MULTI-PRODUCT CAPACITATED LOCATION ROUTING INVENTORY PROBLEM

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Abstract

The efficient design/redesign of the logistics network is essential for companies to maintain their competitive advantage. This paper presents a novel non-linear mixed-integer programming model for the multi-product capacitated location routing inventory problem (MPCLRIP), where a two-layer distribution system consisting of depots and customers was studied. The supply decisions such as location of depots, allocation of customers to the depots, finding routes to serve customers, inventory levels at depots, and product order intervals can be simultaneously and optimally made. A heuristic solution tackles the proposed combinatorial optimization problem, where a set of generated problems were solved in a reasonable computational time.

Introduction

A supply network deals with the distribution of product(s) from a source to the final customers. Smart design/redesign of the supply network is a key to cutting enterprise costs in today's competitive market. As Melo et al. [1] indicated, some typical supply chain decisions include capacity, inventory, procurement, production, routing and the choice of transportation modes. Knowing that such decisions can be classified into strategic, tactical and operational levels, a common decision process is to follow a sequential decision approach from strategic level to operational level when designing/redesigning the logistics network [2]. Due to supply chain decision interactions, such a sequential approach, however, may result in sub-optimal networks [3], [4]. Therefore, there has been a tendency by researchers and practitioners to combine two or more supply decisions. Presented by Laporte et al. [5], capacitated location routing problem (CLRP) is a classical logistics problem which takes location and routing problems into account simultaneously. When using a CLRP model, the decision maker considers three simultaneous decisions: location of facility, allocation of customers to facilities and determining routes to serve the customers. Nagy and Salhi [6] considered some real-world applications of LRP and its variant which include postal delivery, newspaper delivery, blood distribution, food distribution, school bus pick-up and delivery, waste disposal transportation and mobile healthcare. While the LRP models have been well studied, the integration of other supply decisions such as inventory and packaging problems have been dropped from the literature [6]. Moreover, the multi-commodity problem, which represents a real-life aspect of facility location and supply chain networks [1], was dropped from most LRP studies as well. Integrating inventory into the LRP model results in more complexity as more decisions are incorporated into the model. In fact, most of the integrated location, routing and inventory studies concentrate on dual combinations of location, routing and inventory including location-routing, location-inventory and routing-inventory, ignoring all of the three decisions simultaneously. As such, a body of research is also available on integrated routing-inventory and location-inventory models.

A review of the inventory-routing (IR) literature was done by Kleywegt et al. [7], [8], who reviewed the inventory-routing (IR) literature in terms of demand, vehicle, horizon, delivery and inventory characteristics. They then formulated IR as the Markov decision process and provided approximation methods for it. Their study indicates that when the inventory and routing problems are integrated, the total cost of the system is reduced. Moin and Salhi [9] classified the IR models based on their planning horizon: single period, multiple period and infinite. They also categorized the IR solution approaches into two classes: the theoretical method, where a derivation of the lower bound is determined; and, the heuristic method, where a near-optimal solution is sought. Shen et al. [10] proposed a joint inventory-location model for a network consisting of one supplier and a set of retailers to minimize the location-inventory-shipment costs in a blood-products distribution system. The study, in fact, improves the current network configuration from a decentralized system, where each retailer has a blood warehouse, to a centralized inventory system, where some retailers are identified as distribution center (DC)/depot and products (with variable demand) are shipped from there to their assigned retailers. It was assumed that the inventory policy at DCs/depots follows an approximation of the (Q,r) system, where Q is the order quantity and r is the reorder level.

In this paper, the authors present a mixed-integer programming model for a two-layer multi-product capacitated location routing inventory problem (MPCLRIP), where the classical capacitated LRP (CLRP) model is integrated with the inventory system that runs under fixed-order interval policy for multiple products at the depot level. Since CLRP is a combinatorial optimization problem [11], an efficient heuristic solution methodology is presented to tackle the proposed model.

Literature Review

Shen [12] presented a survey on integrated supply chain models. The study proposed various integrated model formulations for location, allocation, transportation and inventory decisions. It also considered routing cost approximation when a subset of customers is assigned to the same route. The combination of location, routing and inventory literature can be classified into two categories: direct delivery and multiple delivery. Direct delivery studies assume that products are directly transported from depots to customers. In other words, each individual customer's requirement is equal to or greater than one truck load. Erlebacher and Meller [13] developed an analytical model for a stylized version of a three-layer supply chain system with capacitated plants. They considered non-linear inventory costs for the proposed continuous review policy. Moreover, the rectilinear distance is assumed from plants to DCs/depots and from there to customers.

