COMPARISON OF TIME DELAY CONTROLLERS FOR A CLASS OF NETWORKED CONTROL SYSTEMS

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Abstract

In a networked control system (NCS), sensors, controllers and actuators are connected to the communication network as nodes instead of hardwiring them with point-to-point connections. The communication network may introduce time delays while exchanging data among devices connected to the shared network medium. These delays can degrade system performance. Traditional controllers that do not consider time delays in the design may not perform adequately when applied to the NCS. The traditional linear quadratic regulator (LQR) design with a delayed state control was shown to degrade the performance for a rotating base pendulum system with control over a wireless network. Based on this current study, it is recommended to use an LQR design that takes a class of delays into account, does not degrade performance, and illustrates the improved results for the rotating-base pendulum system with MATLAB simulations. This study also evaluated the robust stability properties of these controllers against parameter variations.

Introduction

Of late, the use of digital communication networks—such as AS-i, Devicenet, Ethernet, Foundation Fieldbus and Profibus that are commonly referred as fieldbuses—and wireless networks are becoming popular in the implementation of process control systems [1]. As shown in Figure 1, these networks allow sensors, controllers and actuators to be connected to the network as nodes instead of hardwiring the devices with point-to-point connections. Some of these devices, e.g., sensor 2 and actuator 2 in Figure 1, also use wireless communication. These networks provide several advantages such as reduced system wiring and improved flexibility and interoperability, among others. However, the communication network may induce time delays while exchanging data among devices connected to the shared network medium. These time delays can degrade system performance and even affect the stability of the control system [2].

Recently, theoretical results have been reported in the literature addressing these time delays and other aspects of networked control systems [3-16]. Gupta & Chow [11], Tipsuwan & Chow [6] and Yang [12] surveyed recent articles in the NCS area. A special issue on NCS was edited by Antsaklis & Baillieul [13]. Some researchers analyzed the effect of time delays on traditional controllers such as LQR [10], while others introduced new control designs to take the delays into account [8], [16]. Some of the analytical results gave bounds on the allowable delays that would preserve stability of the networked control systems [3-5]. Liu & Goldsmith [7] studied the effects of wireless network medium access control protocols on the performance of the networked control systems. Robust H_∞ control design for NCS was presented by Yue et al. [14]. Li et al. [15] investigated the delays associated with the use of Profibus-PA networks within control loops.

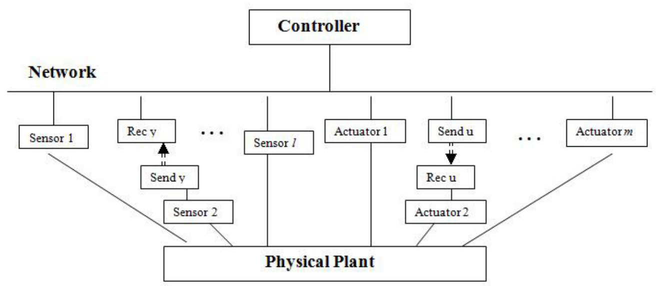

Figure 1. A Typical Networked Control System

This study compared LQR controller designs for a class of networked control systems with time delays. Presented here is a review of the traditional LQR design, followed by a look at the effect of a delayed state on this controller, as used in a wireless implementation by Ploplys et al. [10]. An alternate LQR design that takes delays into account is suggested for NCS. The performance of these two controllers is illustrated using a MATLAB simulation of a rotating-base pendulum system. Finally, the robust stability properties of these controllers are compared with the parameter variations. These robustness results are applied to the rotating-base pendulum system.

Time Delay Controllers for Networked Control Systems

Consider the discrete-time system

$$x(k+1) = Ax(k) + Bu(k) \qquad (1)$$

where x is the n-dimensional state vector, A is the nxn time-invariant asymptotically stable matrix, u is the m-dimensional control vector, and B is the nxm constant matrix. It is of interest when designing controllers that the overall system be stable and meet certain performance measures.

