facilities. Although electric vehicles that come from large manufacturers are well characterized and provide range estimate updates to their operator, the underlying model that is used to provide this information is typically proprietary. Furthermore, vehicles such as the test vehicle that do not come from large manufacturers are less well

characterized so that comprehensive energy usage data is unavailable. One current approach to energy usage prediction is based on fixed duty cycles such as SAE J1711 [2] or Federal Urban Driving Schedules [3] which can be run on dynamometers. Results from such tests provide a way to compare different vehicles under identical conditions. While comparative data are valuable, the duty cycles applied during the tests may not be well correlated to the duty cycle that a particular vehicle will experience when it is deployed. The primary advantages of the approach described herein are that it accounts for vehicle road loads along the specific route that the vehicle will be deployed along, and it is based on vehicle parameters that are readily available.

The approach to energy usage modeling described here requires knowledge of driveline efficiency and the external and inertial forces that are aligned with the direction of travel. The forces aligned with the travel direction are used to predict the tractive force required; these forces depend on both route-specific and vehicle-specific parameters. The route parameters required by the model are vehicle speed and road gradient. In tests, vehicle speed and road gradient were determined using two different sources: Global Positioning System (GPS) tracking devices and the United States Geological Survey (USGS) database. The vehicle parameters required by the model were either directly measured or supplied by book values. For instance, vehicle weight was directly measured, while drag coefficient was based on a book value.

The approach was applied to model an electric utility vehicle on a route on the campus of James Madison University (JMU) in Harrisonburg, VA. Road testing along the chosen route was also performed in order to validate model results. The test vehicle was a Vantage [4] EVX1000 utility truck with a 72V x 192Ah lead acid battery pack, a Curtis controller and a High Performance Electric Vehicle Systems AC-50 three-phase electric motor. Although this system supports regenerative braking, regenerative braking was suppressed during tests to simplify modeling. During road tests, battery pack voltage and current were monitored to provide a running tally of energy usage.

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## Methodology

Prediction of energy usage requires a model of the vehicle and the route that the vehicle will travel. After the model was developed and a prediction obtained, the predicted energy usage was compared to the measured energy usage along the route. A measure of energy usage along the route was obtained by monitoring battery energy output while driving the route. This section describes the vehicle model, route model, energy usage prediction and energy usage measurement.

### Vehicle Model

The vehicle model used in this study was based on the external and inertial forces acting on the vehicle that are aligned with the direction of travel as described by Gillespie [5]. These external forces are in equilibrium with the tractive force applied by the wheels:

$$F_T = F_R + F_D + F_G + F_A \quad (1)$$

where,

- $F_T$  = tractive force
- $F_R$  = rolling resistance force
- $F_D$  = force due to drag
- $F_G$  = force due to gravity
- $F_A$  = force due to acceleration in the direction of travel

Rolling resistance addresses forces related to contact between tire and road surfaces, drag is due to wind resistance, the force of gravity accounts for the force required to change elevation, the acceleration force accounts for the force required to change speed. Equation 1 can be expanded to show the terms associated with each force:

$$F_T = W f_r + (1/2) \rho c_d A v^2 + W \sin \theta + (W/g) a \quad (2)$$

where,

- $W$  = weight of loaded vehicle
- $f_r$  = rolling resistance coefficient
- $\rho$  = air density
- $c_d$  = drag coefficient
- $A$  = frontal area
- $v$  = air speed
- $\theta$  = angle of road
- $g$  = acceleration due to gravity
- $a$  = acceleration in direction of travel