Daskin et al. [14] formulated a non-linear integer program for a three-layer supply chain system (facility, DC/depot and retailer). They converted the model to the Lagrangian relaxation sub-problem and proposed a number of heuristics to solve it. The study applied the (Q,r) model with type I service as the inventory policy and approximated Q* using the EOQ model. The type I service assumes that the service level in the (Q,r) model is approximately equal to the probability of demand during lead time being less than the reorder point (r). Ghezavati et al. [15] studied a three-tiered distribution network that considers service level constraints and coverage radius assumptions, where demand at each depot follows the Poisson distribution. In this model, products are shipped from a single supplier to multiple potential DCs/depots and then from there to the customers. The service level constraint assumption takes the safety stock and demand costs into consideration simultaneously. The coverage radius assumption considers that each DC/depot can serve a subset of customers based on their distances from that DC/depot. Representing the solution in a matrix structure, the authors considered a genetic algorithm (GA) as the solution methodology.

In contrast to direct delivery, the multi-delivery system assumes that customer demand is less than truck load (LTL), allowing multiple customer visits on a route. An initial version of one such model that considered variable warehousing costs in the objective function of the LRP model was the work of Perl and Daskin [11], where the variable warehousing costs were involved in the objective function. Sajjadi and Cheraghi [16] proposed a multi-product LRIP model that considered a periodical inventory policy (T, I_{max}), assuming that the demand follows a stochastic distribution. The proposed model was then solved by a two-phase simulated annealing (SA) approach. Liu and Lee [17] integrated the inventory problem running under the (Q,r) policy with LRP. Liu and Lin [18] proposed a heuristic solution based on the SA and tabu search (TS) approaches to deal with the model that had been previously proposed by Liu and Lee [17]. Ambrosino and Scuttela [19] proposed four-layer static and dynamic LRP models in which there were two types of customers, while warehousing costs were considered in the objective function.

Forouzanfar and Tavakkoli-Moghaddam [20] presented a non-linear mixed-integer model for integrating location, routing and inventory decisions in a supply network where demand is uncertain. A genetic algorithm was proposed to solve various problem sizes. Klibi et al. [21] proposed the stochastic multi-period location-transportation problem (SMLTP). This study modifies three LRP assumptions in order to adopt more realistic situations by considering other transportation modes in the model such as single-customer partial truckloads, full truckloads and multi-drop truckloads. The model assumed that orders are received at depots on a daily basis, which must be fulfilled the next day. The order intervals and quantity of demand follow a probabilistic distribution.

In contrast to many transportation models that minimize the logistics costs, the proposed objective function maximizes profit as it subtracts the depot opening cost as well as inventory holding, customer shipment and production/procurement costs from sales revenue. A heuristic nested method that hierarchically combines location-allocation and transportation problems was proposed as the solution methodology. Javid and Azad [22] proposed a mixed-integer model for location-routing-inventory decisions in a supply network design. Their model considers single-product distribution, while the DCs/depots run under a continuous review system. Furthermore, the study assumed that customer demand would follow a normal distribution, the safety stock would be available to prevent depot stock out, and the DC/depot opening cost would depend on the capacity level of the DC/depot. The proposed two-phase heuristic approach considers a randomly generated constructive algorithm in the first phase, which is improved by TS and SA procedures. The improvement phase is broken down into location and routing sub-problems, where multiple procedures are proposed to improve location and routing phases sequentially. Yang et al. [23] modeled the just-in-time (JIT) delivery strategy using a two-stage integrated location-routing-inventory problem assuming exponential distribution for customer demand. The study determined the best order interval inventory policy at depots, while it took a penalty cost into account in the objective function for late deliveries. A numerical example was solved by the proposed particle swarm optimization (PSO) approach as the solution. Results indicated that the JIT strategy reduced network costs. Other related works were conducted by Xuefeng [24], Zhang et al. [25] and Liu et al. [26].

Model Description

The following notations are used in this paper.