Assuming that the system pair (A, B) is controllable, a state feedback controller to stabilize the system can be designed. This controller can be obtained in several ways [17]. Using the LQR design, the controller is given by

$$u(k) = G\,x(k) \tag{2}$$

where

$$G = -(R + BTKB)-1BTKA \tag{3}$$

and K is the positive definite solution of the algebraic discrete-time Riccati equation

$$K = Q + ATKA - ATKB(R + BTKB)-1BTKA \tag{4}$$

The matrices Q and R are weighting matrices in the performance index

$$J = \sum_{k=0}^{\infty}[x^T(k)Qx(k) + u^T(k)Ru(k)] \tag{5}$$

which can be used as a design parameter to get different stabilizing controllers. It is well known that the resulting closed-loop system

$$x(k+1) = (A + BG)\,x(k) \tag{6}$$

is stable with eigenvalues located inside the unit circle [17].

Controller 1: LQR Design using Delayed State Vector as the Input

The controller of Equation (2) assumes that there are no delays in control implementation. However, in networked control systems, this assumption may not be valid. Recently, Ploplys et al. [10] used the following modified controller with unit delay, suitable for control over wireless networks:

$$u(k) = G\,x(k-1) \tag{7}$$

The controller gain, G, in Equation (7) is the same as in Equation (3), except that the state vector in Equation (2) is delayed. The closed-loop system of Equations (1) and (7) is given by

$$\begin{bmatrix} x(k+1) \\ x(k) \end{bmatrix} = \begin{bmatrix} A & BG \\ I & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ x(k-1) \end{bmatrix} \tag{8}$$

In general, the stability and performance of Equation (8) cannot be guaranteed with a controller, as given by Equation (7) [18]. For a rotating-base pendulum system, Ploplys et al. [10] showed that the dynamics of the closed-loop system—see Equation (8)—are adversely affected by the feedback delay with additional eigenvalues, though the stability is maintained, as illustrated in the following section.

Controller 2: LQR Design using Delayed State and Control Vectors as the Inputs

The following alternative implementation of the controller with unit delay is suggested for control over wireless networks [18]:

$$u(k) = GAx(k-1) + GBu(k-1) \tag{9}$$

The controller gain, G, in Equation (9) is the same as in Equation (3), except that the state vector in Equation (2) is determined from previous state and control vectors. The closed-loop system of Equations (1) and (9) is given by

$$\begin{bmatrix} x(k+1) \\ u(k+1) \end{bmatrix} = \begin{bmatrix} A & B \\ GA & GB \end{bmatrix} \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \tag{10}$$

Mita [18] performed a detailed analysis of this controller and showed that the closed-loop system matrix of Equation (10) has eigenvalues of $(A+BG)$ plus m zero eigenvalues. The stability of the closed-loop system is therefore maintained with the unit delay controller given in Equation (9). The performance index of Equation (5) is modified for this unit delay case [18]. This unit delay controller may perform better than the controller suggested by Ploplys et al. [10] for control over wireless networks, as will be illustrated in the next section.

If the time delay in the networked control system is more than one sample, the controller of Equation (9) can be extended to L sample delays given by

$$u(k) = GA^L\,x(k-L) + GA^{L-1}Bu(k-L) + \ldots + GBu(k-1) \tag{11}$$

The closed-loop system of (1) and (11) has eigenvalues of ($A+BG$) plus mL zero eigenvalues. The stability of the closed-loop system is also maintained with the L-step delay controller Equation (11). The performance index of Equation (5) is modified for this L-step delay case as well [18].