## Determination of Rolling Resistance Force

The force of rolling resistance is due to the deformation of tires rolling along the road surface. Rolling resistance is affected by factors such as tire pressure, pavement characteristics and tread conditions. During road testing, tire pressure was maintained at a consistent level while variation in road surface and tread conditions were noted to be negligible. The weight of the loaded vehicle was measured using a drive-on scale at a local lumber company; the measured weight was 14,055N (3,160lb). The rolling resistance coefficient was determined from coast-down tests. In a coast-down test, the vehicle is driven at a constant speed and then allowed to “coast down” to a stop [6]. Vehicle speed is recorded at known time intervals during the coast-down period so that deceleration can be calculated. If the road is flat and the speed is relatively low, gravitational force and drag force can be neglected and all of the deceleration can be attributed to rolling resistance. As a perfectly flat test surface was not available for this effort, coast-down tests were performed in two directions on a reasonably flat test surface in order to determine the rolling resistance coefficient. The results from each direction were used to provide a two-way average of deceleration. Using this approach, the rolling resistance coefficient was determined to be 0.014, which lies within typical values reported by Gillespie [5].

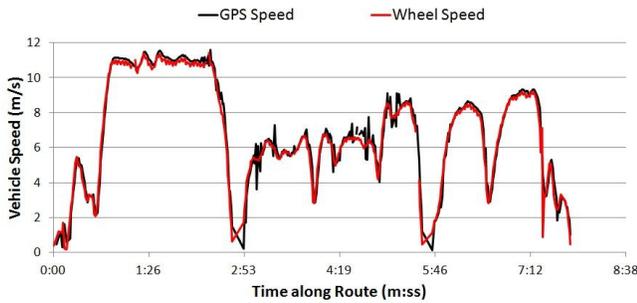
## Determination of Drag Force

The force due to drag accounts for wind resistance and is dependent upon the density of air, the shape of the vehicle, the frontal area of the vehicle and the air speed of the vehicle. Air density was determined based on temperature, pressure and humidity conditions as provided by a handheld weather station at the time of testing. Drag coefficients can be directly measured in wind tunnels or determined from high-speed coast-down tests. The vehicle speed was limited to ~11.5m/s (25MPH), which was determined to be too slow to get reliable coast-down data for drag force. A book value of 0.45 associated with similarly shaped pickup trucks [5] was used for modeling purposes. The frontal area was determined by direct measurement of the vehicle to be 2.16m<sup>2</sup>. Testing was done in calm conditions so that air speed could be well approximated by ground speed. Ground speed along the route was determined by a GPS tracker and verified by comparison to wheel speed, as shown in Figure 1.

The wheel speed shown in Figure 1 was determined by monitoring the motor speed signal from the controller while traveling the route. A combination of the motor speed with known driveline ratios and tire size results in wheel speed. Figure 1 shows that the GPS-tracker-based speed and the wheel speed compare well along the route.

**Table 1. Road-Load Variables, Values and Sources**

Label	Variable	Value	Unit	Source
W	weight of loaded vehicle	14055 (3160)	N (lb)	drive-on scale
$f_r$	rolling resistance coefficient	0.014	N/A	coast down test
$\rho$	air density	varies	kg/m <sup>3</sup>	equation based on parameters measured at time of test
$c_d$	drag coefficient	0.45	N/A	book value for similar vehicle
A	frontal area	2.16	m <sup>2</sup>	measurement of vehicle
v	air and ground speed	varies continuously	m/s	route model
$q$	angle of road	varies continuously	degrees	route model
g	acceleration due to gravity	9.81	m/s <sup>2</sup>	book value
a	acceleration	varies continuously	m/s <sup>2</sup>	route model



**Figure 1. Comparison of GPS-Speed Measurement to Wheel-Speed Measurement**

## Determination of Gravitational Force

The force due to gravity accounts for changes in elevation and is dependent upon vehicle weight and change in elevation. The weight of the loaded vehicle was measured as described above; the road angle was based on sequential GPS measurements of position and elevation.

## Determination of Force Due to Acceleration in the Direction of Travel

The force due to acceleration in the direction of travel accounts for changes in vehicle speed and is dependent on vehicle mass and change in travel speed. The weight of the loaded vehicle was measured as described above; changes in travel speed were based on sequential GPS-tracker-based speed measurements. Table 1 indicates the values of the variables in Equation (2).