I	=	$\{i; i=1, 2,\ldots, n\}$: Set of customers (second layer)
J	=	$\{j; j= n+1, n+2,\ldots, n+m\}$: Set of depots (first layer)
V	=	$\{v; v=1,\ldots, V\}$: Set of vehicles
P	=	$\{p; p=1,\ldots, P\}$: Set of products
H	=	$I \cup J$ $\{h; h=1, 2\ldots, n+m\}$: Set of nodes on the network including depots and customers
L_{gh}	:	Distance traveled from point (node) g to point (node) h, assumed symmetric
A	:	Ordering cost for the family of products (major setup cost)
a_p	:	Ordering cost for product p (minor setup cost)
v_p	:	Unit variable cost of product p
r	:	Annual inventory cost rate
H_p	:	Holding cost of product p per unit per year ($H_p=r^* v_p$)
D_i	:	Annual demand of customer i
D_p	:	Annual demand for product p
D_{ip}	:	Annual demand of customer i for product p
D_{pj}	:	Annual demand of product p at depot j
F_j	:	Fixed depot opening cost of depot j
K_j	:	Capacity space of depot j. Products can share the space based on a standard unit volume.
K_v	:	Capacity of vehicle v. Products can share the space based on a standard unit volume.
T	:	Basic order interval time
t.	:	Service order interval of the network per year
m_p	:	Order interval multiplier of product p
α_p	:	Volume/Size unit coefficient

Inventory Model

The inventory model proposed in this study was adopted from the model presented by Silver et al. [27]. The model represented the order interval inventory system where one orders multiple items for the warehouse in a specific time interval, T. Referring to Silver et al. [27], the annual inventory cost (IC) and optimal order interval (T*) can be obtained from Equations (1) and (2), respectively.

$$IC(T, m_p) = \frac{A + \sum_{p\in P}(a_p/m_p)}{T} + \sum_{p\in P}\frac{D_p m_p T H_p}{2} \quad (1)$$

$$T^* = \sqrt{\frac{2(A + \sum_{p\in P}(a_p/m_p))}{\sum_{p\in P} m_p H_p D_p}} \quad (2)$$

The total annual inventory cost of the system can be defined as the sum of the ordering cost and holding cost, respectively. Equation (1) indicates that there are two types of ordering costs, the ordering cost for all products together (family), known as major ordering cost (A), and the ordering costs considered for each individual product, known as minor ordering cost (a_p). The demand is known and fixed; no shortage and safety stock are allowed. Such a model is applicable when inventory cannot be continuously monitored and/or items from same supplier may yield savings in ordering, packing and shipping costs.

Moreover, when demand levels for multiple items are highly variable (e.g., low-demand product versus high-demand product), it may not be economical to replenish all items at all order times. Instead, at every time interval T, only those items that are in high demand are ordered. In fact, the time interval, T, is defined as the basic (family) order interval, and a product p can be ordered at a positive integer coefficient of interval T, namely product order interval, based on its annual demand size. As such, while a high-demand product is ordered every T units of time, a low-demand product may be ordered every mT (2T, 3T or more). The coefficient m was considered as a variable in the model. In a network such as the LRP, where a subset of customers is assigned to a depot j, Equation (2) can be rewritten as follows:

$$T_j^* = \sqrt{\frac{2(A + \sum_{p\in P}(a_p/m_p))}{\sum_{i\in I}\sum_{p\in P} m_p H_p D_{ip}}} \quad (3)$$

Equation (3) calculates the optimal basic order interval time for a multiple-product multiple-customer inventory system.

Mathematical Model

The problem under consideration was a two-layer distribution system consisting of depots at the first layer and customers at the second layer. The basic assumptions of CLRP are held by the proposed model: at each interval, a set of homogenous vehicle(s) holding limited capacity begin their tour from selected depot(s), serves a subset of customers in full and returns to the same depot(s). Each customer is served through only one vehicle, where the vehicle itself is assigned to only one depot. While the capacities of the depots and vehicles are limited, there is no individual limit on each product capacity, meaning that the products can share depot and vehicle spaces.

Depots hold inventory, and the demand for products is deterministic and the inventory policy follows the time-interval strategy explained in the previous section. It was assumed that there is no inventory cost at the customer level. The annual cost of the network includes the fixed-depot opening cost, routing cost and inventory cost. The network has a fixed and known service interval that means the network is served *t* times per year. However, this is different from the scheduling problem in which the time of the service is identified. Therefore, each route may have a different dispatching schedule during a year; however, the total annual number of dispatches is the same for all routes (1/t).