Application to a Rotating-Base Pendulum System

Referring again to Equation (1), consider the discretized linear model of the rotating-base pendulum system—shown in Figure 2 and described by Ploplys et al. [10]—with the following matrices:

$$A = \begin{bmatrix} 1.0008 & 0.005 & 0 & 0 \\ 0.3164 & 1.0008 & 0 & 0 \\ -0.0004 & 0 & 1 & 0.005 \\ -0.1666 & -0.0004 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} -0.0065 \\ -2.6043 \\ 0.0101 \\ 4.0210 \end{bmatrix}$$

Here, the control input, u, is the torque applied to the rotating base to move the horizontal arm through a yaw angle of ϕ. A pendulum mounted at the end of the horizontal arm freely swings about a pitch angle θ. For the state vector:

$$x = [\theta\ \dot{\theta}\ \phi\ \dot{\phi}]^T$$

Swing up and stabilization are two modes of control used to balance this system. Swing up can be obtained using different methods such as energy control [10]. The stabilization mode of the control for this system can be achieved using several methods such as LQR [10]. The effects of time delay Controllers 1 and 2, due to the delay introduced by wireless communication implementation of controllers and LQR, are illustrated in this section for the stabilization mode control with MATLAB simulation.

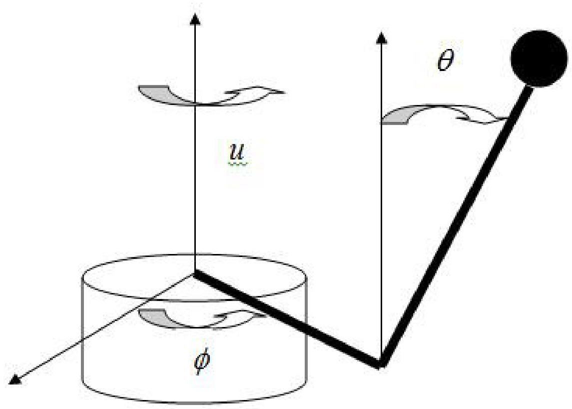

Figure 2. Structure of the Rotating Base Pendulum

For this system, Ploplys et al. [10] gave an LQR controller design of Equation (2) with

$$G = [2.094\ 0.378\ 0.117\ 0.092]$$

The eigenvalues of the closed-loop system without delays are 0.414, 0.987 ± i0.0119, 0.9866. As expected, the closed-loop system was stable with all eigenvalues located inside the unit circle, although an unstable eigenvalue at 1.04 was present in the open-loop system. Figure 3 shows the initial condition response of state x_2 for the closed-loop system of Equation (6).

Ploplys et al. [10] gave the eigenvalues of the closed-loop system for Equation (8) that used Controller 1 given in Equation (7) as 0.5205 ± i0.5961, 0.987 ± i0.0119, 0.9866, 0.0, 0.0, 0.0. They further stated that the complex pair 0.5205 ± i0.5961 contributes to slower and more oscillatory state convergence when the feedback delay of Equation (7) is included and compared with the control Equation (2) that has no feedback delay. Figure 4 shows the initial condition response of state x_2 for the closed-loop system of Equation (8) which exhibits more oscillations than state x_2 for the closed-loop system of Equation (6).

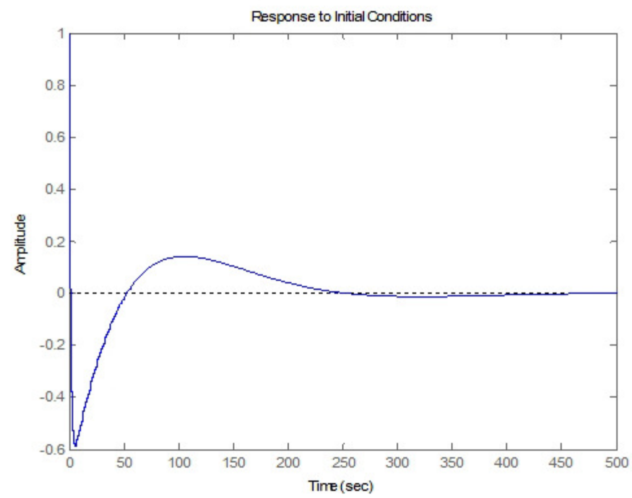

Figure 3. Initial Condition Response of State x_2 for the LQR Closed-Loop System (6) [Time scale should be multiplied by sampling time 5ms]