Table 1 shows that a book value was relied upon for one variable (drag coefficient) while values for the other variables were measured. Some of the variables remained constant during testing while vehicle speed, road angle and acceleration varied continuously. The continuous variables were measured at a frequency of 1Hz which is the collection rate of the GPS trackers used in this study.

## Route Model

The route model consists of the elevation profile of the path of travel as well as an expected speed profile of the vehicle along the path. The elevation profile of the JMU route used in the energy calculations came from two sources: GPS trackers and the USGS database. The source of the speed profile was again the GPS trackers.

## Determination of Elevation Profile

For this study, GPS trackers and USGS data were the sources of elevation data. In order to determine the elevation profile of the path, four different methods were used which were then compared. The four methods were:

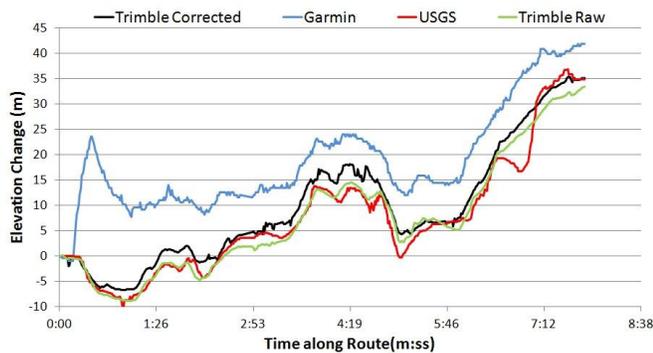
- Raw elevation data from Trimble Geo XH GPS tracker
- Raw elevation data from Garmin eTrex Vista GPS tracker
- Post-processed (differentially corrected) elevation data from the Trimble Geo XH GPS tracker
- Elevation data from USGS database

The GPS-tracker-based elevation data were collected by traveling the route with two active GPS trackers, which provided elevation data at 1Hz intervals. The GPS trackers used to conduct this study were a Trimble Geo XH and a Garmin eTrex Vista. The Trimble Geo XH is survey grade, while the Garmin eTrex Vista is recreational grade. The Garmin eTrex Vista produces lower accuracy position data but has a built-in barometric pressure sensor which allows relative elevation readings to be taken without relying on satellites.

The Trimble GPS data were also post-processed using differential correction to provide a third set of elevation data. Differential correction methods remove anomalies due to issues in the interaction between satellites and GPS receivers (typically due to atmospheric conditions) based on readings from nearby GPS receivers that are in a fixed location [7]. Because the Trimble tracker is survey grade, differentially corrected route data from the Trimble tracker is considered to be the most reliable source of route model data used in this study.

The fourth set of elevation data was sourced from a USGS database. The USGS has image-based digital elevation models (DEMs) published online that have a resolution of 10 meters. This means that in an image, every 10-meter by 10-meter pixel is assigned an elevation value [8]. For the case described here, the route was entered into ArcMap (geospatial analysis software) and the elevation data for points along the route were extracted from the appropriate DEM.

Elevation data from each of the four methods were processed to provide elevation relative to a start point. Figure 2 shows the relative elevation profiles resulting from the application of the four methods discussed above.



**Figure 2. Elevation Profiles from Four Methods**

Figure 2 shows that the Trimble raw, Trimble corrected and USGS elevation data are similar in value and shape. The Garmin elevation data display a ~25m jump near the

beginning of the run, but then follow a similar shape to the others. One potential problem with relying on USGS data is observed as a dip near the 7:00 minute mark. The USGS DEM has 10-meter resolution which is not small enough to resolve a bridge that was part of the route. In this case, the USGS DEM provided elevation data for the ravine that the bridge crosses.

## Determination of Speed Profile

The speed profile of the route was determined by traveling the route at typical speeds with an active GPS tracker. The GPS trackers provided position data at 1Hz intervals, vehicle speed was calculated from the difference in position between sequential position points. The speed profile for the route is shown in Figure 1. The speed profile used in this study is based on differentially corrected route data from the Trimble tracker.