The following decision variables were used in the model:

Z_{ghv}	:	Routing decision: 1 if there is an immediate connection from point *g* to point *h* on route *v*, otherwise 0
X_j	:	Location decision: 1 if depot *j* is open, otherwise 0
Y_{ij}	:	Allocation decision: 1 if customer *i* is assigned to depot *j*, otherwise 0
T_j	:	Basic order interval at depot *j*
m_{pj}	:	Order interval multiplier of product *p* at depot *j*

The mathematical MPCLRIP model formulated as a non-linear mixed-integer programming model is presented as follows:

$$\text{Min} \sum_{j\in J} F_j X_j + \frac{1}{t}\sum_{\substack{h\in H\\ g\#h}}\sum_{\substack{g\in H\\ g\#h}}\sum_{v\in V} L_{gh} Z_{ghv} + \sum_{j\in J}\frac{(A+\sum_{p\in P} a_p/m_{pj})}{T_j}X_j + \frac{1}{2}\sum_{j\in J}\sum_{i\in I}\sum_{p\in P} D_{ip} m_{pj} T_j H_p Y_{ij} \quad (4)$$

subject to:

$$\sum_{v\in V}\sum_{h\in H} Z_{ihv} = 1 \quad i\in I \quad (5)$$

$$\sum_{g\in H, g\#h} Z_{hgv} - \sum_{g\in H, g\#h} Z_{ghv} = 0 \quad v\in V, \boldsymbol{h}\in \boldsymbol{H} \quad (6)$$

$$\sum_{i\in I}\sum_{j\in J} Z_{jiv} \le 1 \quad v\in V \quad (7)$$

$$\sum_{g\in R}\sum_{h\in \bar{R}}\sum_{v\in V} Z_{ghv} \ge 1 \;\forall (R,\bar{R}), R\subset H, J\subseteq R \quad (8)$$

$$-Y_{ij} + \sum_{h\in H}\left(Z_{ihv} + Z_{jhv}\right) \quad v\in V, i\in \boldsymbol{I}, \boldsymbol{j}\in \boldsymbol{J} \quad (9)$$

$$\sum_{i\in I}\sum_{p\in P}\frac{\alpha_p D_{ip}}{t}\sum_{h\in H} Z_{ihv} \le K_v \quad \boldsymbol{v}\in \boldsymbol{V} \quad (10)$$

$$\sum_{j\in J}\sum_{i\in I}\sum_{p\in P}\alpha_p D_{ip} m_p T_j Y_{ij} - \boldsymbol{K_j} X_j \le \boldsymbol{0} \quad \boldsymbol{j}\in \boldsymbol{J} \quad (11)$$

$$Z_{ghv} = 0,1 \quad g = 1,\ldots, n+m;\; h = 1,\ldots, n+m;\; v = 1,\ldots, v \quad (12)$$

$$X_j = 0,1 \quad j = n+1,\ldots, n+m \quad (13)$$

$$Y_{ij} = 0,1 \quad i = 1,\ldots, n,\; j = n+1,\ldots, n+m \quad (14)$$

$$T_j \ge 0 \quad j = n+1,\ldots, n+m \quad (15)$$

$$m_p \in \text{natural number} \quad (16)$$

The objective function, Equation (4), can be explained as follows: The first term indicates the fixed opening cost of depots. The second term is the annual routing cost where the coefficient 1/t indicates the number of services per year for the network. The rest of the terms show the inventory cost: the order cost and holding cost, respectively. As explained, it was assumed that there is a base-order interval time (T_j) for a depot and then each product can be ordered every other $m_{pj} * T_j$ time, meaning that for a product with a lower demand, it is not required to order every T_j time. The constraints can be detailed as follows. Equation (5) indicates that each customer is on only one route. Equation (6) shows that every vehicle that enters a node should exit from that node. Equation (7) indicates that any vehicle on the network can depart a depot only once. Equation (8) notes the subtour elimination. Equation (9) shows that if a customer is assigned to a depot, there should be a route from that depot passing through the customer. Equations (10) and (11) are capacity constraints on the vehicle and depot, respectively.

Equations (12)–(15) illustrate the integrity constraints; Equations (12)–(14) are binary assumptions, while Equation (15) ensures non-negative values for the order interval decisions and Equation (16) restricts the value of m_ps to natural numbers.

Solution Methodology

The heuristic solution algorithm consists of two procedures which are performed sequentially: location-allocation-routing procedure and inventory procedure. The location-allocation-routing procedure is a modified version of the procedure developed by Sajjadi et al. [28]. The structure of this procedure is shown in Figure 1. The algorithm is composed of two components: initial solution generation and solution improvement. The improvement itself is divided into three subsections: location-allocation, allocation-routing and routing improvements.