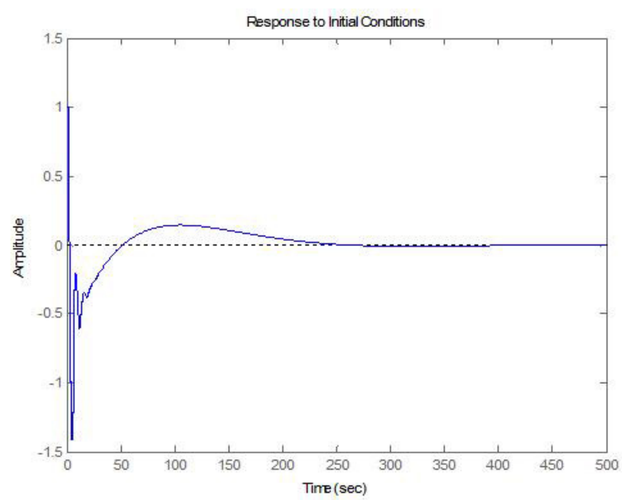

Figure 4. Initial Condition Response of State x_2 for Controller 1 Closed-Loop System (8) [Time scale should be multiplied by sampling time 5ms]

The eigenvalues of the closed-loop system of Equation (10) that uses Controller 2 given by Equation (9) are 0.414, 0.987 ± i0.0119, 0.9866, 0.0. This controller does not change the eigenvalues of the delayless controller and adds extra zero eigenvalues, thus giving better performance than Controller 1. Figure 5 shows the initial condition response of state x_2 for the closed-loop system of Equation (10). This response looks similar to the non-delay controller of Equation (2) and exhibits fewer oscillations than Controller 1 of Equation (7).

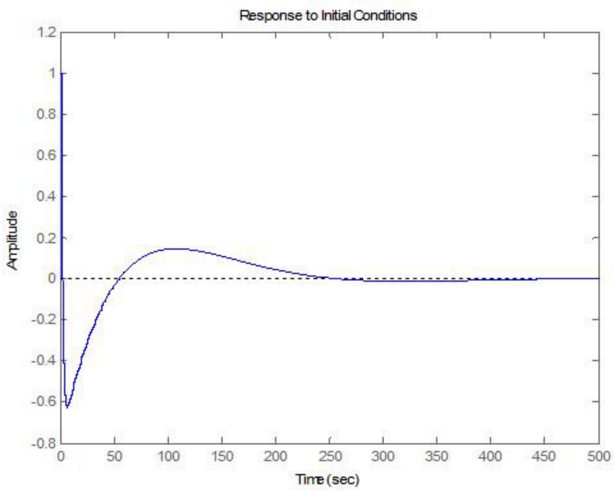

Figure 5. Initial Condition Response of State x_2 for Controller 2 Closed-Loop System (10) [Time scale should be multiplied by sampling-time 5ms]

Stability Robustness of Time-Delay Controllers for NCS

In this section, the stability robustness properties of the NCS in the presence of parameter variations in the system matrices are presented. Consider the linear discrete-time system described by

$$x(k+1) = [A_c + E]x(k) \qquad (12)$$

where x is the n-dimensional state vector, A_c is an nxn time-invariant asymptotically stable matrix, and E is a perturbation matrix. Define constants εij and ε such that

$$eij\,(k) \le max\,|eij(k)| = \varepsilon ij \text{ and } \varepsilon = max\,\varepsilon ij \qquad (13)$$