## Energy Usage Prediction

The goal of this effort was to predict the amount of energy that must be supplied by the battery pack. The amount of energy required from the battery pack is a function of the amount of energy required at the wheel and vehicle drivetrain efficiency. Energy requirements at the wheel are based on tractive force predictions while drivetrain efficiency is based on RPM-specific values published by the motor manufacturer.

Predictions of energy usage at the wheel are based on the tractive force model as calculated in Equation (2). Since some of the tractive force parameters (speed, road angle and acceleration) change with each successive time interval, the energy calculation addresses a single interval. In order to use tractive force to determine energy usage for a given interval, the distance across which the tractive force is applied during the interval must be known, as shown in Equation (3):

$$\Delta E_{wheel,i} = F_{T,i} \Delta d_i \quad (3)$$

where,

$$\begin{aligned} \Delta E_{wheel,i} &= \text{energy usage at the wheel to travel} \\ &\quad \text{the interval distance } \Delta d_i \\ F_{T,i} &= \text{tractive force during interval } i \\ \Delta d_i &= \text{distance traveled during interval } i \end{aligned}$$

Equation (3) can be rewritten in terms of speed, as shown in Equation (4):

$$\Delta E_{wheel,i} = F_{T,i} v_i \Delta t_i \quad (4)$$

where,

$$\begin{aligned} \Delta E_{wheel,i} &= \text{energy usage at the wheel to travel for the} \\ &\quad \text{interval time } \Delta t_i \\ F_{T,i} &= \text{tractive force during interval } i \\ v_i &= \text{average speed along path during time} \\ &\quad \text{interval } i \\ \Delta t_i &= \text{length of time interval } i \end{aligned}$$

Equation (5) is an expansion of Equation (4) and shows the tractive force terms from Equation (2). In Equation (5), terms that are updated for each time interval are given the interval subscript  $i$ .

$$\Delta E_{wheel,i} = [W f_r + (1/2) \rho c_d A v_i^2 + W \sin(\theta_i) + (W/g)(v_i - v_{i-1})/\Delta t_i] v_i \Delta t_i \quad (5)$$

The amount of energy required from the battery pack depends on the energy required at the wheel as well as vehicle drivetrain efficiency. In order to determine the energy required from the battery pack, an efficiency factor based on published data from the motor manufacturer that correlates efficiency to motor RPM at peak load was applied. Motor RPM was determined from vehicle speed and known drive ratios; motor RPM is continuously variable so that drivetrain efficiency must be updated for each time interval, as shown in Equation (6):

$$\Delta E_{battery,i} \times \eta_i = \Delta E_{wheel,i} \quad (6)$$

where,

$$\begin{aligned} \Delta E_{battery,i} &= \text{energy provided from battery pack during} \\ &\quad \text{interval } i \\ \eta &= \text{drive efficiency during interval } i \\ \Delta E_{wheel,i} &= \text{energy required at wheel during interval } i \end{aligned}$$

This method results in a prediction of the incremental energy provided by the battery pack for each time interval; the total energy provided by the battery pack along a particular route is predicted by the summation of the  $\Delta E_{battery,i}$  values from all of the intervals.

## Road Tests

In order to validate the energy usage predictions of the model, vehicle energy usage was monitored (measured at the battery pack) while traveling along the route. This was done by logging battery pack voltage and current at 1Hz time intervals along the route. The amount of energy removed from the battery during each time interval is the product of the battery pack voltage, current and time interval, as shown in Equation 7.

$$E_{measured,i} = V_{measured,i} I_{measured,i} \Delta t_i \quad (7)$$

where,

$$\begin{aligned} E_{measured,i} &= \text{energy removed from battery pack} \\ &\quad \text{during interval } i \\ V_{measured,i} &= \text{battery pack voltage during interval } i \\ I_{measured,i} &= \text{battery pack current during interval } i \\ \Delta t_i &= \text{length of time of interval } i \end{aligned}$$

This approach results in a measurement of the incremental energy provided by the battery pack for each time interval; the total energy provided by the battery pack along a particular route is the summation of the  $\Delta E_{measured,i}$  values from all of the intervals.