Step one generates the initial solution; it determines which depot(s) should be open and allocates customers to the opened depots. The second step deals with the location-allocation structure; it improves the initial solution by changing the open depots and allocating the customers to the selected depots. The third step constructs tours from open depots to customers. The fourth step attempts to reduce the routing costs by changing the customer allocation; if this step changes the allocation of customers, it also constructs another set of tours from open depots to customers. The fifth step selects the best solution generated in the third and fourth steps. Finally, the sixth step improves the position of customers among the routes which are assigned to a depot.

While the model by Sajjadi et al. [28] deals with a single-product LRP, the model presented in this paper considers a multi-product LRP and so the customer may demand several types of products with different sizes and volumes. The α_p coefficient converts each product volume to a standard volume. To update the remaining capacity of depots and remaining capacity of vehicles in the above procedure, a standard unit volume (e.g., one cubic foot) must first be selected and then the total volume of each customer's demand is calculated using α_p. The depot capacity and vehicle capacity are also expressed based on the standard unit.

Using the solution from the location-allocation-routing procedure (Steps 1-6), the inventory decisions are obtained by the inventory procedure. The procedure consists of four additional steps adopted from Silver et al. [27] to find the order intervals for each product. The four steps are as follows.

Step 7. For each open depot, based on the customers allocated to the depot, calculate the total demand for each product, D_{pj}.

Step 8. Set m_{pj} =1 for the product that has the minimum:

$$\frac{a_p}{D_{pj} v_p}$$

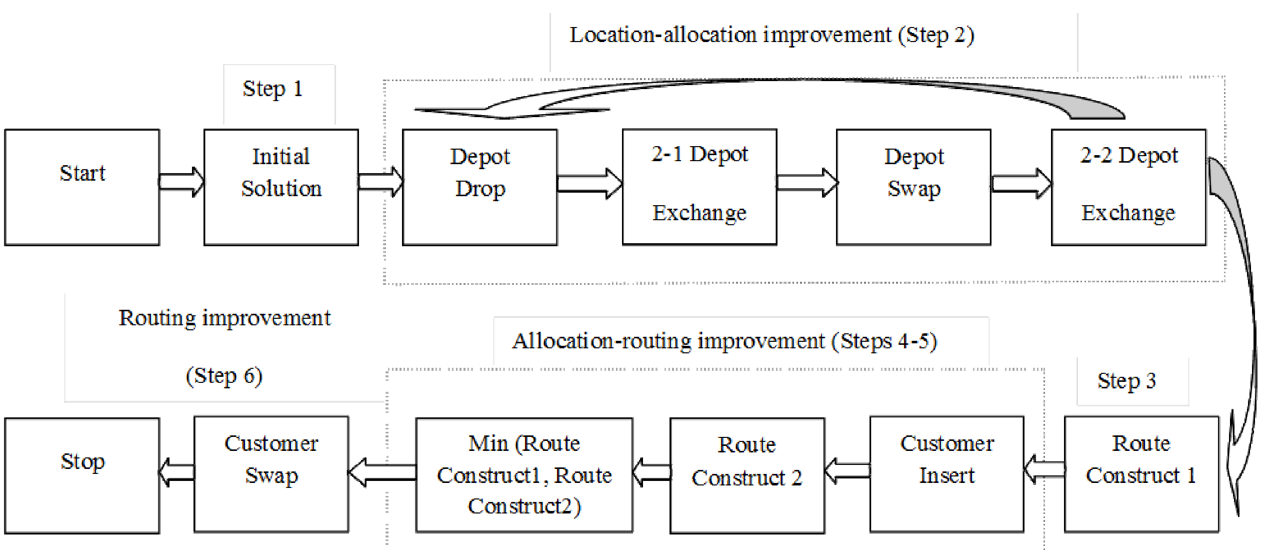

Figure 1. The structure of location-allocation-routing procedure [14]

For other products calculate m_{pj} based on Equation (17) and round it to the nearest integer greater than zero.

$$m_{pj} = \sqrt{\frac{a_p}{D_{pj} v_p} * \frac{D_{1j} v_1}{A + a_1}} \quad (17)$$

Step 9. Calculate the basic order interval by Equation (18):

$$T_j = \sqrt{\frac{2\left(A + \sum_p {}^{a_p}/_{m_{pj}}\right)}{r \sum_p m_{pj} D_{pj} v_p}} \quad (18)$$

Repeat Steps 7–9 for all open depots.