Let $U = [uij]$, $uij = \varepsilon ij/\varepsilon$. Note that $0 \le uij \le 1$, with $uij = 0$ if the perturbation eij of $acij$ is known to be zero. The following bound μ on ε of the E matrix that maintains the stability of the A_c matrix was presented by Kolla et al. [19]:

$$\varepsilon < \mu = -\left[\frac{\sigma_{\max}(U^T|PA_c|)_s}{\sigma_{\max}(U^T|P|U)}\right] + \sqrt{\left[\frac{\sigma_{\max}(U^T|PA_c|)_s}{\sigma_{\max}(U^T|P|U)}\right]^2 + \left[\frac{\sigma_{\min}(\overline{Q})}{\sigma_{\max}(U^T|P|U)}\right]} \qquad (14)$$

where $P > 0$ is the solution of the discrete-time Lyapunov equation $A_c^T P A_c - P + \overline{Q} = 0$ for any given symmetric $\overline{Q} > 0$, and $\sigma(.)$ is the singular value of (.). The notation |(.)| represents the matrix whose elements are the magnitudes of the elements of (.), and $(.)_s$ represents the symmetric part of (.). This bound can be used to study the robust stability properties in the presence of parameter variations for the closed-loop NCS with delays by taking A_c as any of the closed-loop matrices given in Equations (6), (8) or (10). These results are applied to the rotating-base pendulum system and illustrated with the following MATLAB simulation.

Application to a Rotating-Base Pendulum System

Consider the rotating-base pendulum system [10] discussed in the previous section. For this system, as described earlier, the LQR controller design of Equation (2) gives G = [2.094 0.378 0.117 0.092]. As expected, the closed-loop system is stable with all eigenvalues located inside the unit circle. For the closed-loop system of Equation (6), the stability robustness bound of Equation (14) with

$$U = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \text{ and } \overline{Q} = \mathrm{I} \text{ gives } \mu = 2.1409x10^{-6}.$$

This means that the closed-loop system remains stable as long as variation in each element of A_c is less than μ.

Controller 1, given in Equation (7) provides the closed-loop system matrix in Equation (8) for NCS. For this closed -loop system, the stability robustness bound of Equation (14) with an 8x8 U matrix with all ones, and $\overline{Q}$ = I, gives $\mu = 7.3749x10^{-7}$. This means that the closed-loop system remains stable as long as variation in each element of A_c is less than μ. The closed-loop system with Controller 1 still has some robustness properties, though the allowable parameter variations are less than the controller without time delay due to the lower value for μ.

Controller 2, given by Equation (9) provides the closed-loop system matrix in Equation (10) for NCS. For this closed-loop system, the stability robustness bound of Equation (14) with a 5x5 U matrix with all ones, and $\overline{Q}$ = I, gives $\mu = 3.8557x10^{-6}$. As before, this means that the closed-loop system remains stable as long as variation in each element of A_c is less than μ. It can be observed that the closed-loop system with Controller 2 given in Equation (10) showed better robustness properties against parameter variations than the closed-loop system with Controller 1 given in Equation (8) due to the higher value for μ.

Conclusion

In this paper, a comparison of the time-delay controller designs for a class of NCS were presented. One of these controllers (Controller 1) uses the traditional LQR state-feedback gain matrix with a unit delay state vector [10]. The performance of Controller 1 for NCS may not be satisfactory based on the eigenvalue analysis of the closed-loop system and the initial condition response, illustrated using a rotating-base pendulum system with a MATLAB simulation. It is suggested that a different LQR controller [18] with time delays (Controller 2) be used for NCS and obtain better performance based on eigenvalue analysis of the closed-loop system and initial condition response, illustrated using the rotating-base pendulum system. The stability robustness properties of both these time-delay controllers for NCS against parameter variations were studied. These controllers' robustness properties were illustrated using the rotating -base pendulum system. For this system, Controller 2 showed better robustness properties against parameter variations than Controller 1. In addition to the type of time delays considered in this paper, other issues such as information packet loss during communication in NCS and random time delays are critical, and their effect on performance needs further investigation [11-13].

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Biography

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