## Results

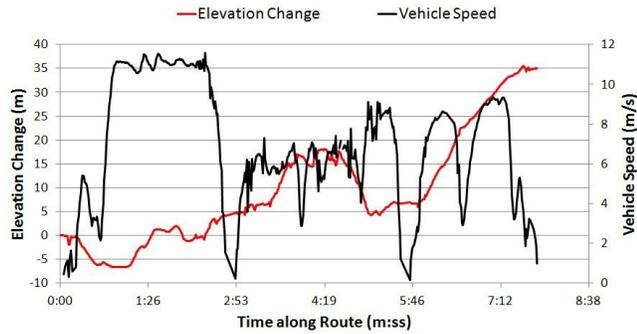
The results of this effort include the raw speed and elevation data used to construct the route model, battery energy usage predictions based on tractive force prediction, and measured energy usage from road testing.

Figure 3 shows a typical set of elevation and speed profile data for a particular route along with the corresponding energy usage prediction (based on differentially corrected Trimble tracker elevation data). Energy usage is graphed as cumulative energy usage so that the final point on the graph represents the total energy prediction for the route. The dependency of energy usage predictions on route elevation and speed profiles can be seen in Figure 3. For instance, note that near the 2:50 minute mark, the vehicle speed is near zero and there is little elevation change which suggests that little energy input is required. The cumulative energy graph is nearly flat at this point, which indicates little predicted energy usage at that time. On the other hand, note that near the 7:00 minute mark, the road increases in elevation while speed is maintained at a constant ~9m/s (20MPH), which suggests that significant energy input is required. The cumulative energy graph has a steep slope at this time which indicates high predicted energy usage.

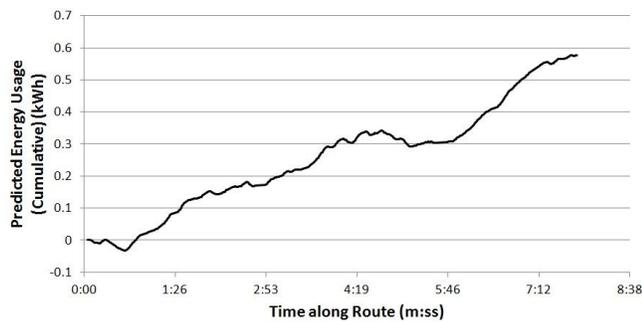
Recall that four different methods were used to determine elevation in the route model. Each method results in a different route elevation profile, as shown in Figure 2. The elevation profile variation affects energy usage predictions, as shown in Figure 4.

The energy values in Figure 4 are graphed as cumulative energy usage so that the final points on the graph represent the total energy predictions for the route. Figure 4 also includes the measured energy from road testing for comparison. It can be seen that the lines representing the four energy usage predictions are similar in shape and value to the line that represents measured energy usage. The prediction

with the worst agreement is the one that relies on the Garmin GPS tracker for elevation profile. All predictions over-predicted the energy usage required for the entire route, although not at all points along the route. Table 2 summarizes the predictions of energy usage along the route from the four methods, as well as their variation from the measured energy usage.