Step 10. The annual inventory cost for each depot (IC_j) is calculated from Equation (19). The total inventory cost will be the sum of the inventory costs of all open depots.

$$IC_j = \frac{A + \sum_{p \in P} {}^{a_p}/_{m_{pj}}}{T_j} + \sum_{p \in P} \frac{D_{pj} m_{pj} T_j v_p r}{2} \quad (19)$$

Numerical Example and Computational Analysis

The above heuristic algorithm was coded in MATLAB R2012b on an Intel(R) Core(TM) i7-3770S CPU @ 3.10 GHz machine with 8.00 GB of RAM. A numerical example consisting of 100 customers, 10 depots and 5 products was solved using the proposed algorithm. The location and capacity of the depots and vehicles were obtained from the dataset by Prins et al. [29].

As the data set takes single products into consideration, the problem was broken down in order to represent multiple -product problems (five products). The input inventory assumptions were assumed as follows: 1) The major setup cost for the family of products, A, is \$1,000; 2) The annual inventory cost of one dollar of inventory, *r*, is \$0.25; and, 3) The minor setup cost, a_p, and the unit variable cost, v_p, for each product are shown in Table 1. Moreover, the volume/ size unit coefficient, α_p, which converts different product sizes to a standard unit volume, is shown in Table 1. As proposed, the algorithm consists of 10 steps of which Steps 1-6 solve the location-routing problem and Steps 7-10 take the inventory decision into account. Figure 2 plots the location-routing solution, indicating how the initial LRP solution would be improved from Step 1 to Step 6. The total demand for each product at each depot, D_{pj}, is shown in Table 2. Tables 3-6 indicate the inventory solution for the problem (output), which is the result of Steps 7-10 of the proposed heuristic algorithm.

Table 1. Minor Setup Cost and Unit Variable Cost (dollars)

Product	a_p(\$)	v_p(\$)	α_p
Product 1	200	4000	10
Product 2	240	500	6.67
Product 3	280	2500	2
Product 4	320	500	0.67
Product 5	360	3500	0.36

Table 2. Demand at Open Depots

Product	Depot 3	Depot 5	Depot 8	Depot 9
Product 1	6	9	9	8
Product 2	8	14	14	12
Product 3	28	46	46	42
Product 4	83	137	138	125
Product 5	152	252	253	229

Table 3. Basic Order Interval

Opened Depot	T_j , in years	T_j , in days
Depot 3	0.1439	53
Depot 5	0.1121	41
Depot 8	0.1119	41
Depot 9	0.1177	43

Table 4. Order Interval Multipliers

Product	m_{pj}
Product 1	2
Product 2	5
Product 3	1
Product 4	2
Product 5	1

Table 5. Order Interval Time and Inventory Cost

Product\Depot	Depot 3	Depot 5	Depot 8	Depot 9
Product 1	106	82	82	86
Product 2	265	205	205	215
Product 3	53	41	41	43
Product 4	106	82	82	86
Product 5	53	41	41	43
Inventory Cost (\$), ICj	27,082	34,767	34,830	33,109

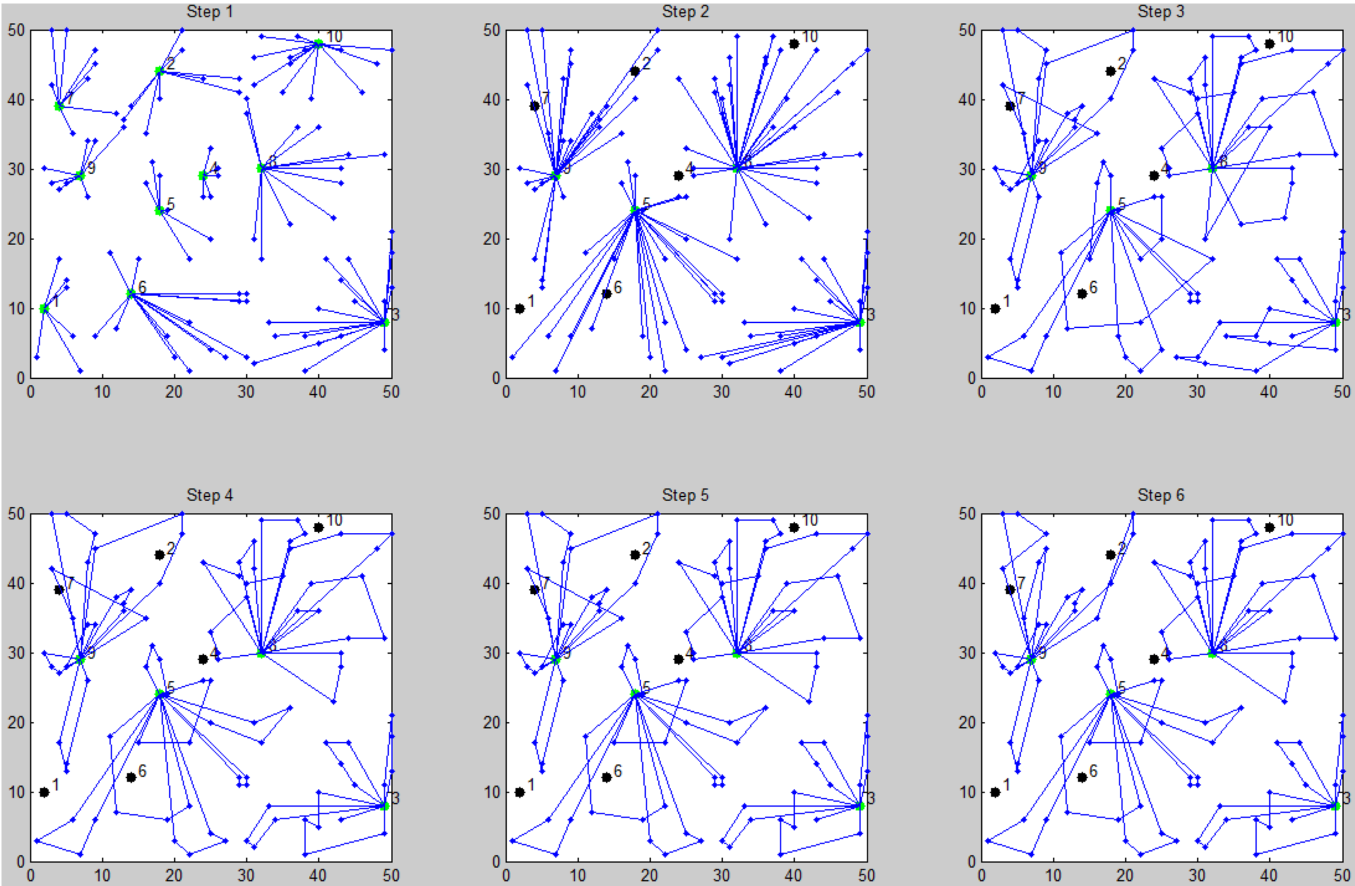

Figure 2. Location-Routing Solution for 100-10-1 Benchmark Problem

Table 6. Solution Results

Benchmark Problem	T_j , in days, at opened depots				Cost ($)		
					Location-Routing	Inventory	Total
20-5-1	74	112	82	-	55,908	49,201	105,109
20-5-2	111	80	77	-	49,403	48,944	98,347
50-5-1	56	57	53	-	92,484	77,462	169,946
50-5-2	58	61	47	-	92,501	77,948	170,449
100-5-1	56	33	35	-	281,820	109,485	391,305
100-5-2	32	31	-	-	199,159	90,697	289,856
100-10-1	53	41	41	43	323,577	129,788	453,365
100-10-2	39	40	37	-	249,332	110,272	359,604
200-10-1	25	32	26	-	485,749	155,837	641,586
200-10-2	29	25	28	-	456,304	156,383	612,687

The proposed algorithm determined the opening of depots 3, 5, 8 and 9 (Table 3). The basic order intervals, T_j, are presented in the table as well. The order interval multipliers of products at open depots, m_{pj}, are shown in Table 4. Also, the order interval times for each product at each depot, $m_{pj}*T_j$, and the inventory costs, IC_j, are presented in Table 5.

In addition to the numerical example, using the developed algorithm, two sets of problems were solved. First, in order to verify the accuracy of the algorithm, a small-size problem consisting of two depots, two products and three customers was solved and then the results were compared to the optimal solution obtained from the enumeration method. It was observed that the proposed algorithm is capable of achieving the optimal solution for the problem under study. Second, a set of problems with different sizes was solved. To form the problems, the inventory data (same as the numerical example) were added to a set of LRP benchmark problems obtained from the dataset by Prins et al. [29]. From each problem size in the dataset, two problems were solved. Similar to the numerical example, the number of product types was set to five.