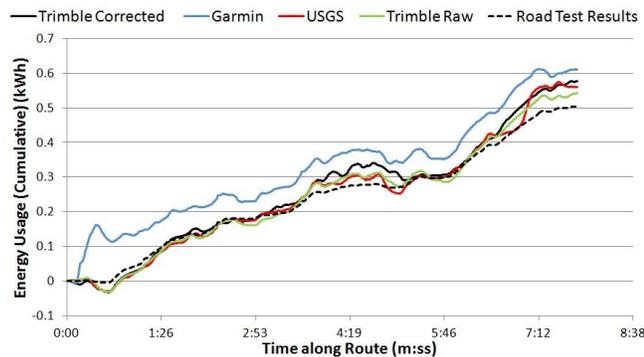


(a) Elevation and Speed Profiles of Route



(b) Prediction of Energy Usage along Route

**Figure 3. Energy Usage Prediction along Route and the Underlying Elevation and Speed Profiles**



**Figure 4. Energy Usage Prediction and Road Test Results**

**Table 2. Total Energy to Travel Route**

Data Source	Total Energy to Travel Route (kWh)	% Variation from Road Test
Trimble (corrected)	0.577	+ 14.8
Garmin	0.611	+ 21.4
USGS	0.560	+ 11.3
Trimble (raw)	0.542	+ 7.9
Road Test	0.503	-

Table 2 shows that variation between energy usage measured during the road test and energy usage predictions varies from 8% to 22%. The major contributor to variation is the source of the elevation model used to predict energy usage. The Garmin tracker, which reported an elevation profile dissimilar to the Trimble tracker and the USGS model, is the source of the elevation model that varies by nearly 22% from the road test values. Although the raw Trimble tracker data show the lowest variation (8%) relative to road test data, the differentially corrected output from the Trimble tracker (Trimble corrected) is likely a better source of elevation data.

Figure 4 demonstrates that the model over predicts the measured energy usage. At the end of the route, the total energy usage predicted by the most reliable model (Trimble corrected) was 0.58kWh in comparison to a measurement of 0.50kWh; this represents a variation of ~15%.

## Discussion

The selected route included ~20m total descent and ~55m total ascent, which is representative of typical routes traveled on the campus of JMU. It can be seen from Figure 4 that energy usage predictions and measured energy usage exhibit similar form although differences do exist. If the variation between predicted and measured energy usage is characterized as ~14% (the average variation), different conclusions could reasonably be drawn. For some purposes, 14% variation may be considered acceptable in which case an avenue for future work would be to further test and validate the current approach on other routes and with other vehicles to gain confidence that the approach is universally reliable. For other purposes, 14% variation may be considered unacceptable in which case further refinement of the model would be in order.

Efforts directed at model refinement could begin with a comparison of Figures 4 and 3a which suggest that while in

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general the energy usage models tend to over predict energy usage, this is especially noticeable from 3:45 - 4:40 minutes (minimal elevation change combined with varied speeds) and from 6:20 - 7:47 minutes (extended deceleration while making an ascent). While such observations provide some insight, it would be ill-advised to make adjustments to the model in order to better match the measured energy usage without understanding the interactions between vehicle model parameters and route model parameters. This suggests that an avenue for further work would be to continue to work with the test vehicle under controlled conditions of slope, speed and payload in order to scrutinize the model.

The main strength of using a GPS-tracker approach to developing a route model is that elevation measurements along the route are made directly. This means that the model does not depend on DEMs from the USGS which may not resolve critical route features. The main weakness of using the GPS-tracker approach is that the route must be driven while collecting tracker data. For relatively long routes, the effect of route features that are not properly resolved by DEMs may not be significant, in which case predictions of total energy usage should be reasonably close to actual energy requirements. This suggests that another avenue for further work would be development and validation of USGS-based route models with the intent of developing a reliable prediction of energy usage that does not require physical presence along the route.

## Conclusions

An energy usage model that accounts for the route of an electric vehicle was applied to a small, electric-powered utility vehicle traveling a route with significant elevation variation. A model of tractive force as well as elevation and speed profiles of the route provided energy usage predictions along the route. The predictions were validated by a road test in which the route was traveled while energy usage from the battery pack was monitored. At the end of the route, the total predicted energy usage varied from the measured energy usage by ~15%.

Paths for future work include continued testing of the current model along other routes and with other vehicles, refinement of the model, and further development of USGS-based route models.

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