Compared to the work by Silver et al. [27] and Sajjadi et al. [28], the proposed algorithm generates quality solutions for Steps 1-6 and Steps 7-8, respectively (see Table 6). Note also that the multipliers, m_{pj}, for each product at all depots in all problem sizes are the same (see Table 4). The reason is that the multiplier value is greatly affected by the inventory data, which were the same for all of the problems considered in this study. In each problem, the algorithm balanced the depots and tried to use as much capacity as possible from the open depots. Since the demands for each product at the open depots were not very far from each other, the algorithm computed the same multipliers at the open depots. However, as expected, T_j was more sensitive to demand levels than m_{pj}. That is why the T_j values, shown in Table 6, were different for each depot and in each problem. In fact, as demand increased, T_j decreased. For example, consider the T_j values for the 20-5-1 problem. The solution indicated that there should be three open depots in the network with T_j of 74, 112, and 82 days. Product 1, in open depot 1, should be ordered every 2*74 days, while product 2 should be ordered every 5*74 days, etc.

In order to show the capability of the proposed algorithm, the effect of using different order intervals for different products on the inventory, cost was considered in Table 7. If each depot wants to order all products at the same order interval (m_{pj}=1), as the classical order interval policy assumes, the inventory cost increases. It is indicated in Table 7 that for the benchmark problems, the inventory cost increases by almost 5%. This shows that the proposed algorithm is effective in decreasing the cost. Since the proposed model is novel, there is no exact benchmark problem that can be compared to this computational time. However, the computational time for the benchmark problems in this study were compared with the time obtained from the LRGTS algorithm proposed by Prins et al. [30]. Figure 3 shows the plots of the computational time for the considered problems. It was shown that the CPU time for the proposed algorithm was much less than that of the LRGTS.

Table 7. Classical versus Proposed Order Intervals Inventory Cost Effect

Benchmark Problem	Inventory Cost with *Different* Order Intervals	Inventory Cost with *One* Order Interval	Increase in Inventory Cost
20-5-1	49,201	51,662	5.00%
20-5-2	48,944	51,322	4.86%
50-5-1	77,462	81,217	4.85%
50-5-2	77,948	81,732	4.85%
100-5-1	109,485	114,837	4.89%
100-5-2	90,697	95,131	4.89%
100-10-1	129,788	136,158	4.91%
100-10-2	110,272	115,628	4.86%
200-10-1	155,837	163,427	4.87%
200-10-2	156,383	164,053	4.90%

Considering that the proposed algorithm in this study takes both the inventory decision and multiple product system into consideration, while the LRGTS only considers a CLRP model, it can be concluded that the solutions are generated in a reasonable amount of time.

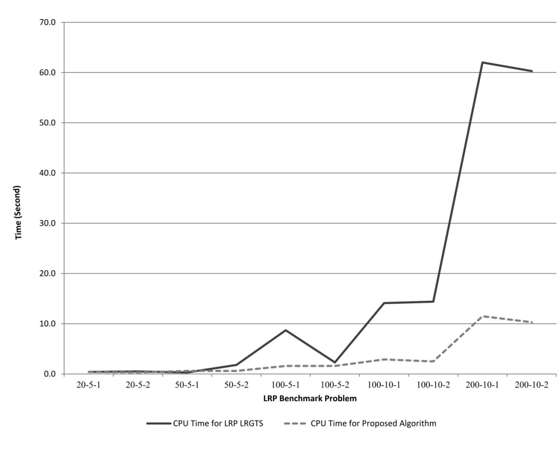

Figure 3. CPU-Time Comparison

Conclusion

A novel mathematical model for multiple-product capacitated location routing problems that takes into account inventory decisions was proposed in this study. The inventory procedure takes the time interval policy into account where the basic time interval is determined for each open depot, and computed multipliers indicate the time interval order for each product. Such a model is especially appropriate when demands for different products vary greatly, making a wide customer request range. A heuristic solution that combines a modified version of the work by Silver et al. [25] and Sajjadi et al. [26] was proposed as the solution approach. Benchmark problems obtained from the LRP literature were combined with inventory parameters set by the authors and then solved by the proposed algorithm. A numerical example was also solved to show how the proposed algorithm tackles the problem. Future research is needed to consider demand uncertainty in the model. Furthermore, heuristic solution approaches that consider inventory decision and location routing problems simultaneously rather than sequentially should also be considered in future studies.

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